Kinetic Theory and Nonlinear Perturbative Particle Simulations of High-Intensity Beams

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Beam physics at Princeton Plasma Physics Lab, Princeton University

- ❑ Develop advanced analytical and numerical models describing the nonlinear dynamics and collective processes in intense charged particle beams and beam-plasma interactions. (OHEP, OFES).
- ❑ Carry out experimental studies of intense charged particle beam and beam-plasma interactions in high-leverage areas of heavy ion fusion science that make effective use of Princeton's established experimental capabilities (OFES).

Nonneutral plasmas ⇔ High intensity beams (Davidson 1980s)

Methods and techniques of plasma physics ➔ beam physics.

- ❑ Vlasov-Maxwell theory for high intensity beams.
- ❑ Perturbative (delta-f) particle simulation method for collective effects.
- ❑ Paul trap simulator experiment.

Presentation outline

- Vlasov-Maxwell kinetic system – self-consistently description of collective effects of high intensity beams.
- Perturbative (δf) particle simulation method greatly reduces simulation noise.
- Stable eigenmode excitations
- Instabilities
- Two-stream interactions and instabilities

Nonlinear Vlasov-Maxwell equations for high intensity beams

Collective dynamics described by the Vlasov equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m_b\left(\omega_{\beta b}^2\mathbf{x}_\perp + \omega_z^2 z\mathbf{e}_z\right) + e_b(\nabla\phi - v_z c\nabla_\perp A_z)\right]\cdot\frac{\partial}{\partial \mathbf{p}}\right\} f(\mathbf{x},\mathbf{p},t) = 0$$

Smooth focusing field:

$$\mathbf{F}_{foc} = -m_b\omega_{\beta b}^2\mathbf{x}_\perp - m_b\omega_z^2 z\,\mathbf{e}_z$$

distribution function in phase space

Self-electric and self-magnetic fields self-consistently determined from Maxwell's equations.

$$\nabla^2\phi = -4\pi e_b\int d^3p f(\mathbf{x},\mathbf{p},t),$$

$$\nabla^2 A_z = -4\pi c e_b\int d^3p v_z f(\mathbf{x},\mathbf{p},t).$$

$f(\mathbf{x},\mathbf{p},t)$ and self-field (ϕ, A_z) nonlinearly coupled

δf particle simulation method reduces noise

$$f = f_0 + \delta f,$$

$$\phi = \phi_0 + \delta\phi,$$

$$A_z = A_{z0} + \delta A_z.$$

Fully nonlinear

(f_0, ϕ_0, A_{z0}) – equilibrium

$(\delta f, \delta\phi, \delta A_z)$ – perturbation

$$\frac{dw_i}{dt} = -(1 - w_i)\frac{1}{f_0}\frac{\partial f_0}{\partial \mathbf{p}} \cdot \delta\left(\frac{d\mathbf{p}_i}{dt}\right),$$

$$\delta\left(\frac{d\mathbf{p}_i}{dt}\right) \equiv -e_b\left(\nabla\delta\phi - \frac{v_{zi}}{c}\nabla_\perp\delta A_z\right),$$

Weight function $w \equiv \dfrac{\delta f}{f}$ dynamically determines δf

Statistical noise **significantly** reduced by a factor of $\dfrac{\delta f}{f}$

Beam Equilibrium Stability and Transport (BEST) code

Physics

- Perturbative particle simulation method to reduce noise.
- Linear eigenmodes and nonlinear evolution.
- 2D and 3D equilibrium structure.
- Multi-species; electrons and ions; accommodate very large mass ratio.
- Multi-time-scales, frequency span a factor of 10^5.
- 3D nonlinear perturbation.

Computation

- Message Passing Interface
  - Particle-domain-decomposition.
- Large-scale computing: particle x time-steps ~ 0.5×10^{12}.
- Scales linearly to 512 processors on IBM-SP3 at NERSC.
- NetCDF, HDF5 parallel I/O diagnostics.

$$H = \frac{p^2}{2m_b} + e_b\phi + \frac{1}{2}m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right),$$

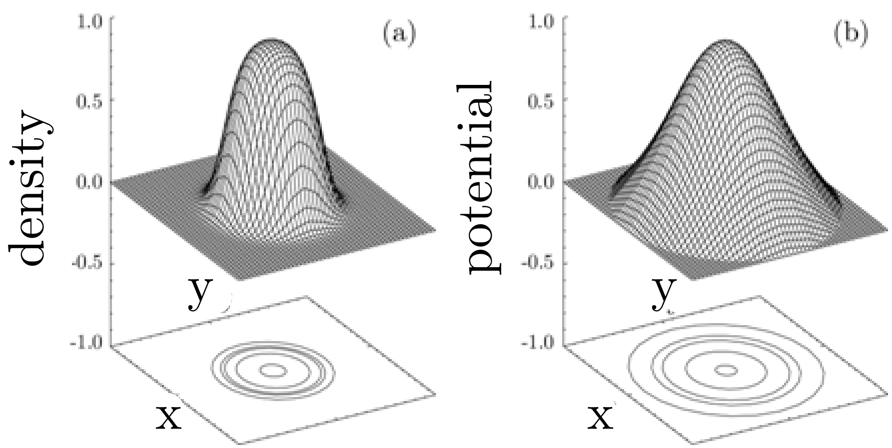
Nonlinear equilibrium

$$f_0 = f_0(H) = \frac{\hat{n}_b}{(2\pi m_b T)^{3/2}} \exp\left(\frac{-H}{T}\right),$$

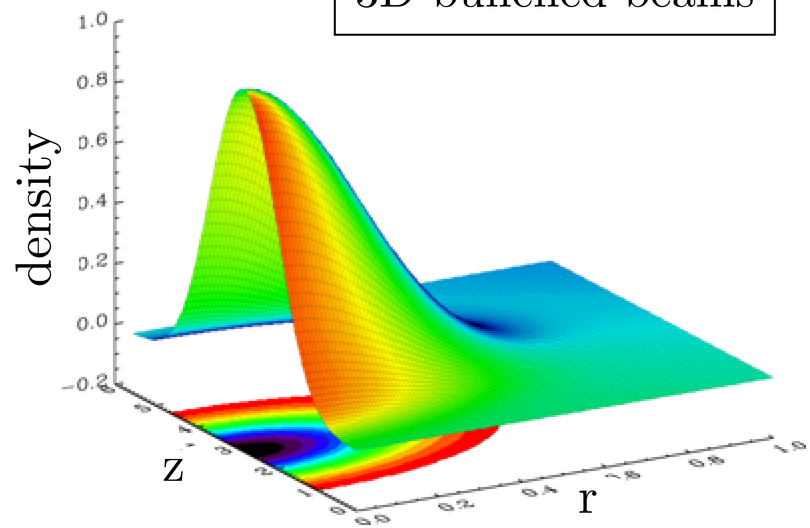
$$\nabla^2\phi_0 = -4\pi e_b \hat{n} \exp\left[-\frac{m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)}{2T} - \frac{e_b\phi_0}{T}\right],$$

Chaotic orbits.
Linear stability theory not possible.

2D coasting beams



3D bunched beams



Nonlinear stability theorem

- Global conservation constrains for the nonlinear Vlasov-Maxwell equations.
- Determine Class of beam distributions that are stable at high intensity.

A sufficient condition for linear and nonlinear stability is:

$$\frac{\partial f(H)}{\partial H} \leq 0$$

Self-field

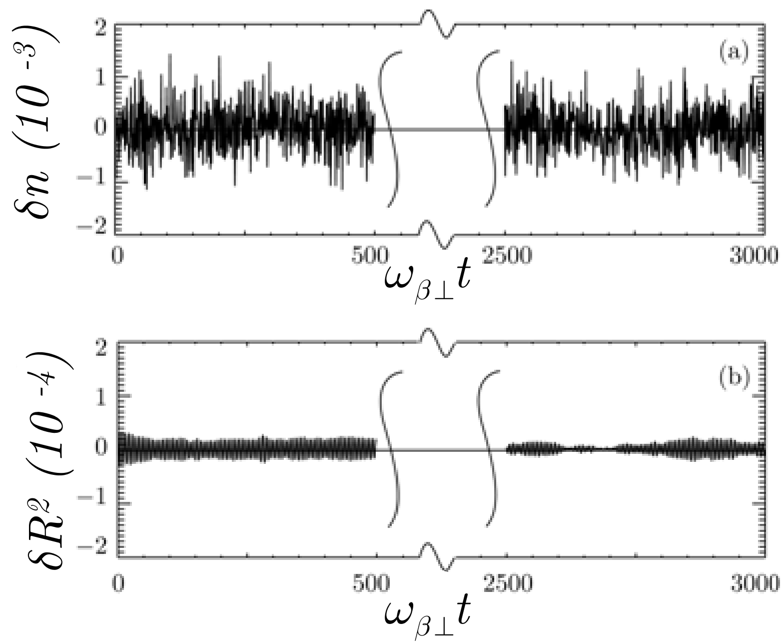
where $$H \equiv \frac{1}{2}\mathbf{p}^2 + \psi(\mathbf{x}) + e_b\phi(\mathbf{x})$$

Focusing field

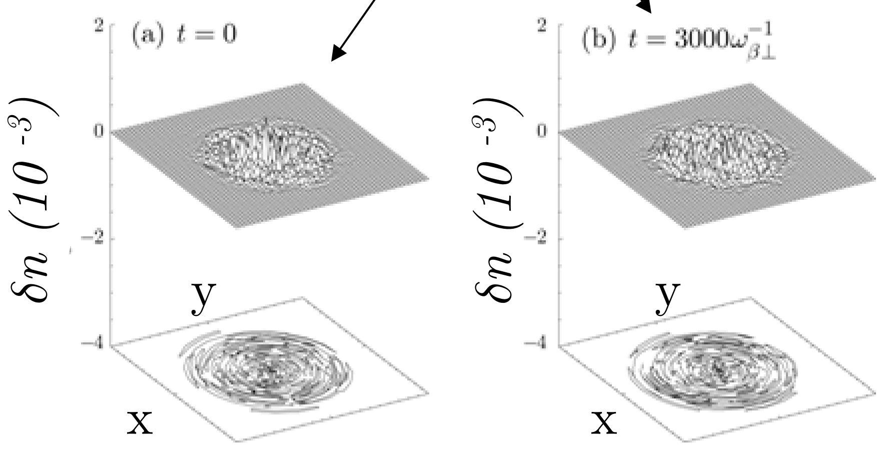
$$\psi(\mathbf{x}) = \frac{1}{2} m_b \gamma_b \left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)$$

- R. C. Davidson, Physical Review Letters **81**, 991 (1998).
- *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, 2001), R. C. Davidson and H. Qin, Chapter 4;

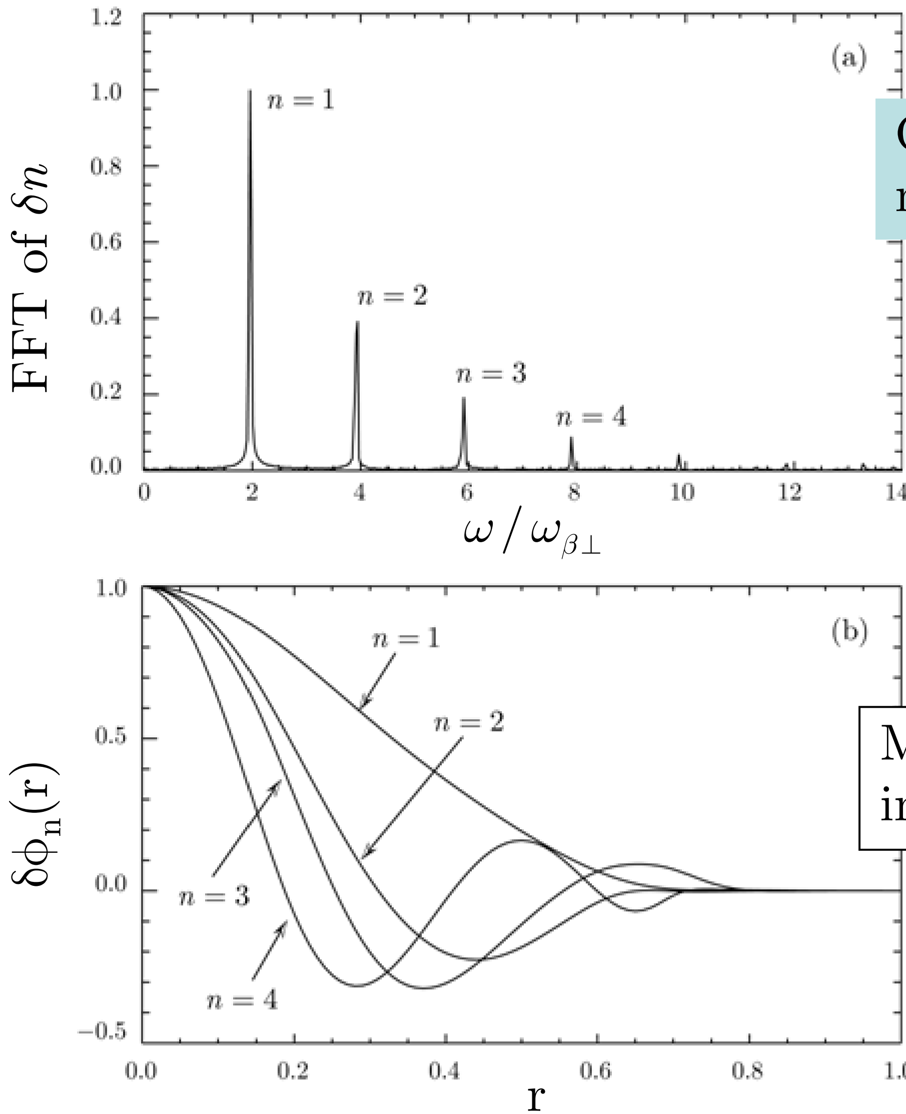
Long-time nonlinear perturbations



contain perturbation spectrum



Collective interior mode excitation



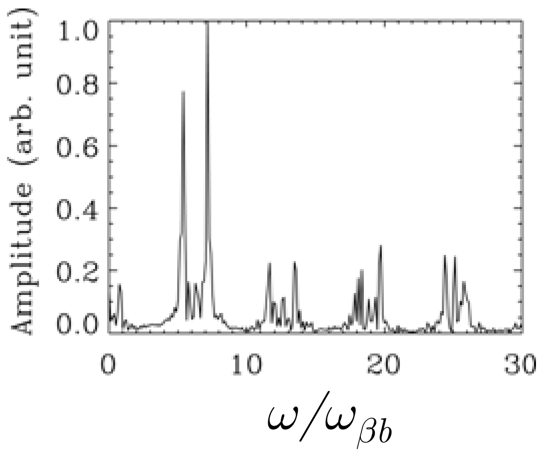
Collective dynamics manifest by eigenmodes

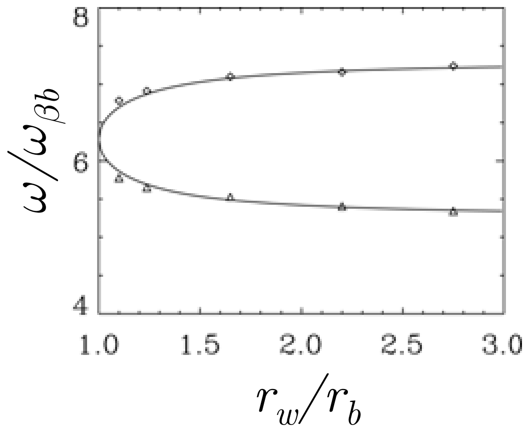
Mode structure confined inside the beam

Collective surface mode excitation

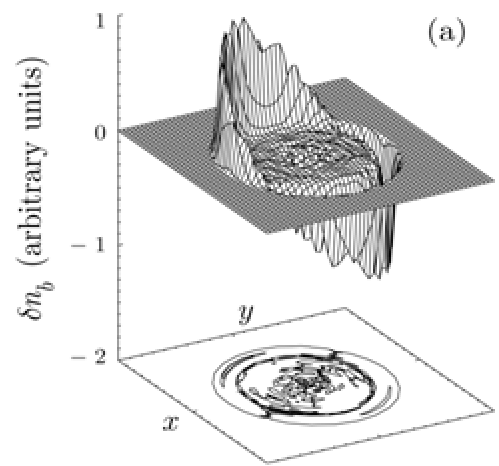
Dispersion relation:

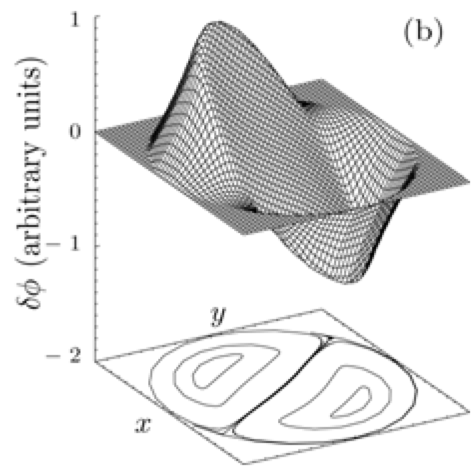
$$\omega = k_z V_b \pm \frac{\hat{\omega}_{pb}}{\sqrt{2}\gamma_b}\sqrt{1 - \frac{r_b^2}{r_w^2}}$$





Dipole mode structure





Collective instabilities in intense charged particle beams

One-Component Beams

- Harris and Weibel instability driven by temperature anisotropy

$$T_\perp \gg T_\| \; .$$

- Resistive wall instability

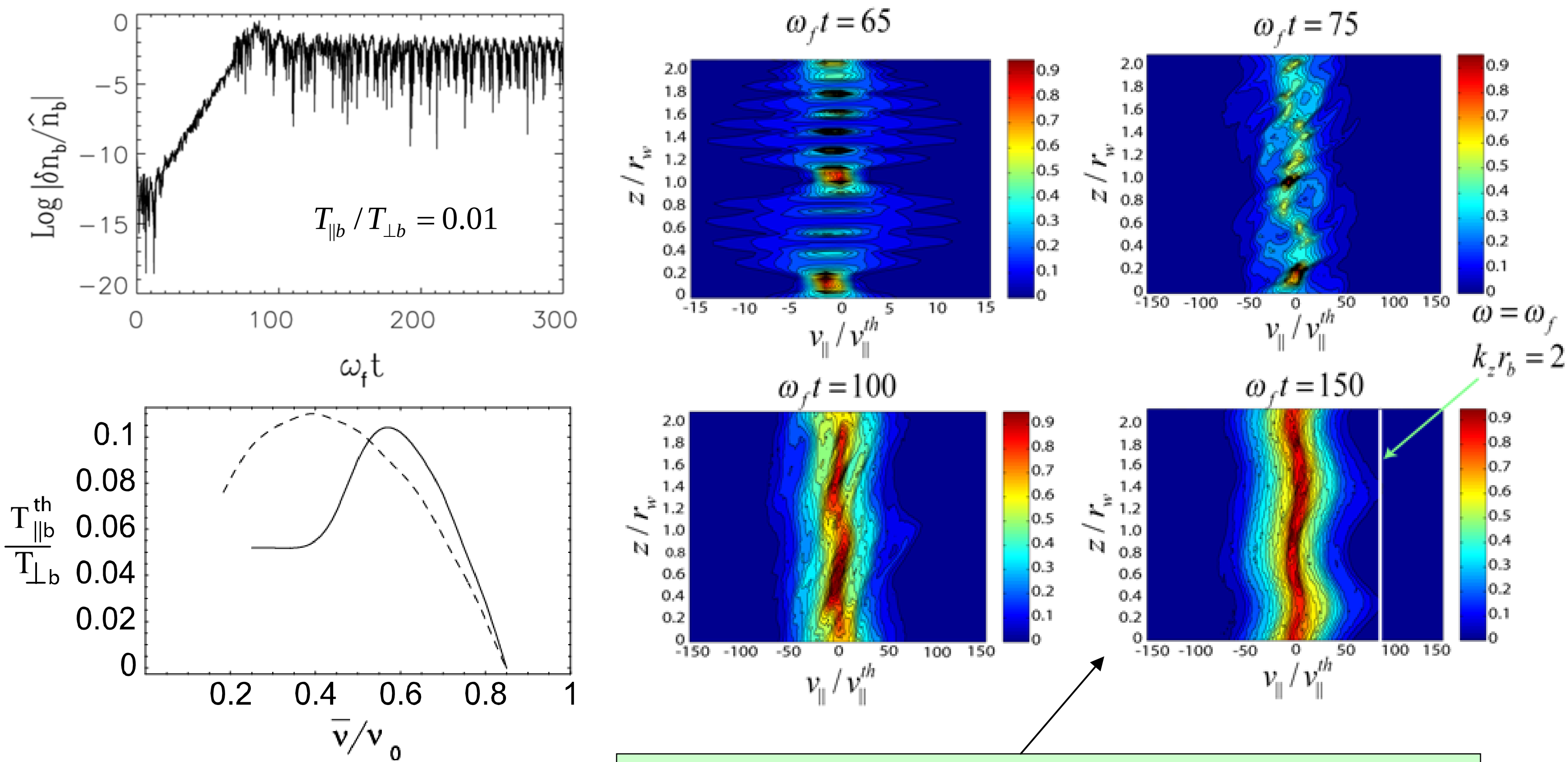
Propagation Through Background speices

- Two-stream instability
- Ion-electron (Electron cloud) instability

Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

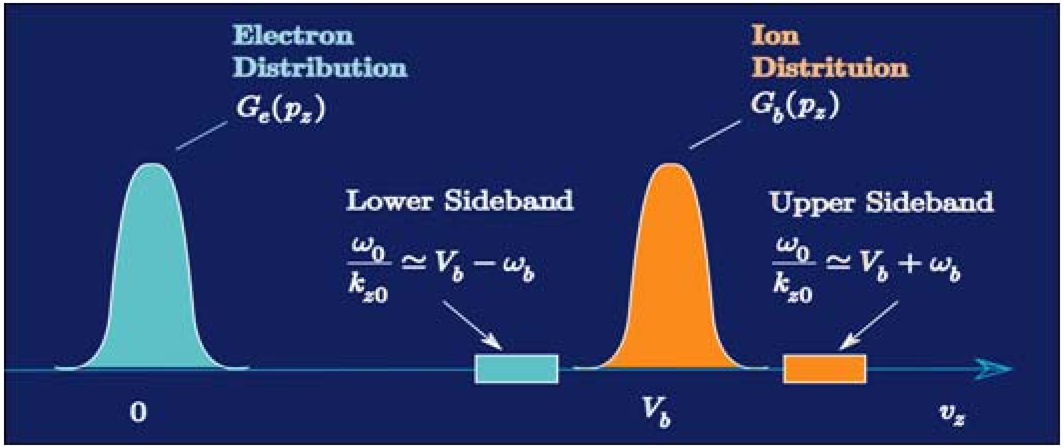
Harris instability by large temperature anisotropy

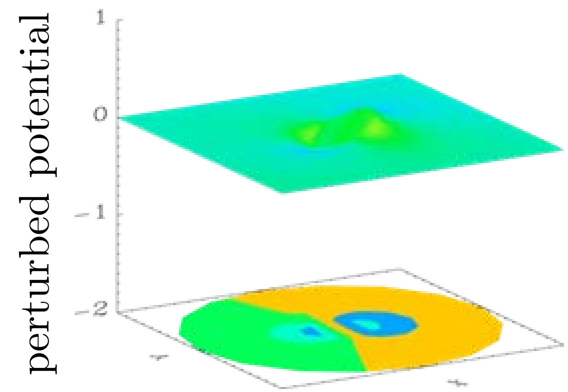


Nonlinear saturation by particle trapping

E. A. Startsev, et al., Physical Review Special Topics Accel. & Beams **8,** 124201(2005).

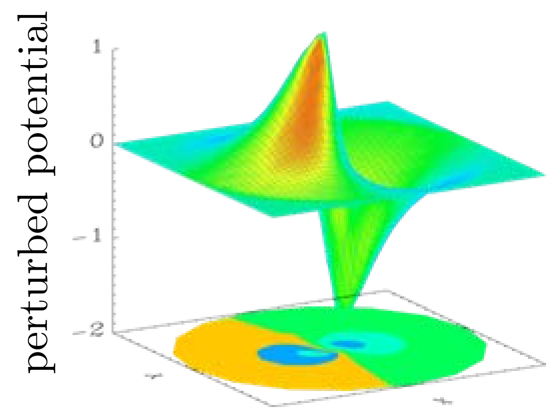
Electron-ion two-stream instability





$t = 0$

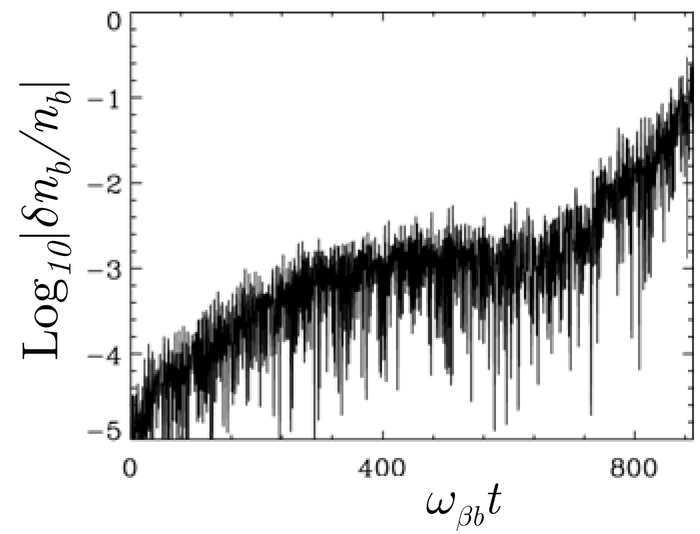
- Surface mode destabilized by background species.
- Observed in high intensity proton beams (PRS).
- Could be a show stopper for high intensity accelerators, e.g. SNS.
- Transverse geometry and dynamics are important.
- Damping mechanisms (Landau damping) are important.



$t = 200 / \omega_{\beta b}$

Instability properties predicted by BEST simulations

- **Agrees well with experiment observations** (Proton Storage Ring)
  - Mode structure
  - Growth rate
  - Real oscillation frequency



- Late time nonlinear growth observed.
- There are two-phases to the instability.

Possible two-stream instabilities for Cornell ERL

- No self-space-charge effect for beam. $\gamma_b \sim 10000$
- Possible two-stream interactions:
  - ✓ Small mass ratio is a disadvantage.
  - ✓ Fast electron- slow electron in ERL
  - ✓ Electron-ion (background)

Fractional charge neutralization $f = 0.1\%$ (Hoffstaetter, et al, PRST-AB)

$$\omega_{pb}^2 \equiv \frac{4\pi n_b e^2}{m_b \gamma_b^3}$$

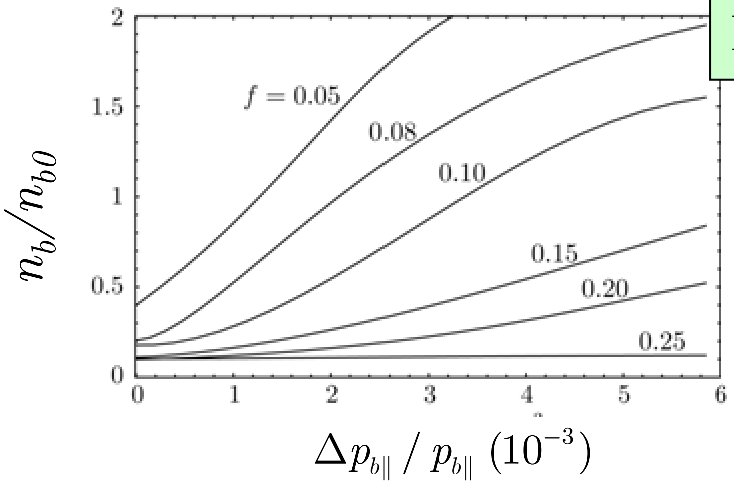
$$\frac{(\mathrm{Im}\,\omega)_{\max}}{\omega_\beta} = \frac{1}{2^{7/4}} \frac{f^{1/2}(\gamma_b m_b / m_i)^{1/4}(\omega_{pb}^2/\omega_\beta^2)^{3/4}}{\left(1+\frac{f}{2}\frac{\omega_{pb}^2}{\omega_\beta^2}\right)^{1/4}} \sim 0.092$$

R. C. Davidson, et al, PRST-AB **2**, 054401(1999).

Damping mechanisms for two-stream instabilities

- Landau damping by momentum spread.
- Landau damping induced by transverse turn spread.
    - Nonlinear space charge field.
    - Chromaticity induced.
- Theoretical growth rate is greatly reduced.
- Instability threshold observed both numerically and experimentally.

Instability threshold
Proton Storage Ring



References

General references

- R. C. Davidson and H. Qin, *An Introduction to the Physics of Intense Charged Particle Beams in High Energy Accelerators*, 583 pp., World Scientific (2001).
- R. C. Davidson, Physical Review Letters **81**, 991 (1998).

Two-stream instability

- R. C. Davidson, et al, PRST-AB **2**, 054401(1999).
- H. Qin, et al, PRST-AB **3**, 084401 (2000).
- R. C. Davidson and H. Qin, Physics Letter A **270**, 177 (2000).
- H. Qin, et al, PRST-AB **6**, 014401 (2003).
- E. A. Startsev and R. C. Davidson, Physics of Plasmas **13**, 062108 (2006).

Other publications available at http://nonneutral.pppl.gov/

Conclusions

- **Vlasov-Maxwell equations provide a very effective tool to understand collective processes in intense charged particle beams.**

- **Nonlinear δf particle simulation method significantly reduces noise.**

- **Major progress has been made in simulation studies of collective effects.**
  - Electron-cloud-induced two-stream instability.
  - Temperature anisotropy instability.