Study of Reconnection Using the Two-Fluid Magnetic Reconnection Code (MRC)

Graduate Student Seminar

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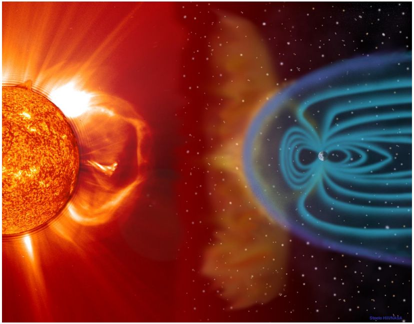
Outline

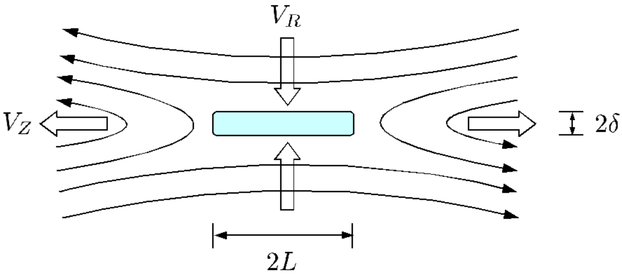
Magnetic Reconnection Defined



Magnetic reconnection is a topological rearrangement of magnetic field lines that converts magnetic field energy to particle kinetic energy.

Reconnection occurs in sawtooth relaxations in tokamaks, solar flares and CMEs in the solar corona, and in the magnetosphere, amongst other places.

What Does Reconnection Look Like?



- Oppositely directed field lines flow into a reconnection region, then reconnect and move away
- The reconnection region is known as the current sheet due to the out of plane currents flowing through it

Sweet-Parker Reconnection

Magnetic Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

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Continuity Equation

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + (\nabla \cdot \mathbf{v})\,\rho$$

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Pressure Balance

$$P + \frac{\rho v^2}{2} + \frac{B^2}{2\mu_0} = C$$

Sweet-Parker Reconnection

Magnetic Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} \quad \longrightarrow \quad V_R = \frac{\eta}{\mu_0 \delta}$$

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$$S = \frac{\mu_0 L V_A}{\eta}$$

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Lundquist number: the ratio of the Ohmic diffusion time to the Alfvén wave crossing time

Solar Flare Parameters

Solar Flare Parameters

$$\begin{aligned} B &= 300 \text{ G} \\ n &= 10^{15} \text{ m}^{-3} \\ T &= 100 \text{ eV} \\ L &= 10,000 \text{ km} \\ V_A &= 2.1 \times 10^7 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \eta_{Spitzer} &= 1.6 \times 10^{-6}\ \Omega\text{m} \\ S &= \frac{\mu_0 L V_A}{\eta} = 1.6 \times 10^{14} \\ V_R &= \frac{V_A}{\sqrt{S}} = 1.7 \text{ m/s} \\ t_R &= \frac{L}{V_R} = 2.3 \text{ months} \end{aligned}$$

For solar flares, predicts time scale of months to years instead of the observed scale of minutes to hours

How Can Reconnection Be Sped Up?

Enhanced Resistivity

- As $t_R \sim \frac{1}{\sqrt{\eta}}$, an anomalously higher resistivity would create faster reconnection
- Several ideas have been developed as to why η might be higher, often involving microturbulence

Globally Enhanced Resistivity

- A resistivity globally higher than the Spitzer value

$$\eta_{eff} = \eta_{Spitzer} + \eta_{anom}$$

- Difficult explain theoretically, but offers a solution for fast reconnection

Locally Enhanced Resistivity

- Resistivity is locally higher near the center of the current sheet

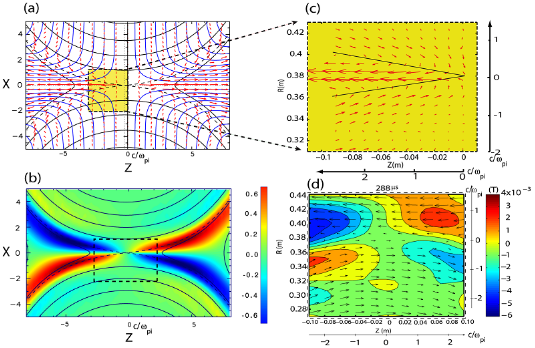
$$\eta_{eff}(\mathbf{r})$$

- This would create slow shocks, but these have never been observed

The Hall Effect

Generalized Ohm's Law:

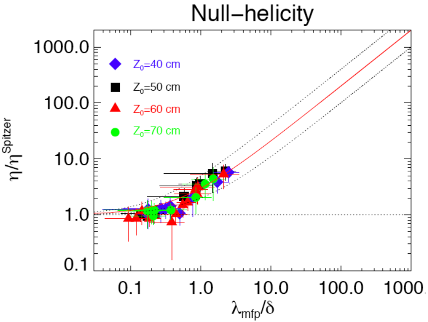
$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} + \frac{1}{ne}\mathbf{j} \times \mathbf{B} + \frac{m_e}{e}\left(\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{v}_e\right) - \frac{1}{ne}\nabla \cdot \mathbf{P}_e$$

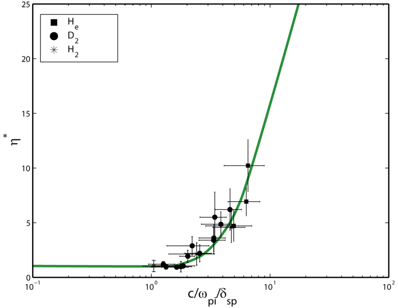


Quadrupolar field is a signature of the Hall effect

Observed in MRX, SSX, and the POLAR satellite, predicted by theory and computation

When Are Two-Fluid Effects Important?





$$\delta = (0.2 - 0.4)\,\delta_i \qquad\qquad \frac{\eta}{\eta_{Spitzer}} \approx 1 + 3\frac{\lambda_{mfp}}{\delta_i}$$

$\eta = \frac{E}{j}$ is referred to as the effective resistivity

Is L Really Predetermined by the System Size?

In our derivation of the Sweet-Parker reconnection rate, we assumed the system size was set at some L, and solved for V_R and δ.

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Perhaps, then, there is a relationship between η and L.

How are η and L Related?

From the Sweet-Parker derivation, recall that

$$V_R = \frac{\eta}{\mu_0 \delta}$$

$$V_Z = V_A$$

$$V_R L = V_Z \delta$$

Solving, we find

$$\eta L = \mu_0 V_A \delta^2$$

If ηL is a constant, then it may be possible to explain fast reconnection, as $S = \frac{\mu_0 L V_A}{\eta}$ will be reduced twice

Can Fast Solar Flare Reconnection Now Be Explained?

Plasma Parameters

$$
\begin{aligned}
B &= 300\ \mathrm{G} \\
n &= 10^{15}\ \mathrm{m}^{-3} \\
T &= 100\ \mathrm{eV} \\
L &= 10,000\ \mathrm{km} \\
V_A &= 2.1 \times 10^{7}\ \mathrm{m/s} \\
\lambda_{mfp} &= 70\ \mathrm{km} \\
\delta_i &= 7\ \mathrm{m}
\end{aligned}
$$

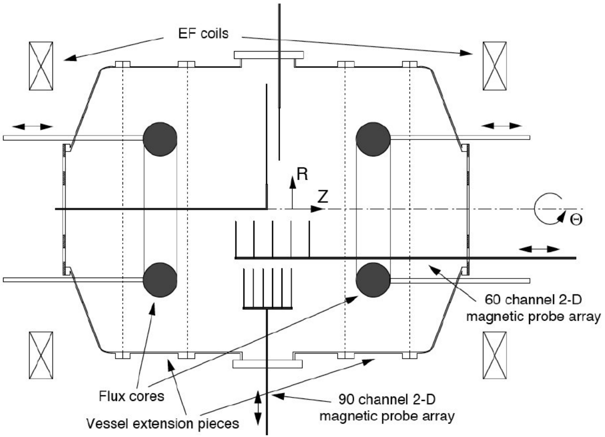
Classical Sweet-Parker Model:

$$
\begin{aligned}
S &= \frac{\mu_0 L V_A}{\eta} = 1.6 \times 10^{14} \\
t_R &= \frac{L}{V_R} = 2.3\ \text{months}
\end{aligned}
$$

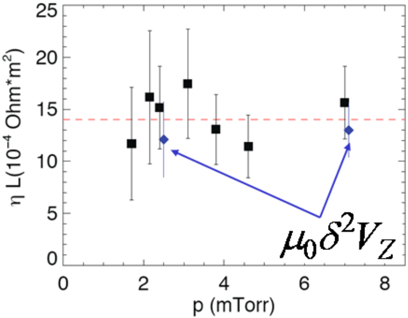
Using Enhanced η and shortened L:

$$
\begin{aligned}
\frac{\eta}{\eta_{Spitzer}} &\simeq 3\frac{\lambda_{mfp}}{\delta_i} = 3 \times 10^{4} \\
S_{eff} &= S\left(\frac{\eta}{\eta_{Spitzer}}\right)^{-2} = 1.8 \times 10^{5} \\
t_R &= 3\ \text{minutes}
\end{aligned}
$$

The Magnetic Reconnection Experiment



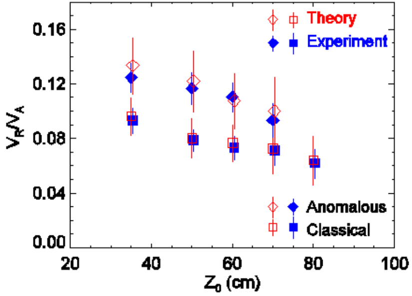
Experimental Data Shows that ηL is Constant

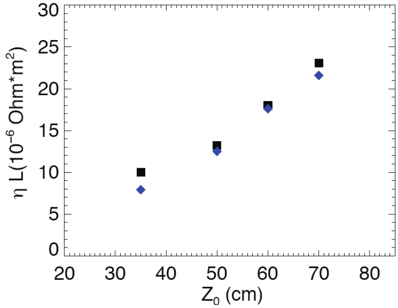


- In MRX, ηL seems to be constant [1].
- Neutral gas fill pressure is varied in order to control collisionality in the experiment
- Similar results seen when fixing pressure and varying driving currents

[1] Kuritsyn *et al.*, Geo. Res. Lett. 34, L16106 (2007).

The Reconnection Rate is also Affected by the System Size





The system sizes could only change by a factor of 2

To confirm this study and allow a larger parameter regime, numerical simulations with proper boundary conditions are necessary...

The Magnetic Reconnection Code (MRC)

Parallel implicit algorithm for simulating reconnection on a 2D grid in either a Cartesian or a cylindrical system.

Solves either single fluid resistive or two-fluid extended MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v}\rho) = 0$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = \mathbf{j} \times \mathbf{B} - \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{M_H}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} + \frac{1}{ne}\mathbf{j} \times \mathbf{B} - \frac{1}{ne}\nabla \cdot \mathbf{P}_e + \mathbf{R_H}$$

$$\frac{\partial P_e}{\partial t} + \mathbf{v_e} \cdot \nabla P_e = -\frac{5}{3} P_e \nabla \cdot \mathbf{v_e} + \frac{2}{3}\left[\eta \left|\mathbf{j}\right|^2 + \nabla \cdot \left(m_i \kappa_e \nabla \frac{P_e}{\rho}\right) - Q\right]$$

$$\frac{\partial P_i}{\partial t} + \mathbf{v} \cdot \nabla P_i = -\frac{5}{3} P_i \nabla \cdot \mathbf{v} + \frac{2}{3}\left[\nu \left\|\nabla \mathbf{v}\right\|^2 + \nabla \cdot \left(m_i \kappa_i \nabla \frac{P_i}{\rho}\right) + Q\right]$$

Where $\mathbf{M_H} \propto -\nabla^4 \mathbf{v}$ is the hyperviscosity term, and $\mathbf{R_H} \propto -\nabla^2 \mathbf{j}$ is the hyperresistivity term, used to dampen grid-scale oscillations

MRC Boundary and Initial Conditions

Boundary Conditions:

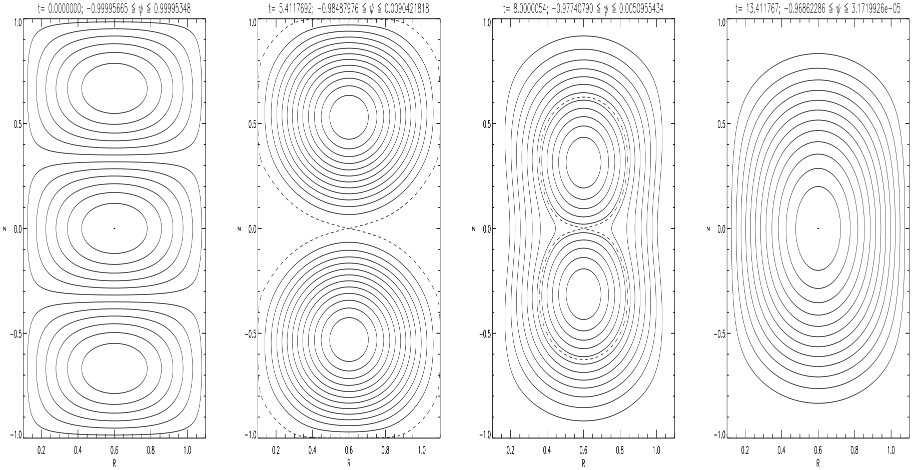
- Conducting walls, such that $\mathbf{E} \times \hat{n} = 0$ and $\mathbf{v} \cdot \hat{n} = 0$ at the walls
- This setup is good for comparison with experiments

Initial Conditions:

- Three flux tubes with out of plane currents in equilibrium with pressure gradients such that $\nabla P = -\nabla^2 \psi \nabla \psi$ and no out of plane B (null helicity)
- The central island has artificially high resistivity in order to make it decay away quickly, destroying equilibrium:

$$\eta(\psi) = \begin{cases} \eta_-, & \psi \leq 0 \\ \eta_- + (\eta_+ - \eta_-)\left(1 - e^{(\psi/0.075)^2}\right), & \psi \geq 0 \end{cases}$$

Phases of Reconnection in MRC



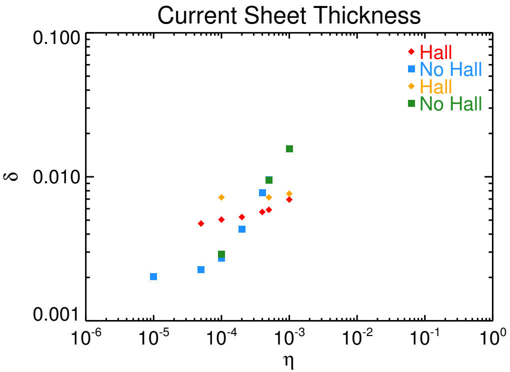
Initial State Middle Island Gone Reconnecting Decaying

References

For more information about MRC, see:

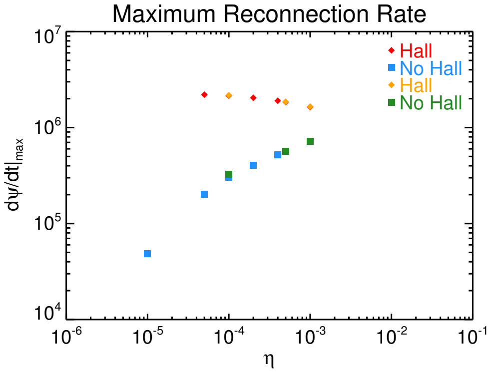
- J. A. Breslau, Ph.D. Thesis, Princeton University, 2001.
- J. A. Breslau and S. C. Jardin, Comput. Phys. Commun. 151, 8 (2003).
- J. A. Breslau and S. C. Jardin, Phys. Plasmas 10, 1291 (2003).

Current Sheet Thickness is Dependent on Two Fluid Effects



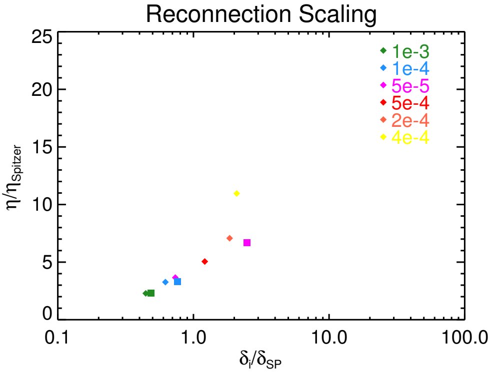
- Current sheet thickness δ depends on whether two fluid effects are included
- Hall current sheet thickness is the ion skin depth δ_i
- The ratio of the thicknesses allow measurement of $\frac{\delta_i}{\delta}$

Hall Reconnection Proceeds Faster



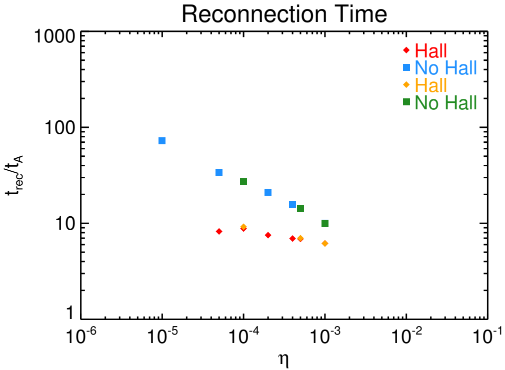
- The reconnection rate is much higher when Hall effects are included
- The ratio of the two fluid and single fluid reconnection rates allow measurement of the enhanced resistivity

When are Two Fluid Effects Important?



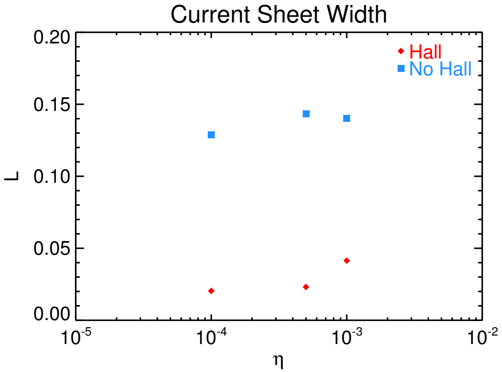
Two fluid effects become important when $\delta_i \sim \delta$

Hall Reconnection Finishes Sooner



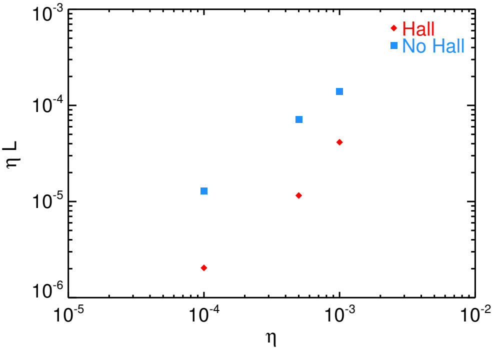
- Reconnection is much faster when Hall effects are included
- Overall, as η is larger, reconnection proceeds faster

The Current Sheet Length is Larger in the Single Fluid Case



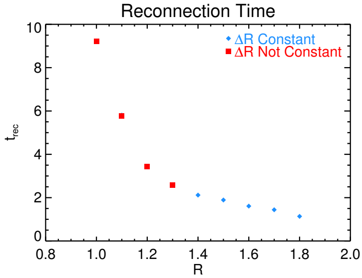
The current sheet width L is defined as the length over which the toroidal current density falls off by 50%

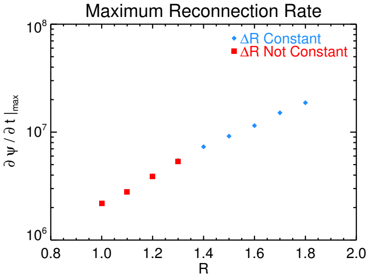
Is ηL Held Constant as η Varies?



ηL is not held constant as η is varied

How Fast does Reconnection Proceed as the Aspect Ratio Changes?



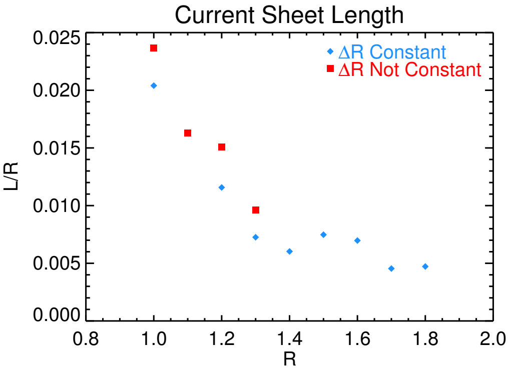


Reconnection time decreases as the aspect ratio becomes larger

The maximum reconnection rate increases with the aspect ratio

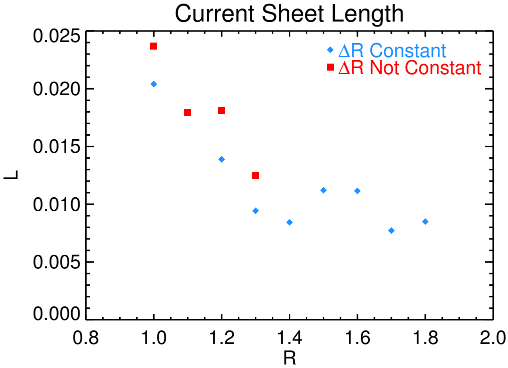
This is in disagreement with previously shown MRX data!

Does the Increasing the System Size Increase the Current Sheet Length?



The current sheet length L actually decreases as the system size R increases

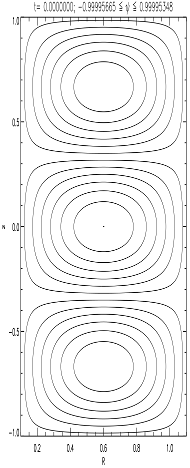
So Then, is ηL Held Constant?

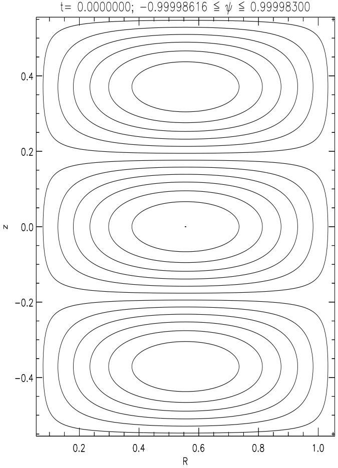


L decreases, and as η is held constant, ηL also decreases

This is in disagreement with experimental data

Initial Equilibrium Relates to Reconnection Drive





$R = 1.0, Z = 2.0$ $R = 1.8, Z = 2.0$

What is to be Done?

- The system size must be increased without affecting the drive
- This should be possible by keeping the aspect ratio fixed and increasing the system size

Summary

- Magnetic reconnection is an important process in astrophysical and laboratory plasmas
- Fast reconnection is not well understood, but some mechanisms for speeding up reconnection, such as enhanced resistivity and two fluid effects, have been identified
- It was experimentally observed that ηL was constant
- MRC runs scanning over aspect ratio did not show that ηL was constant, but this may be an effect related to the driving force

Future Work

- Increase or decrease the system size while keeping the aspect ratio constant in order to learn how L scales when the driving force is kept the same
- For these fixed aspect ratio systems, determine if ηL is then constant
- Determine if much higher system sizes can be accessed without increasing the number of grid points in R too much in order to extrapolate to solar flare regimes
- Vary collisionality by changing the viscosity

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- Hantao Ji

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