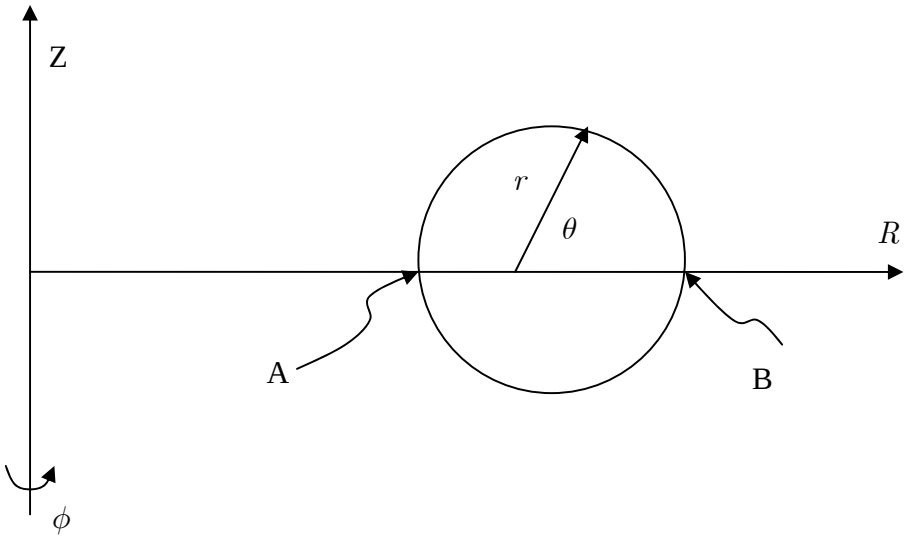
AST 552 – General Plasma Physics II, Part 1 Magnetohydrodynamics

Homework Set 4, Feb. 27, 2006

7. Curvature of magnetic field in a tokamak

For the equilibrium of a low-β, large aspect-ratio tokamak (see figure), where dose the magnetic field line has a larger curvature, in point A$(\theta = \pi)$ or B $(\theta = 0)$?
(Hint: calculate the curvature of magnetic field using the solution for the Grad-Shafranov equation.)



8. Another definition of $q(\psi)$.

Consider a magnetic field with nested flux surfaces, which are symmetric in the toroidal direction (see figure). The safety fact $q(\psi)$ as a flux function is defined as the number of toroidal transit in one poloidal transit of a magnetic field on the flux surface $\psi = const.$,

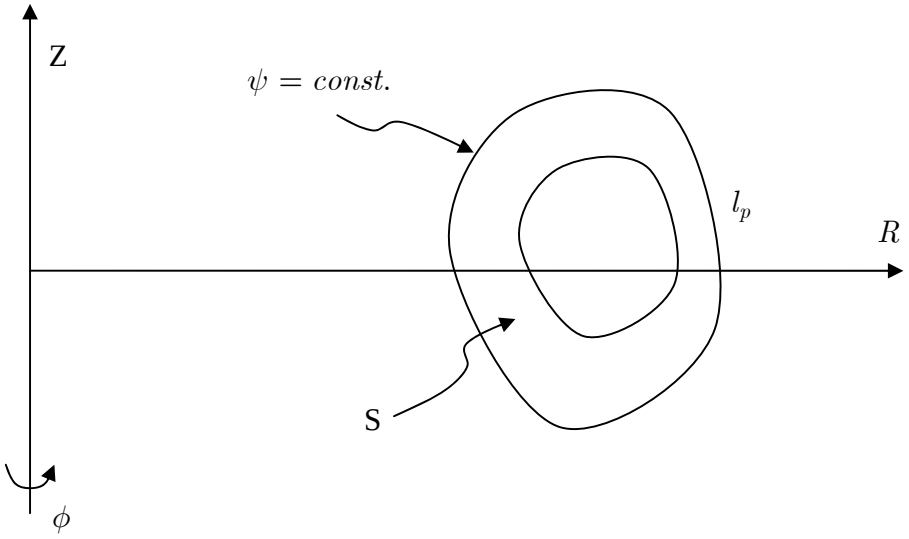
$$q(\psi) \equiv \frac{F(\psi)}{2\pi} \oint_{\psi=const.} \frac{dl_p}{|\nabla\psi|\, R} \ ,$$

where l_p is the poloidal arc-length of a magnetic field line on the flux surface. Show that

$$q(\psi) = \frac{1}{2\pi} \frac{d\psi_t(\psi)}{d\psi} \ ,$$

where $\psi_t(\psi)$ is the toroidal magnetic flux enclosed by the flux surface

$$\psi_t(\psi) \equiv \int_S \boldsymbol{B}_\phi \cdot d\boldsymbol{s}\,.$$



9. Resistive shear Alfvén wave

Use the resistive MHD equations to derive the following dispersion relation for the linear shear Alfvén wave in a homogenous, magnetized plasma with finite resistivity

$$\left(1 + \frac{c^2\eta^2k^2}{4\pi\omega}\,i\right)\omega^2 \;= k_\parallel^2 v_A^2\,.$$