Particle Simulation, Gyrokinetics, Turbulence and Beyond

W. W. Lee

Theory Department Seminar
PPPL

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Acknowledgement
S. Ethier and R. Ganesh (IPR, India)

THE PHYSICS OF FLUIDS VOLUME 5, NUMBER 4 APRIL, 1962

One-Dimensional Plasma Model

JOHN DAWSON
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(Received June 27, 1961; revised manuscript received December 27, 1961)

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A one-dimensional plasma model consisting of a large number of identical charge sheets embedded in a uniform fixed neutralizing background is investigated by following the sheet motions on a high-speed computer. The thermalizing properties and ergodic behavior of the system are examined and found to be in agreement with the assumption that one is equally likely to find the system in equal volumes of the available phase space. The velocity distribution, Debye shielding, drag on fast and slow sheets, diffusion in velocity space, the Landau damping of the Fourier modes, the amplitude distribution function for the Fourier modes, and the distribution of electric fields felt by the sheets were obtained for the plasma in thermal equilibrium and compared with theoretically predicted values. In every case, except one, the drag on a slow sheet, the numerical results agreed with theory to within the statistical accuracy of the results. The numerical results for the drag on a slow sheet were about a factor of 2 lower than the theory predicated indicating that the approximations made in the theory are not entirely valid. An understanding of the cause of the discrepancy might lead to a better understanding of collisional processes in plasmas.

Other pioneers:
Oscar Buneman
Ned Birdsall

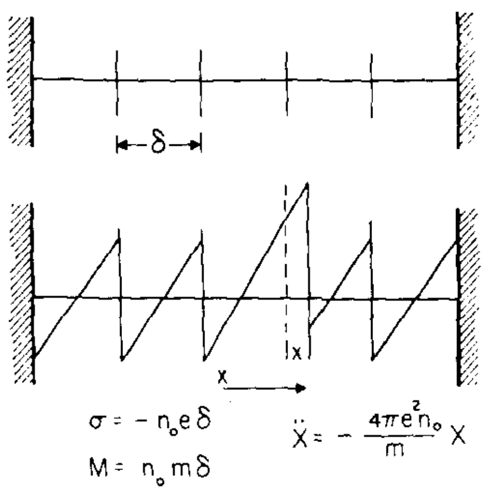
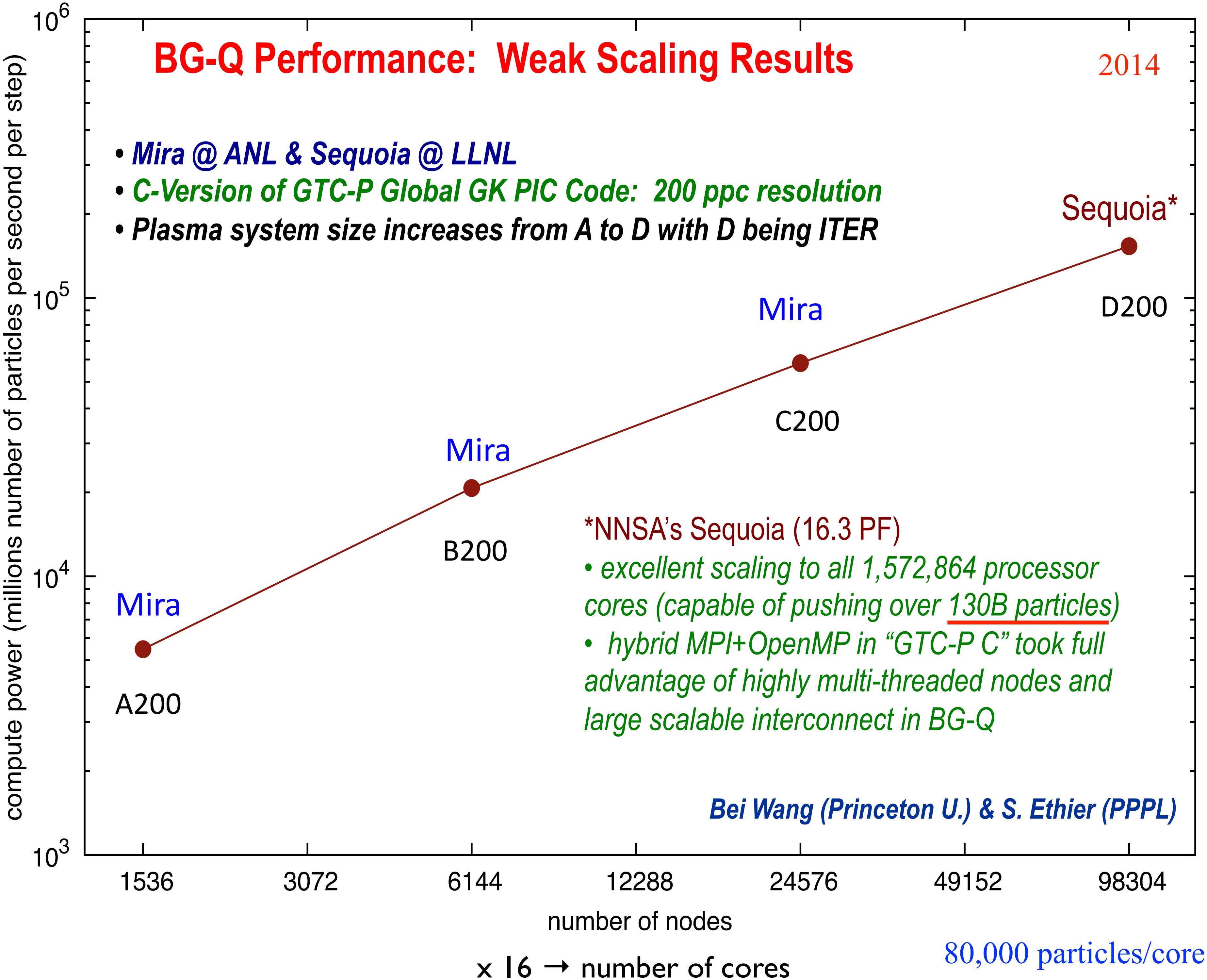


FIG. 1. One-dimensional plasma model.


BG-Q Performance: Weak Scaling Results
2014
• Mira @ ANL & Sequoia @ LLNL
• C-Version of GTC-P Global GK PIC Code: 200 ppc resolution
• Plasma system size increases from A to D with D being ITER
Sequoia*
D200
Mira
C200
Mira
B200
Mira
A200
*NNSA's Sequoia (16.3 PF)
• excellent scaling to all 1,572,864 processor cores (capable of pushing over 130B particles)
• hybrid MPI+OpenMP in "GTC-P C" took full advantage of highly multi-threaded nodes and large scalable interconnect in BG-Q
Bei Wang (Princeton U.) & S. Ethier (PPPL)
compute power (millions number of particles per second per step)
10^6
10^5
10^4
10^3
1536
3072
6144
12288
24576
49152
98304
number of nodes
x 16 → number of cores
80,000 particles/core

Fluctuation-Dissipation Theorem for Weakly Damped Normal Modes

[Klimontovich '67]

- Particle Noise in a Simulation Plasma

$$|e\Phi(k, w_{pe})/T_e|_{th} = \frac{1}{\sqrt{N}k\lambda_{De}}$$ plasma waves - space charge waves

$$|e\Phi(k, w_s)/T_e|_{th} = \frac{1}{\sqrt{N}}$$ ion acoustic waves

- N is the number of simulation particles in the wave, not in the Debye shielding volume.

Fluctuations in Simulations with Finite Size Simulation Particles

[Many papers by Birdsall, Langdon and Okuda in '70's]

- Linear Dispersion Relation $\epsilon \equiv 1 + |S_k|^2[1 + \xi_e Z(\xi_e) + \tau + \tau\xi_i Z(\xi_i)]/(k\lambda_{De})^2 = 0$

- Finite size particles only affects short wavelength modes by reducing their fluctuation (noise) level .

- For long wavelength modes with $k^2\lambda_D^2 \ll 1$ and $k^2a^2 \ll 1$, physics is intact.

The need of a quasineutral simulation model w/o space charge waves is essential

THE PHYSICS OF FLUIDS VOLUME 11, NUMBER 3 MARCH 1968

Drift Instabilities in General Magnetic Field Configurations

P. H. Rutherford and E. A. Frieman
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received 5 October 1967)

A theory of low-frequency drift (universal) instabilities in a nonuniform collisionless plasma is developed for general magnetic field configurations including trapped particle effects, rather than the plane geometry which has previously received most attention. A type of energy principle shows that the special equilibrium distribution $F(\epsilon, \mu)$, of interest in minimum-B mirror configurations, is absolutely stable to these modes provided $\partial F/\partial \epsilon < 0$ together with a second condition on $\partial F/\partial \mu$. For equilibrium distributions not of this special form, in particular for a Maxwell distribution with a density gradient, the case of axisymmetric toroidal configurations with closed poloidal field lines is considered in detail. Three unstable drift modes are found, a flute-like mode, a drift-ballooning mode local to the region of unfavorable curvature, and a drift-universal mode. Stability criteria and growth rates for the modes are given. The equations also describe a recently discussed low-frequency trapped-particle instability.

Nonlinear gyrokinetic equations for low-frequency electromagnetic waves in general plasma equilibria

E. A. Frieman[a] and Liu Chen
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544
(Received 6 October 1981; accepted 6 January 1982)

A nonlinear gyrokinetic formalism for low-frequency (less than the cyclotron frequency) microscopic electromagnetic perturbations in general magnetic field configurations is developed. The nonlinear equations thus derived are valid in the strong-turbulence regime and contain effects due to finite Larmor radius, plasma inhomogeneities, and magnetic field geometries. The specific case of axisymmetric tokamaks is then considered and a model nonlinear equation is derived for electrostatic drift waves. Also, applying the formalism to the shear Alfvén wave heating scheme, it is found that nonlinear ion Landau damping of kinetic shear-Alfvén waves is modified, both qualitatively and quantitatively, by the diamagnetic drift effects. In particular, wave energy is found to cascade in wavenumber instead of frequency.

Plasma Physics, Vol. 20, pp. 719 to 722

0032-1028/78/0701-0719$02.00/0

P. J. Catto

Linearized gyro-kinetics

(*Received 5 December* 1977)

Abstract—Finite gyroradius effects are retained in a far simpler manner than previous treatments by transforming to the guiding center variables and gyro-averaging *before* introducing magnetic coordinates.

Phys. Fluids, Vol. 26, No. 2, February 1983

Gyrokinetic approach in particle simulation

W. W. Lee
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(Received 16 October 1981; accepted 20 October 1982)

A new scheme for particle simulation based on the gyrophase-averaged Vlasov equation has been developed. It is suitable for studying linear and nonlinear low-frequency microinstabilities and the associated anomalous transport in magnetically confined plasmas. The scheme retains the gyroradius effects but not the gyromotion; it is, therefore, far more efficient than conventional ones. Furthermore, the reduced Vlasov equation is also amenable to analytical studies.

Phys. Fluids, Vol. 26, No. 2, February 1983 W. W. Lee

$$\frac{\partial\langle F\rangle}{\partial t} + \mathbf{v}_{\parallel}\cdot\frac{\partial\langle F\rangle}{\partial \mathbf{R}} - \frac{q}{m}\frac{1}{\Omega}\frac{\partial\langle\Psi\rangle}{\partial\mathbf{R}}\times\hat{\mathbf{b}}\cdot\frac{\partial\langle F\rangle}{\partial\mathbf{R}} - \frac{q}{m}\frac{\partial\langle\Psi\rangle}{\partial\mathbf{R}}\cdot\hat{\mathbf{b}}\,\frac{\partial\langle F\rangle}{\partial v_{\parallel}} = 0, \quad (21)$$

Nonlinear gyrokinetic Vlasov equation

$$\langle\Psi\rangle \equiv \langle\Phi\rangle + \frac{1}{2}\frac{q}{T}\langle\Phi\rangle^2 - \frac{1}{2}\frac{q}{T}\langle\Phi^2\rangle \simeq \langle\Phi\rangle - \frac{1}{4}\frac{q}{T}\left(\frac{v_\perp}{\Omega}\right)^2\left|\frac{\partial\Phi}{\partial\mathbf{R}_\perp}\right|^2.$$

$$\nabla^2\Phi - k_{Di}^2(n_i/n_0)(\Phi - \tilde{\Phi}) = -4\pi e(n_i - n_e), \quad (37)$$

Gyrokinetic Poisson's equation

$$-k_{Di}^2(\Phi - \tilde{\Phi}) \sim (\omega_{pi}^2/\Omega_i^2)\nabla_\perp^2\Phi,$$

Density response due to polarization drift

JOURNAL OF COMPUTATIONAL PHYSICS 72, 243–269 (1987)

Gyrokinetic Particle Simulation Model

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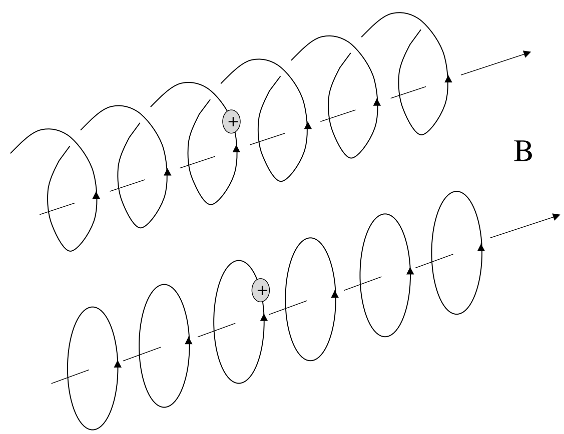


77 Phys. Fluids B 5 (1), January 1993

A fully nonlinear characteristic method for gyrokinetic simulation

S. E. Parker and W. W. Lee
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

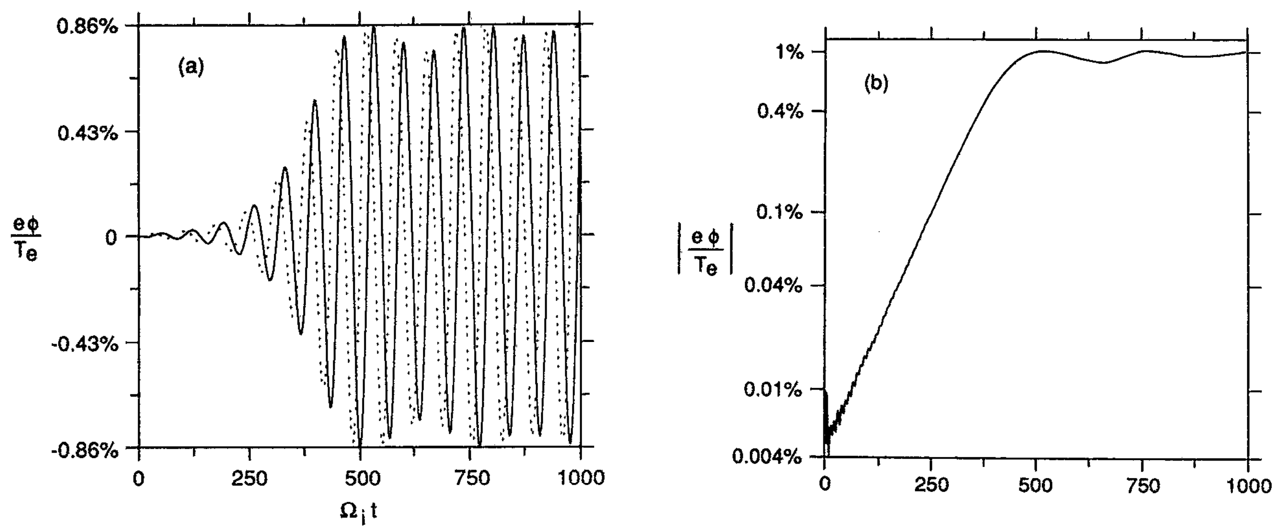
$$F = F_0 + \delta f$$

$$F(\mathbf{x}, \mathbf{v}, t) = \sum_{j=1}^{N} \delta[\mathbf{x} - \mathbf{x}_j(t)]\delta[\mathbf{v} - \mathbf{v}_j(t)]$$

$$\delta f(\mathbf{x}, \mathbf{v}, t) = \sum_{j=1}^{N} w_j(t)\delta[\mathbf{x} - \mathbf{x}_j(t)]\delta[\mathbf{v} - \mathbf{v}_j(t)]$$

$$w = \delta f / F$$

δf simulation for
Noise Reduction



COMPUTATIONAL SCIENCE&DISCOVERY

Nonlinear turbulent transport in magnetic fusion plasmas

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Published 10 December 2008
Computational Science & Discovery **1** (2008) 015010 (14pp)
doi:10.1088/1749-4699/1/1/015010

1. Steady state thermal flux is much smaller than the quasilinear value

2. Formation of global zonal flows

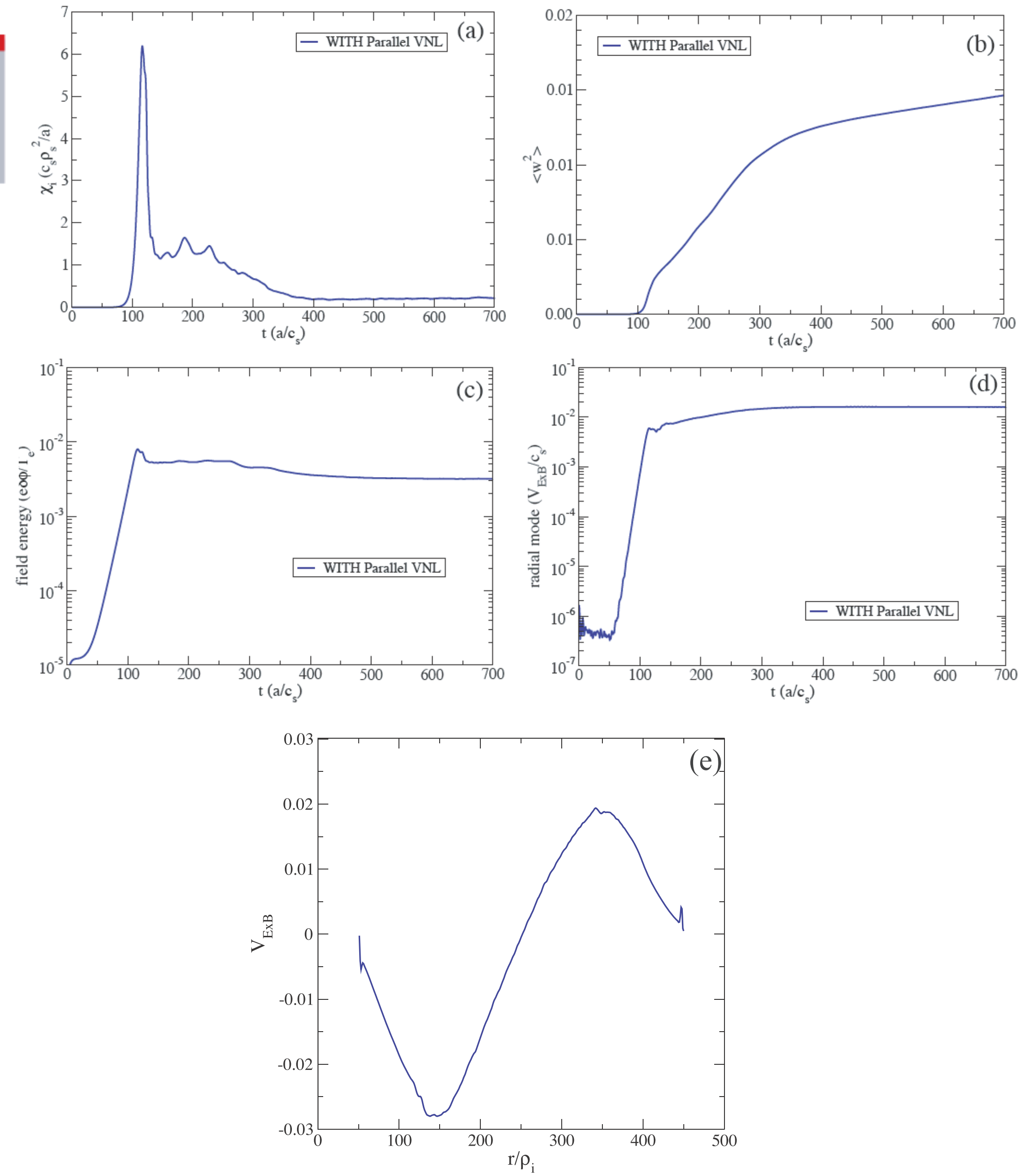

Figure 5. Time evolution of (a) the ion thermal flux, (b) the particle weights, (c) the field energy, and (d) the radial modes, as well as (e) the zonal flow structure for $a/\rho = 500$ including both the nonlinearly generated zonal flows and the velocity space nonlinearity (VNL).

INSTITUTE OF PHYSICS PUBLISHING and INTERNATIONAL ATOMIC ENERGY AGENCY NUCLEAR FUSION

Nucl. Fusion **46** (2006) 24–32 doi:10.1088/0029-5515/46/1/003

• Velocity–space structures of distribution function in toroidal ion temperature gradient turbulence

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- For discretization of the velocity space with 1025 x 65 grid points for

$$-5v_{ti} < v_{\|} < 5v_{ti} \qquad \mu < 12.5v_{ti}^2/\Omega_i$$

- For collisions

$$\nu_{ii} = 0.001 v_{ti}/L_n$$

- Precipitous drop of thermal diffusivity after saturation is apparent.

- Higher thermal flux level due to collisions

• Global PIC and high resolution flux-tube Vlasov simulations give rise to similar results

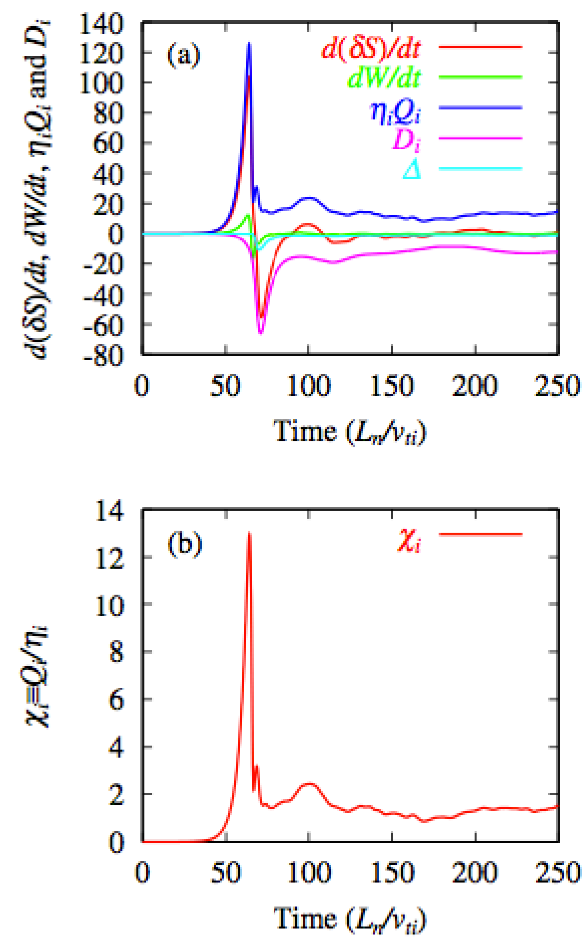


Figure 5. Time-evolutions of (*a*) the entropy balance and (*b*) the ion thermal transport coefficients χ_i obtained by the toroidal ITG turbulence simulation.

Applications to tokamak transport physics based on the two-weight scheme (δf to full-F)

[Lee, Jenkins, and Ethier, CPC 2011; Ganesh, Ethier and Lee, ICPP, 2014]

• Governing equations of motion with electrostatic approximation:

$$\frac{d\mathbf{R}}{dt} = v_{\|}\hat{\mathbf{b}} + \mathbf{v}_d - \frac{\partial\bar{\phi}}{\partial\mathbf{R}} \times \hat{\mathbf{b}}, \qquad \frac{dv_{\|}}{dt} = -\hat{\mathbf{b}} \cdot \left(\frac{v_\perp^2}{2}\frac{\partial}{\partial\mathbf{R}} lnB\right) - \hat{\mathbf{b}}^* \cdot \frac{\partial\bar{\phi}}{\partial\mathbf{R}}$$

$$\mu_B \equiv \frac{v_\perp^2}{2B} \approx const., \qquad \mathbf{v}_d \approx \left(v_\|^2 + \frac{v_\perp^2}{2}\right)\hat{\mathbf{b}} \times \frac{\partial}{\partial\mathbf{R}} lnB$$

Similar to Hu and Krommes (PoP ’94)

$$\hat{\mathbf{b}}^* \approx \hat{\mathbf{b}} + v_\|\hat{\mathbf{b}} \times \frac{\partial}{\partial\mathbf{R}} lnB$$

• The weight equations:

$$\frac{dw}{dt} = (p-w)\left[(\mathbf{v}_d + \mathbf{v}_{E\times B})\cdot\kappa + \frac{T_e}{T_i}(v_\|\hat{\mathbf{b}} + \mathbf{v}_d)\cdot\mathbf{E}\right]$$

-- Ramping up $\mathbf{v}_d$ associated with inhomogeneity slowly at the beginning of the simulation to avoid numerical problem

$$\frac{dp}{dt} = (p-w)(\mathbf{v}_d + \mathbf{v}_{E\times B})\cdot\kappa \qquad \frac{d(p-w)}{dt} = -(p-w)\frac{T_e}{T_i}(v_\|\hat{\mathbf{b}} + \mathbf{v}_d)\cdot\mathbf{E}$$

• delta-f to full-F schemes - entropy weighting:

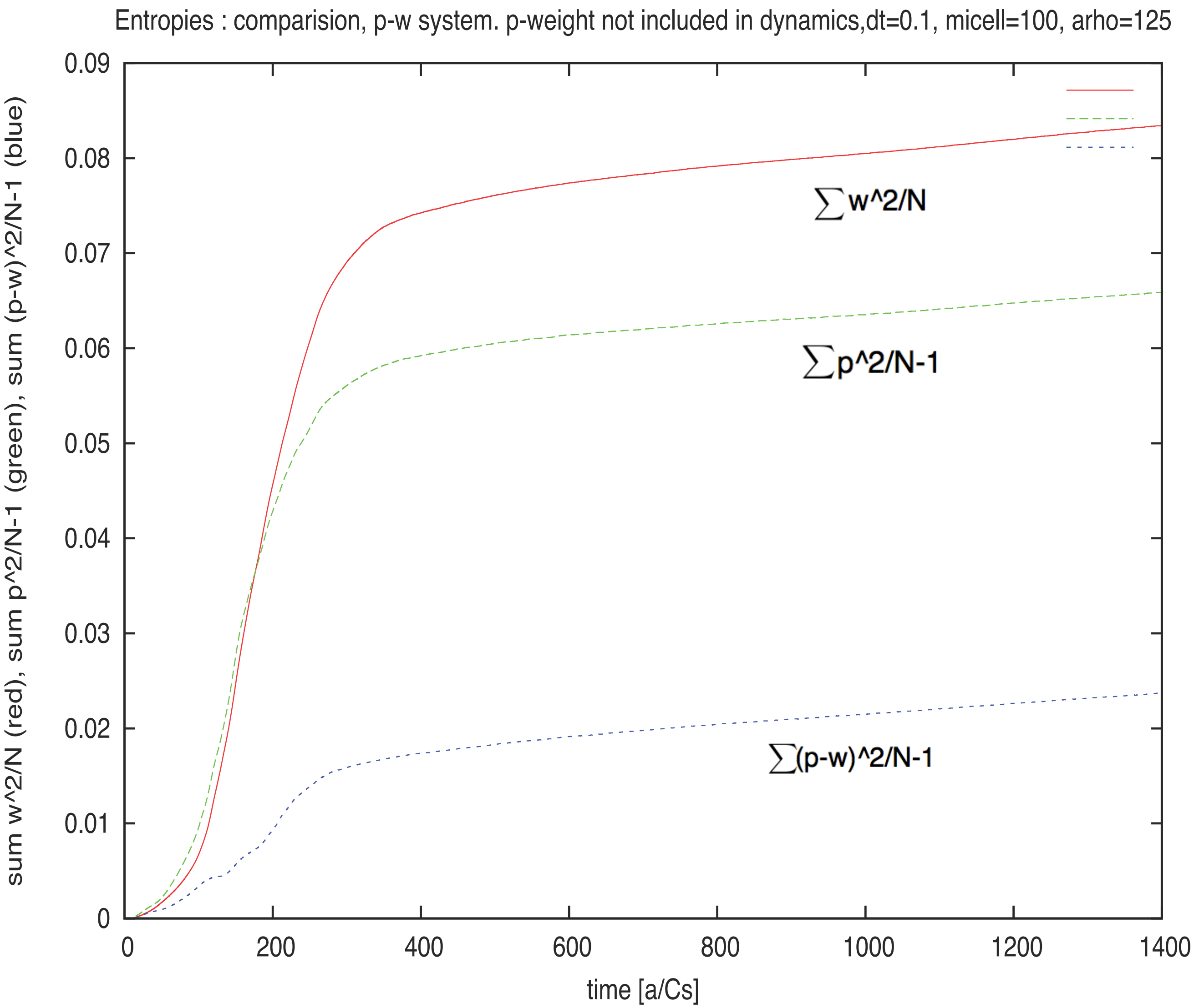
$$\rho = e\sum_j [\alpha_1 p_j + \alpha_2 w_j]\delta(\mathbf{x} - \mathbf{x}_j)]$$

$$\alpha_1 + \alpha_2 = 1$$

$$\alpha_1 = \frac{|w_j|^2}{|p_j|^2 + |w_j|^2} \qquad \alpha_2 = \frac{|p_j|^2}{|p_j|^2 + |w_j|^2}$$

• Time history of the weights in delta-f simulation

$$\frac{a}{\rho_s} = 125$$



w-weight: related to thermal diffusivity

p-weight: related to imhomogeneity

(p-w)-weight: related to wave-particle interaction

1. Solve dynamics for both p and w weight.
2. No p weight in the poisson equation
3. Calculate sum w^2/N, sum p^2/N and sum (p-w)^2/N.
4. v_D term included in the dynamics

Understanding of steady state thermal flux

• From the plots here for the time rate of change for w^2, p^2 and $(p-w)^2$ in the saturated state, we can made the following observations:

-- p^2 is increasing in time, which means the drift terms associated the inhomogeneity contribute in the NL stage.

-- $(p-w)^2$ is increasing in time, which, deducing from $dv_\parallel^2 /dt$ term, means $v_\parallel$ is losing energy in the saturated state

-- w^2 is increasing in time in the saturated state, which is the result of radial diffusion and energy exchange between the waves and particle parallel velocity.

• Thus, the non vanishing energy flux in the saturated state is the result of radial diffusion and the energy exchange between the particles and the waves in this collisionless case. How to access this level of energy flux is a theoretical challenge.

• These levels of the energy flux are orders of magnitude lower than their quasilinear values as shown in these figures.

THE PHYSICS OF FLUIDS VOLUME 11, NUMBER 12 DECEMBER 1968

Nonlinear Theory of Low-Frequency Instabilities

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Department of Physics, Department of Nuclear Engineering,
and
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 11 April 1968)

It is proposed that the dominant nonlinear effect of low-frequency instabilities is an incoherent scattering of particle orbits by waves. The scattering is considered to be incoherent even when only a single mode is present. Because it is incoherent, one can include wave scattering in the theory just as collisional scattering is included in the linear wave theory. A nonlinear dispersion relation is obtained from the linear dispersion relation simply by adding wave scattering terms to the collisional scattering terms. The wave scattering causes particle diffusion over the transverse wavelength and in the theory appears as an enhanced viscosity. This wave induced viscosity can be computed from the amplitude of the wave. Waves which are linearly unstable grow until the enhanced scattering causes their nonlinear growth rates to vanish. Theoretically computed values of density fluctuations, phase shifts, and flux are in good agreement with experimental values in the cases considered.

- Prof. Dupree may be right after all, since particle trapping and scattering are similar physical processes !

- PIC is the most efficient way to capture this type of physics on present-day high performance computers.

Darwin Electromagnetic (finite-β) Gyrokinetic Equations

- Original Vlasov Equation $F \equiv F(\mathbf{x}, \mathbf{v}, \mathbf{t})$

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[\mathbf{E} + \frac{1}{c}\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B})\right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} = 0$$

$$\mathbf{E} = -\nabla\phi - (1/c)\partial\mathbf{A}/\partial t \qquad \delta\mathbf{B} = \nabla \times \mathbf{A}$$

- Using the Lagragian of $L = \frac{1}{2}mv^2 - q\phi + \frac{q}{c}\mathbf{v} \cdot \mathbf{A}$ to obtain

(see, for example, Corben and Stahle, 1966)

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v} \cdot \mathbf{A}) + \frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right] \cdot \frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha \mathbf{A}/m_\alpha c)} = 0$$

- Using $\mathbf{v} \rightarrow \mathbf{v} + \frac{q_\alpha}{m_\alpha c}\mathbf{A}_\perp$

to re-write it as

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v}_\perp \cdot \mathbf{A}_\perp) - \frac{1}{c}\frac{\partial \mathbf{A}_\parallel}{\partial t} + \frac{1}{c}\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B}_\perp)\right] \cdot \frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha \mathbf{A}_\perp/m_\alpha c)} = 0$$

$$F \equiv F(\mathbf{x}, v_\parallel, \mu/B, t)$$

$$\mathbf{v} \approx v_{\|}\mathbf{b} + \frac{c}{B_0}\mathbf{E}\times\mathbf{b}$$

$$\mathbf{E} = -\nabla(\phi - \mathbf{v}_\perp \cdot \mathbf{A}_\perp/c) - (1/c)\partial \mathbf{A}_{\|}/\partial t$$

$$\mathbf{b} = \hat{\mathbf{b}}_0 + \delta\mathbf{B}_\perp/B_0 \qquad \hat{\mathbf{b}}_0 = \mathbf{B}_0/B_0 \qquad \delta\mathbf{B}_\perp = \nabla\times\mathbf{A}_{\|}$$

- Gyrokinetic Vlasov Equation

$$\frac{\partial F_\alpha}{\partial t} + \left[v_{\|}\mathbf{b} - \frac{c}{B_0}\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp})\times\hat{\mathbf{b}}_0\right]\cdot\frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m}\left[\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp})\cdot\mathbf{b} + \frac{1}{c}\frac{\partial \overline{A}_{\|}}{\partial t}\right]\frac{\partial F_\alpha}{\partial v_{\|}} = 0$$

$$\nabla^2\phi + \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\phi = -4\pi\sum_\alpha q_\alpha\int \overline{F}_\alpha dv_{\|}d\mu$$ -- for $k_\perp^2\rho_i^2 \ll 1$

$$\nabla^2\mathbf{A} - \underline{\frac{1}{v_A^2}\frac{\partial \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\sum_\alpha q_\alpha\int \mathbf{v}\overline{F}_\alpha dv_{\|}d\mu$$ Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\mu = v_\perp^2/2 \qquad \mathbf{v}_p^L = -(mc^2/eB^2)(\partial\nabla_\perp\phi/\partial t) \qquad \mathbf{v}_p^T = -(mc/eB^2)(\partial^2\mathbf{A}_\perp/\partial^2 t)$$

- Energy Conservation

$$\frac{d}{dt}\left\langle \int (\frac{1}{2}v_{\|}^2 + \mu)(m_eF_e + m_iF_i)dv_{\|}d\mu + \frac{\omega_{ci}^2}{\Omega_i^2}\frac{|\nabla_\perp\Phi|^2}{8\pi} + \frac{|\nabla A_{\|}|^2}{8\pi}\right\rangle_{\mathbf{x}} = 0$$

$$\Phi \equiv \phi - \overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp}/c$$

- Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic, PhD thesis (2010)]

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt} \frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0} \nabla \bar{\Phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2} \mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_\alpha}{m_\alpha} \left(\mathbf{b}^* \cdot \nabla \bar{\Phi} + \frac{1}{c} \frac{\partial \bar{A}_\parallel}{\partial t} \right)$$

$$\Omega_{\alpha 0} \equiv q_\alpha B_0 / m_\alpha c$$

$$\bar{\Phi} \equiv \bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} / c \qquad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} = -\frac{1}{2\pi} \frac{eB_0}{mc} \int_0^{2\pi} \int_0^{\rho} \delta B_\parallel r dr d\theta$$

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

$$F_\alpha = \sum_{j=1}^{N_\alpha} \delta(\mathbf{R} - \mathbf{R}_{\alpha j}) \delta(\mu - \mu_{\alpha j}) \delta(v_\parallel - v_{\parallel \alpha j})$$

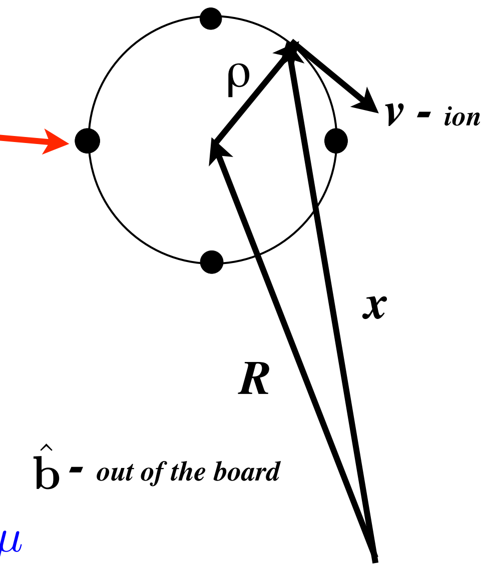
- Including only the parallel vector potential, Startsev et al. have studied low (m,n) as well as high (m,n) tearing modes [APS 2004, Sherwood 2005] using GTS [Wang et al., PoP 2003].

Gyrokinetic Current Densities

[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\|gc}(\mathbf{x}) + \boxed{\mathbf{J}^M_{\perp gc}(\mathbf{x})} + \mathbf{J}^d_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_\| + \mathbf{v}_\perp + \mathbf{v}_d)\delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_\| d\mu \rangle_\varphi$$



FLR calculation

$$\mathbf{v}_d = \frac{v_\|^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$

$$\mathbf{J}^M_{\perp gc}(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp} \qquad p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2) F_{\alpha gc}(\mathbf{x}) dv_\| d\mu$$

$$\mathbf{J}^d_{\perp gc} = \frac{c}{B}\sum_\alpha \left[p_{\alpha\|}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB) \right] \qquad p_{\alpha\|} = m_\alpha \int v_\|^2 F_{\alpha gc}(\mathbf{x}) dv_\| d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^M_{\perp gc} + \mathbf{J}^d_{\perp gc} \quad = \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\|} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp \right]$$

$$p_\alpha = p_{\alpha\|} = p_{\alpha\perp}$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_\alpha}$$

- **Gyrokinetic MHD Equations:** a reduced set of equations but in full toroidal geometry

-- For $k_\perp^2 \rho_i^2 \ll 1$ $\quad \bar{F} \to F \quad \bar{\phi} \to \phi \quad \bar{A}_\parallel \to A_\parallel \quad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \to 0$

-- Ampere's law

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} J_\parallel$$

$$\nabla_\perp^2 \mathbf{A}_\perp - \underline{\frac{1}{v_A^2} \frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c} \mathbf{J}_\perp$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\delta \mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Pressure Driven Current: $\mathbf{J}_\perp = \frac{c}{B} \sum_\alpha \mathbf{b} \times \nabla p_\alpha$

-- Vorticity Equation: $\frac{d}{dt} \nabla_\perp^2 \phi - 4\pi \frac{v_A^2}{c^2} \nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0 \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B} \nabla \phi \times \mathbf{b} \cdot \nabla$

-- Ohm's law: $E_\parallel \equiv -\frac{1}{c} \frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla \phi = \eta J_\parallel \to 0$

-- Equation of State: $\frac{dp_\alpha}{dt} = 0$

-- Normal modes: $\omega = \pm k_\parallel v_A$

• MHD Equilibrium

1. For a given pressure profile, we obtain the pressure driven current from

$$\boxed{\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b}\times\nabla p_\alpha}$$

2. We then solve the coupled equations of

$$\boxed{\begin{aligned} &\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\mathbf{b}\cdot\nabla)\nabla_\perp^2 A_\parallel - 4\pi\frac{v_A^2}{c^2}\nabla\cdot\mathbf{J}_\perp = 0 \\ &E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0 \end{aligned}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\dfrac{\partial A_\parallel}{\partial t} \to 0$,

this is then the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0}$$

4. The GK vorticity equation retains all the toroidal physics, different than Strauss' equation [PF 77]

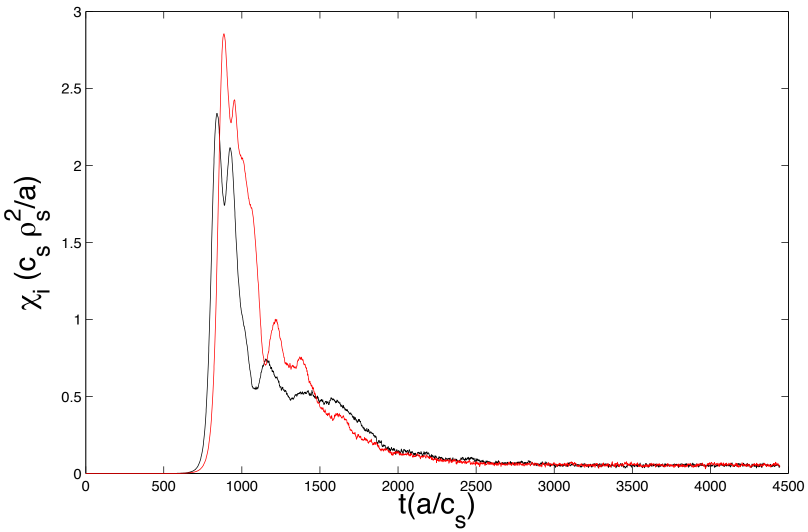
5. Perpendicular current is consisted of both a divergent free diamagnetic current and a magnetic drift current. Only the latter was originally included in Lee and Qin [PoP, 2003].

Linear Upshift of ITG Gradient due to Equilibrium Ion Density

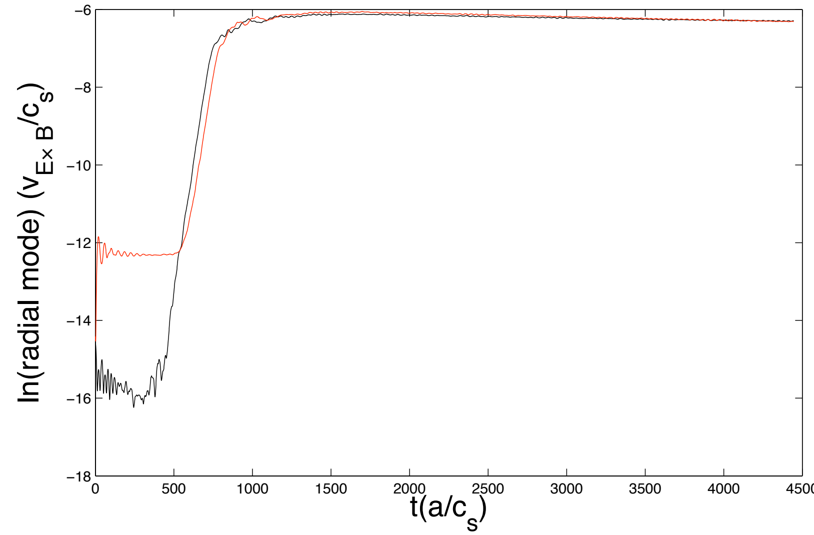
• Gyrophase-Averaged Equilibrium Ion Number Density [W. W. Lee and R. A. Kolesnikov, PoP 2008]

$$\bar{n}(\mathbf{x}) = n + \underline{\frac{1}{2}\rho_t^2 \frac{1}{T}\nabla_\perp^2 nT}$$

• ITG simulation using GTC (a/rho =125)



Heat Flux with (red) and without (black) the equilibrium density term



Zonal flow amplitude with (red) and without (black) the equilibrium density term

-- Dimits Shift is nonlinear [PoP 7, 969 (2000)]

-- This shift is linear

GTC: Lin et al., Science ’98

Gyrokinetic Pressure Force Balance

$$\bar{\mathbf{J}}_\perp = \frac{c}{B}\hat{\mathbf{b}} \times \nabla p_\perp + qn\mathbf{v}_{E\times B} + qn\frac{\rho_t^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{nT}\nabla_\perp^2 nT\right]$$

$$p_\perp = m\int \mu F_{gc}(\mathbf{x})dv_\parallel d\mu$$

$$\frac{\mathbf{V}_{E\times B}}{c_s} = \frac{1}{2}\kappa_{pi}\rho_s\frac{T_i}{T_e}\hat{\mathbf{b}} \times \hat{\mathbf{x}} \qquad \kappa_{pi}\rho_s \equiv -\rho_s\frac{\nabla p_i}{p_i}$$

$$\mathbf{J} = \frac{c}{B}\hat{\mathbf{b}} \times \nabla(p_{\perp e} + p_{\perp i}) + en_i\frac{\rho_i^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{n_iT_i}\nabla_\perp^2 n_iT_i\right]$$

modification by equilibrium zonal flows

$$\mathbf{J} = \frac{c}{B}\hat{\mathbf{b}} \times \nabla\left(p_{\perp e} + p_{\perp i} - \frac{1}{2}\rho_i^2\nabla_\perp^2 p_{\perp i}\right)$$

- Conventional MHD Equations:

-- Continuity Equation: $$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \rho_m \mathbf{V} = 0$$

-- Momentum Equation: $$\rho_m \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p$$

-- Ohm's Law: $$\mathbf{E} + \frac{1}{c} \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}$$

-- Faraday's law $$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

-- Ampere's law: $$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

-- Normal Modes: $\omega = \pm k v_A$ $\omega = \pm k_{\parallel} v_A$

Pioneering work on GK MHD based on the ordering of $\rho/L \sim \omega/\Omega \sim o(\epsilon)$

The Boltzmann equation and the one-fluid hydromagnetic equations in the absence of particle collisions

By G. F. Chew, M. L. Goldberger and F. E. Low
Los Alamos Scientific Laboratory

(*Communicated by S. Chandrasekhar, F.R.S.—Received 9 March* 1956)

Starting from the Boltzmann equation for a completely ionized dilute gas with no inter-particle collision term but a strong Lorentz force, an attempt is made to obtain one-fluid hydromagnetic equations by expanding in the ion mass to charge ratio. It is shown that the electron degrees of freedom can be replaced by a macroscopic current, but true hydrodynamics still does not result unless some special circumstance suppresses the transport of pressure along magnetic lines of force. If the longitudinal transport of pressure is ignored, a set of self-contained one-fluid hydromagnetic equations can be found even though the pressure is not a scalar.

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Higher-Order Corrections to the Chew–Goldberger–Low Theory

Edward Frieman, Ronald Davidson, and Bruce Langdon
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(Received 18 January 1966)

A complete higher-order set of equations of the Chew–Goldberger–Low theory has been derived. A slight modification of these equations produces the finite Larmor-radius hydromagnetic theory of Kennel and Greene. Some comments on the properties of these equations are given.

Discussions

• Both particle simulation and gyrokinetics started at PPPL in the sixties.

• Nonlinear gyrokinetic equations were developed in the eighties also at PPPL

• First PIC codes for studying microturbulence in tokamaks were products from PPPL [Parker et al. PRL 71, 2042 (1993), Lin et al. Science 281, 1835 (1998).

• Global GK PIC efforts at PPPL for studying turbulence: GTC-P, GTS, XGC.

• Fully electromagnetic physics in GTS is progress [Startsev et al., Sherwood Conference (2015)]

• How feasible is GK PIC for studying MHD physics?

• How useful are the gyrokinetic MHD equations?

• A white paper entitled “A Multiphysics and Multiscale Coupling of Microturbulence with MHD Equilibria,” by Lee, Startsev, Hudson, Wang and Ethier has been submitted to DoE’s Workshop on Integrated Simulations. The purpose is to use GTS and SPEC together in an attempt to minimize turbulence and transport. This approach is based on an iterative procedure, which first “decouple” the transport problem from the equilibrium problem, and then to “couple” them through global parameter exchanges.