Using Dipole and Quadrupole Perturbations in a Linear Paul Trap to Study Magnetic Lattice Errors

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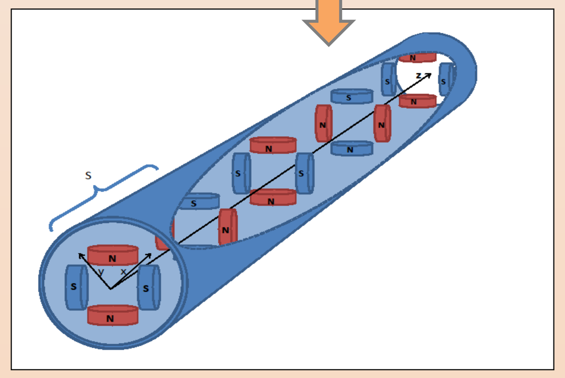
Abstract

The Paul Trap Simulator Experiment (PTSX) simulates the collective dynamics of a charged particle bunch traveling through a long magnetic alternating-gradient particle accelerator. For example, PTSX is used to study the effect of random lattice errors caused by quadrupole magnets that are misaligned or vary in magnetic field strength. A pair of arbitrary function generators was used to apply a trapping voltage waveform for many lattice periods with a range of frequencies and with either a dipole or quadrupole spatial structure. Every 12th or 13th period, the amplitude was changed to create a perturbed full or half waveform to stimulate the bunch. The duration of the perturbation was also varied to simulate different numbers of revolutions in the ring. The experimental results demonstrate the growth in the equivalent beam emittance that occurs due to the perturbation amplitude and duration by measuring the on-axis charge of the bunch. The data does not demonstrate a strictly monotonic decrease in charge but rather a periodic relationship that depends on the perturbation amplitude and duration. In the experiments with dipole and quadrupole perturbations the data revealed a power-law relationship between these parameters. To begin to explain this behavior, models based on the individual particle motion and the envelope equation were studied.

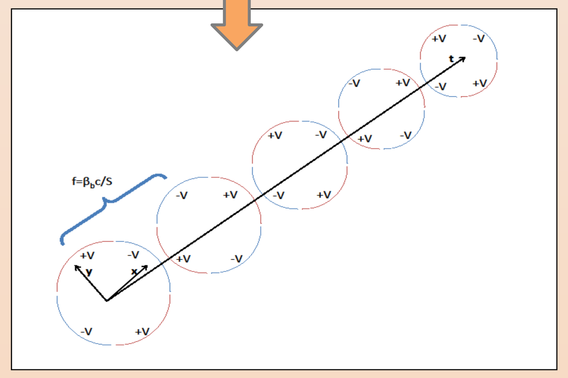
Introduction

The Paul Trap Analogy

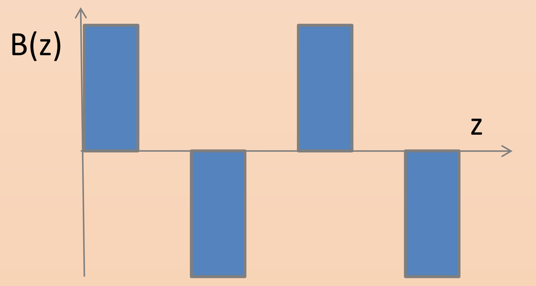
Lorentz Transform

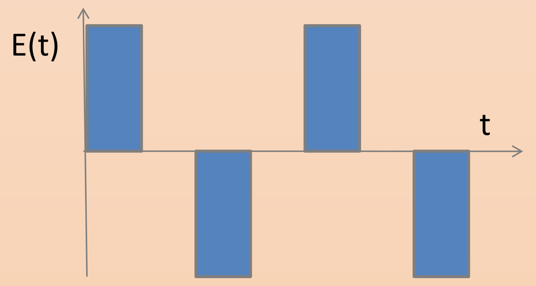


Alternating gradient transport lattice (Focusing-Off-Defocusing-Off or FODO), S=1 m
6000 lattice periods = 6 km



Cylindrical Paul Trap, f=60 kHz
6000 lattice periods=100ms





The transverse equations of motion for an ion are the same in both systems – they are equivalent under a Lorentz transform

Transverse Equations of Beam Transport

Under the assumption that the plasma has uniform charge density, coupled equations describe the transverse dynamics of the beam in terms of the minor and major radii of the "edge," or envelope of the beam where $k_q(s)$ describes the lattice, K_b is the perveance and is proportional to the line charge, and ϵ is the emittance. In the case where K_b or ϵ become too large the solutions to the equations will diverge.

$$\frac{d^2}{ds^2}a(s) + k_q(s)a(s) - \frac{2K_b}{a(s)+b(s)} = \frac{\epsilon_x^2}{a(s)^2}$$

$$\frac{d^2}{ds^2}b(s) - k_q(s)b(s) - \frac{2K_b}{a(s)+b(s)} = \frac{\epsilon_y^2}{b(s)^2}$$

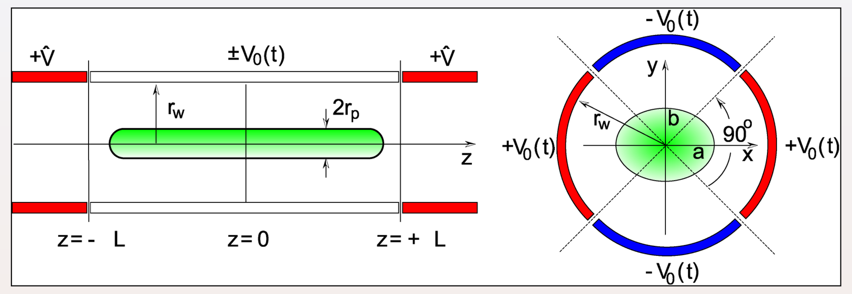
Assuming that the diffuse beam plasma behaves as an ideal gas (p=nkT), we can write the force balance expression below. This equation allows the emmitance, a standard measure of beam quality, to be calculated

Force Balance $$m\omega_q^2 R_b^2 = 2k_b T_\perp + \frac{N_b q^2}{4\pi\varepsilon_o}$$

Emittance $$\varepsilon(t) = 2R_b\left(\omega_q^2 R_b^2 - \frac{N_b q^2}{4\pi\varepsilon_o m}\right)^{1/2}$$

Paul Trap Simulator Experiment

Apparatus Specifications



Plasma length	2 m	Maximum wall voltage	~ 400 V
Wall radius	10 cm	End electrode voltage	< 150 V
Plasma radius	~ 1 cm	Frequency	< 100 kHz
Cesium ion mass	133 amu	Pressure	$5x10^{-9}$ Torr
Ion source grid voltages	< 10 V		

Experiment Concept

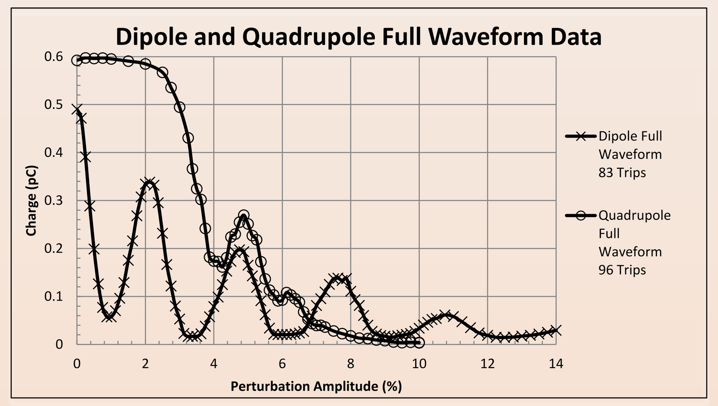
By using a pair of arbitrary function generators, it is possible to create either a dipole or quadrupole spatial structure. The function generators are controlled by a Labview program making it possible to adjust many parameters such as the type of waveform, the voltage of the waveform, the voltage multiplier/frequency, the time the perturbation is applied, and how many periods between the perturbations. Each of these parameters affects the beam and by measuring the charge of the beam at the end of its run cycle it is possible to determine if a mode of the beam is being excited and how destructive the perturbation is to the beam.

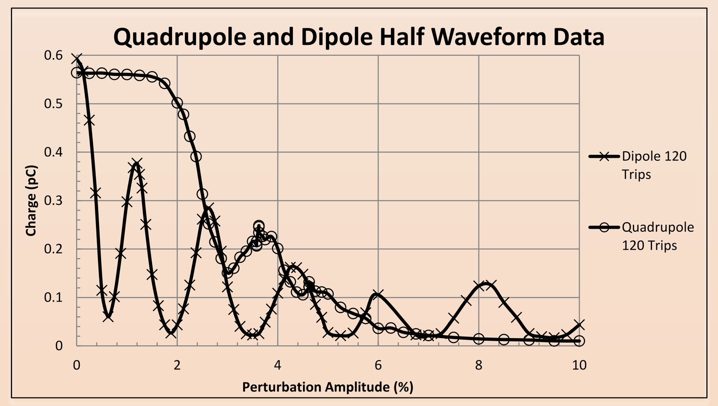
"Tuning" of PSTX and Mode Frequency

When Δa and Δb = -Δa then the perturbation is quadrupolar in shape. In this case Δa and Δb oscillate with a mode frequency ω_q that is proportional to Voltage (V) divided by the trapping waveform frequency (f). Typically $\omega_q/2\pi$ < 20 kHz while f = 60 kHz. By varying V, ω_q can be changed to some desirable value such as 10 kHz (60/6) or 20 kHz (60/3). If one of these ideal values is chosen, then it is possible to make an impulse perturbation every certain number of lattice periods that resonates with the mode. In this resonance, the perturbation is expected to destroy the beam. In PSTX, tuning is done by setting the perturbation amplitude to 2% of the applied waveform voltage and then measuring the charge as the waveform voltage is varied. Since a quadrupole mode is used, the "tune" mentioned can be in increments of half integer values since the quadrupole mode is twice the frequency of the dipole mode.

Results

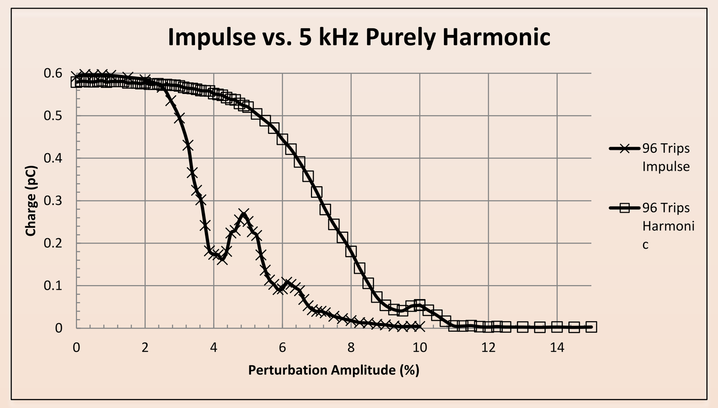
Increasing Perturbation Amplitude

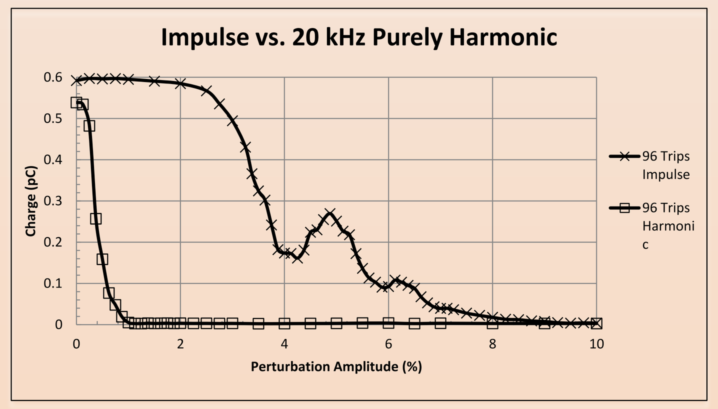




Data taken to determine the effects of one lattice error in a ring. The experiment consisted of quadrupole and dipole spatial structures in which the perturbation amplitude of the trapping waveform is increased and applied for a full period as well as half a period.

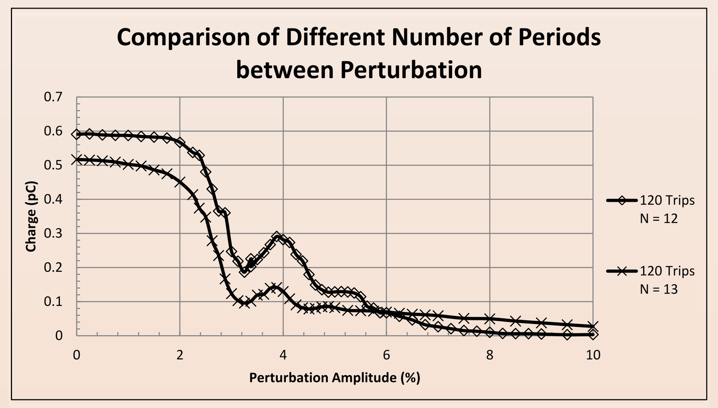
Purely Harmonic Perturbation

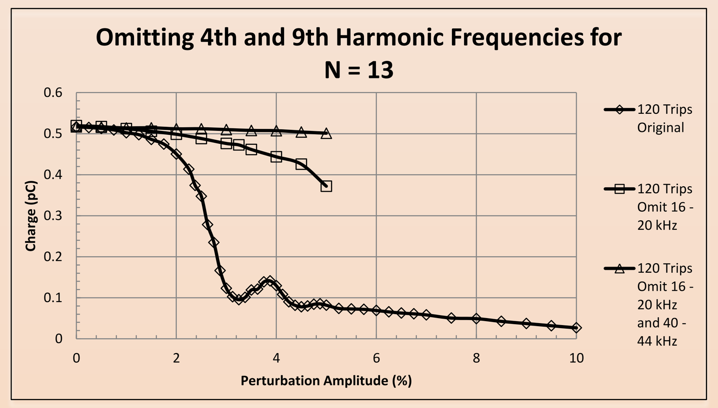




Data taken to determine whether the perturbation is exciting a mode of the beam. The experiment consisted of the summation of two sine waves where one is the 60 kHz driving frequency and the other is the mode frequency or affective frequency of the perturbation.

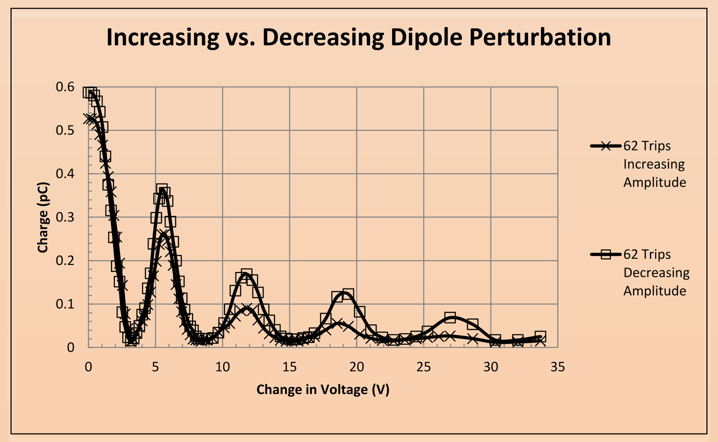
Periods Between Perturbations

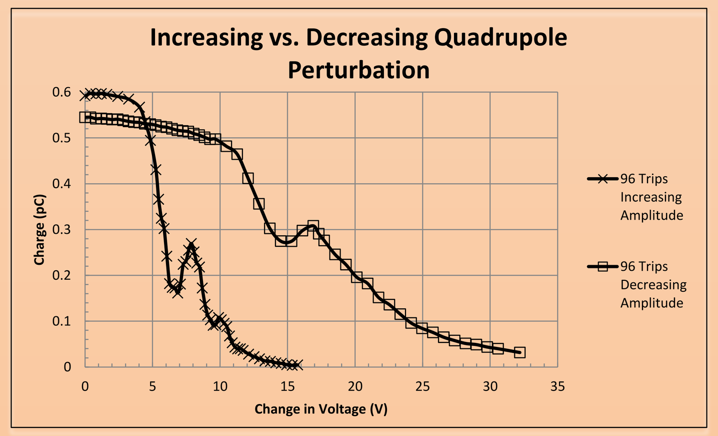




Data taken to determine whether the perturbation is exciting a mode of the beam. The experiment consisted of changing the number of periods between perturbations to determine which harmonics of the trapping waveform were damaging the beam.

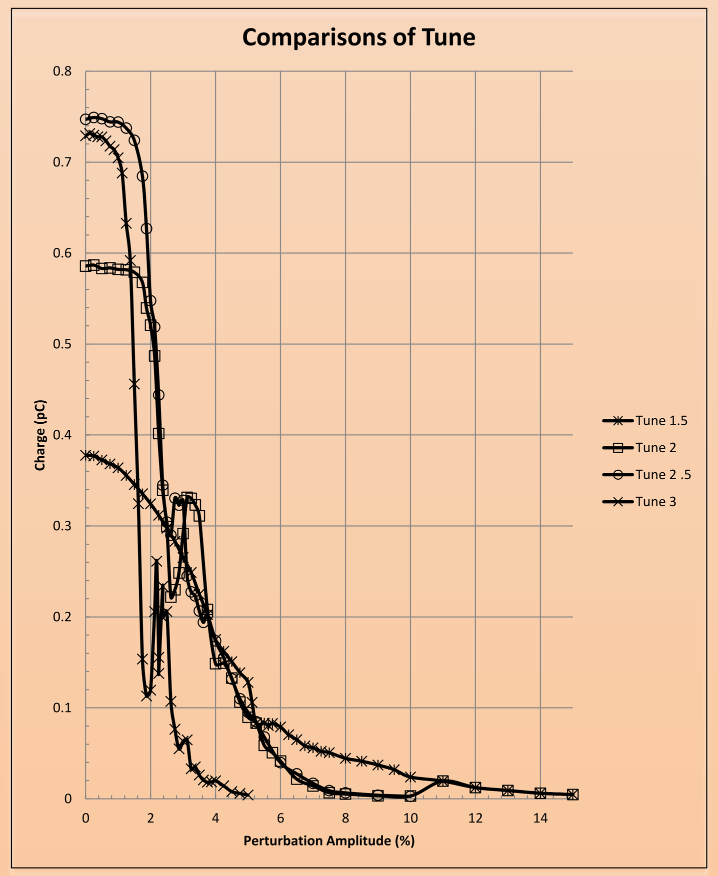
Decreasing Perturbation Amplitude





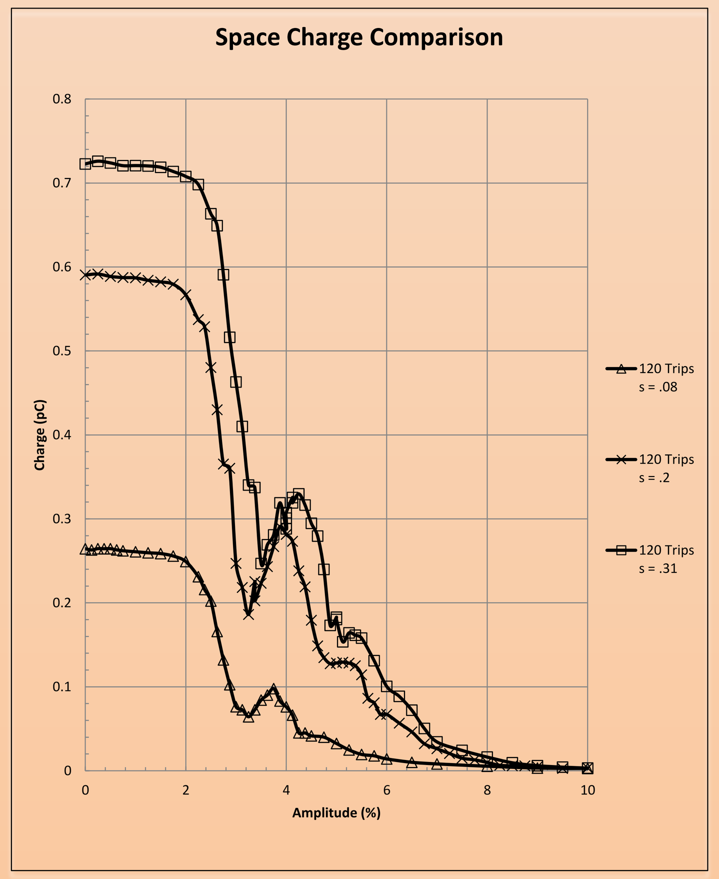
Data taken to determine the effects of one lattice error in a ring. The experiment consisted of quadrupole and dipole spatial structures in which the perturbation amplitude is decreased.

Changing the Tune



Data taken to determine how changing to which tune the system is calibrated affects the beam. The experiment consisted of a quadrupole spatial structure and changing to the overall voltage of the trapping waveform to change the tune and then increasing the perturbation amplitude.

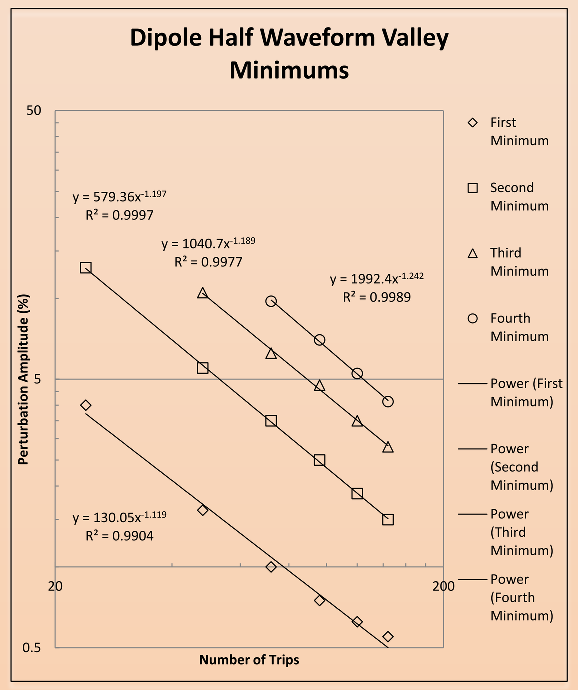
Changing the Space Charge

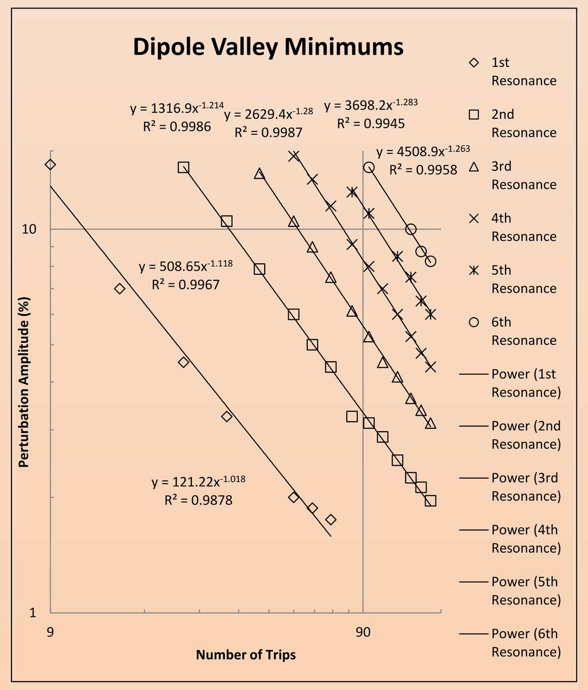


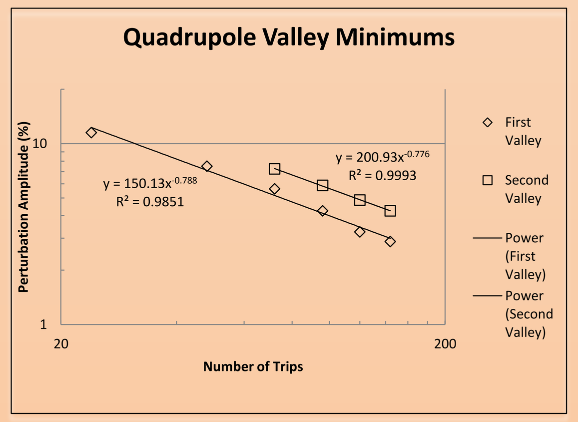
Data taken to confirm that the beam dynamics in a quadrupole spatial structure are dependant on the space charge of the beam. The experiment consisted of changing the bias voltage at extraction to pull more or less cesium ions off of the source.

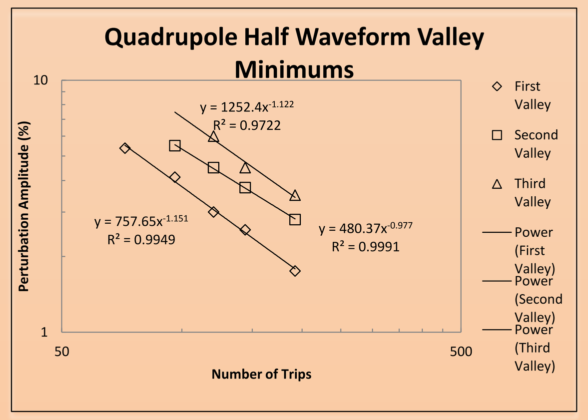
Conclusions

- The Perturbation is exciting the mode of the beam because when applying a purely harmonic 5 kHz perturbation the emittance of the beam increased more slowly than with the purely harmonic 20 kHz perturbation or even the impulse perturbation. Also when applying the perturbation every 13th period instead of every 12th, the 4th harmonic was the frequency causing the emittance of the beam and when removed the emittance grew drastically more slowly when increasing the perturbation amplitude.
- Space charge is a factor in the emittance of the quadrupole spatial structure as suggested by the theory because when space charge of the beam is increased there is trend that the curves shift to the right.
- As the FFT of the trapping wavefrom implies, that as the tune increases the emittance of the beam also increases. This is only a general trend though and there may be more factors affecting the emittance based on the data presented.
- For the dipole case, whether the perturbation is increasing in amplitude or the perturbation is decreasing the amplitude of the trapping waveform the results are the same. For the quadrupole case, increasing or decreasing the perturbation amplitude cause the same trend but the emittance grows at different rates for either case.
- Whether the perturbation is a full waveform or a half waveform both the dipole and quardurpole spatial structures produce the same trend of a non-linear response. The only differences is that the half waveform perturbation is more destructive in that as a part of the perturbation there is no restoring force. This non-linear response is a result of the combination of the perturbation falling out of resonance with the mode of the beam and the state of the beam when the perturbation is turned off.
- There is a power-law relationship between the perturbation amplitude the amount of time that the perturbation is applied. This is seen when plotting the minima of the curves and adding a trendline to the families of minima.









Acknowledgements

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