MHD modes in NSTX

Roscoe White, Nikolai Gorelenkov

Straight field line Boozer coordinates ψ_p, θ, ζ, with $d\zeta/d\theta = q(\psi)$
Normalized parallel velocity $\rho_\parallel = v_\parallel/B$
Covariant representation $\vec{B} = g(\psi_p)\nabla\zeta + I(\psi_p)\nabla\theta + \delta(\psi_p, \theta)\nabla\psi_p$

Canonical momentum, toroidal flux ψ, $d\psi/d\psi_p = q$

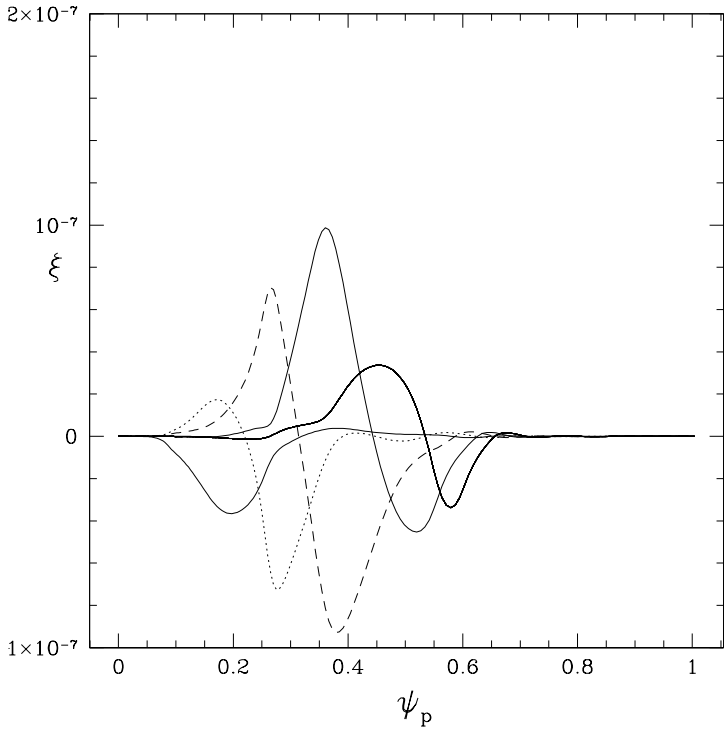
$$P_\zeta = g\rho_\parallel - \psi_p, \qquad P_\theta = \psi + \rho_\parallel I,$$

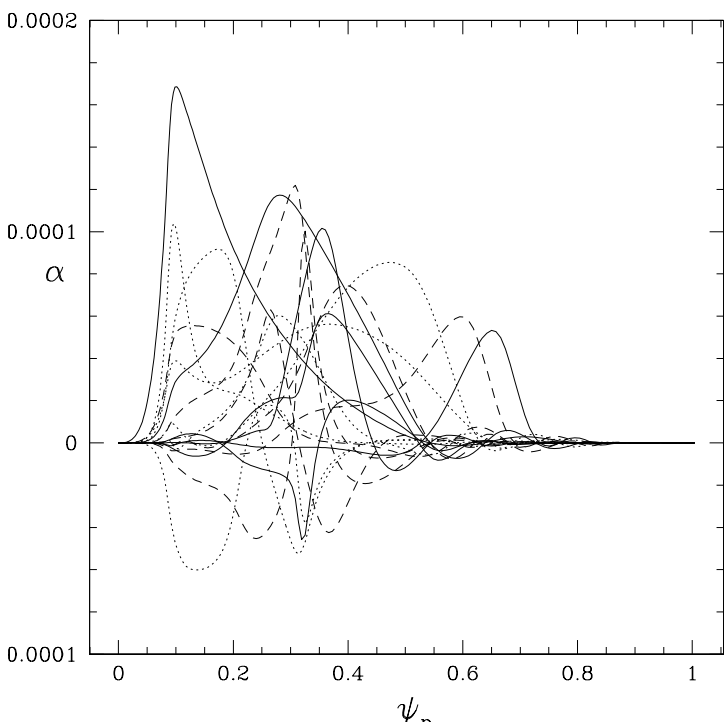
Hamiltonian $H = \rho_\parallel^2 B^2/2 + \mu B + \Phi$,

$$\dot{\theta} = \frac{\partial H}{\partial P_\theta} \qquad \dot{P}_\theta = -\frac{\partial H}{\partial \theta}$$
$$\dot{\zeta} = \frac{\partial H}{\partial P_\zeta} \qquad \dot{P}_\zeta = -\frac{\partial H}{\partial \zeta}.$$

Perturbation $\delta\vec{B} = \nabla \times \alpha\vec{B}$ and $\alpha = \sum_{m,n} \alpha_{m,n}(\psi_p) sin(\Omega_{mn})$
$\Phi = \sum_{m,n} \phi_{m,n}(\psi_p) sin(\Omega_{mn}), \qquad \Omega_{mn} = n\zeta - m\theta - \omega_n t - \phi_n.$

Mode frequency much less than cyclotron frequency, so μ is constant





Typical mode structure in NSTX.
10 modes 38 harmonics

$$\alpha = \sum_{m,n} \alpha_{m,n}(\psi) sin(\Omega_{mn}),$$

$\Omega_{mn} = n\zeta - m\theta - \omega_n t - \phi_n$.
Electric potential Φ to cancel the parallel electric field induced by $d\vec{B}/dt$, with

$$\sum_{m,n} \omega B \alpha_{m,n} cos(\Omega_{mn}) - \vec{B} \cdot \nabla\Phi/B = 0,$$

Φ gives zero parallel electric field,

$$(gq + I)\omega\alpha_{mn} = (nq - m)\Phi_{mn}.$$

α related to the ideal displacement $\vec{\xi}$.

$$\alpha_{mn} = \frac{(m/q - n)}{(mg + nI)} \xi^{\psi}_{mn}.$$

Resonance Determination - Kinetic Poincaré Plot

Single n, Hamiltonian a function of the combination $n\zeta - \omega t$,

then from $\dot{P}_\zeta = -\frac{\partial H}{\partial \zeta}$ and $\frac{dE}{dt} = \frac{\partial H}{\partial t}$, with $P_\zeta = g\rho_{\|} - \psi_p$

Fixed n, μ we have $E - P_\zeta \omega/n = constant$ in time - Energy in mode frame

Kinetic Poincaré plot - plot of positions of orbits in the P_ζ, θ plane

at times when $n\zeta - \omega t = 2\pi k$, k integer

All particles having the same value of $E - P_\zeta \omega/n$ and μ

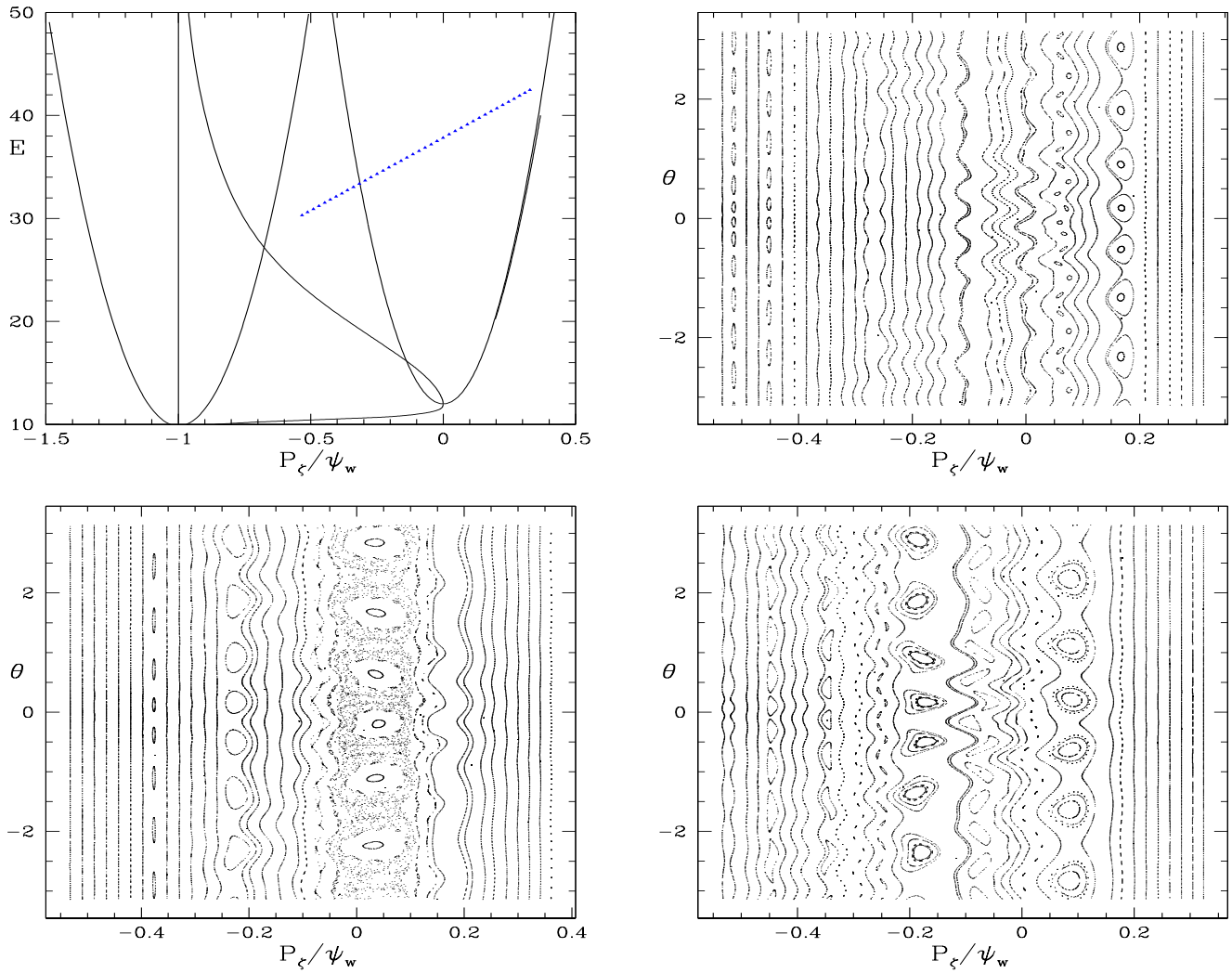
This plot shows mode-particle resonances along $E - P_\zeta \omega/n = constant$

Poincaré plot in the P_ζ, θ or E, θ or ψ_p, θ plane.
Given P_ζ, θ, E is determined , so also $\rho_{\|}$ and ψ_p.
P_ζ is constant without perturbation, but perturbation is function of ψ_p.

Poincaré plots made with 500 transits, or about 0.5 msec

Without resonance there is no irreversible energy transfer

Fine scale mixing produced by rotation in island a function of radius



$$\alpha = \sum_{m,n} A_n \alpha_{m,n}(\psi) sin(\Omega_{mn}),$$
$$\Omega_{mn} = n\zeta - m\theta - \omega_n t - \phi_n.$$

Stepping equations for mode, k=particle index, Yang Chen 1998
<>= time average over rapid time scale, a few mode periods

$$\frac{dA_n}{dt} = \frac{-\nu_A^2}{\omega_n} \sum_{k,m} \left\langle \left[\rho_{\|} B^2 \alpha_{mn} - \Phi_{mn}(\psi_p) \right] cos(\Omega_{mn}) \right\rangle - \gamma_d A_n$$

$$\frac{d\phi_n}{dt} = \frac{-\nu_A^2}{\omega_n A_n} \sum_{k,m} \left\langle \left[\rho_{\|} B^2 \alpha_{mn} - \Phi_{mn}(\psi_p) \right] sin(\Omega_{mn}) \right\rangle$$

.

Orbit described by E, P_ζ, μ. Construct a δf formalism in these variables. Usual δf is in $\vec{x}$, v, μ. After some algebra

$$\dot{E} = -\rho_{\|} B^2 \partial_t \alpha + \partial_t \Phi, \qquad \dot{P}_\zeta = \rho_{\|} B^2 \partial_\zeta \alpha - \partial_\zeta \Phi$$

These terms typically smaller than those for stepping $\vec{x}$, v, by 10^4

The distribution is completely described by $f(E, P_\zeta, \mu, t) = f_0 + \delta f$

$$\delta f(E, P_\zeta, /mu, t) = \sum w_j \delta(E - E_j(t)) \delta(P_\zeta - P_{\zeta,j}(t)) \delta(\mu - \mu_j(t))$$

Markers

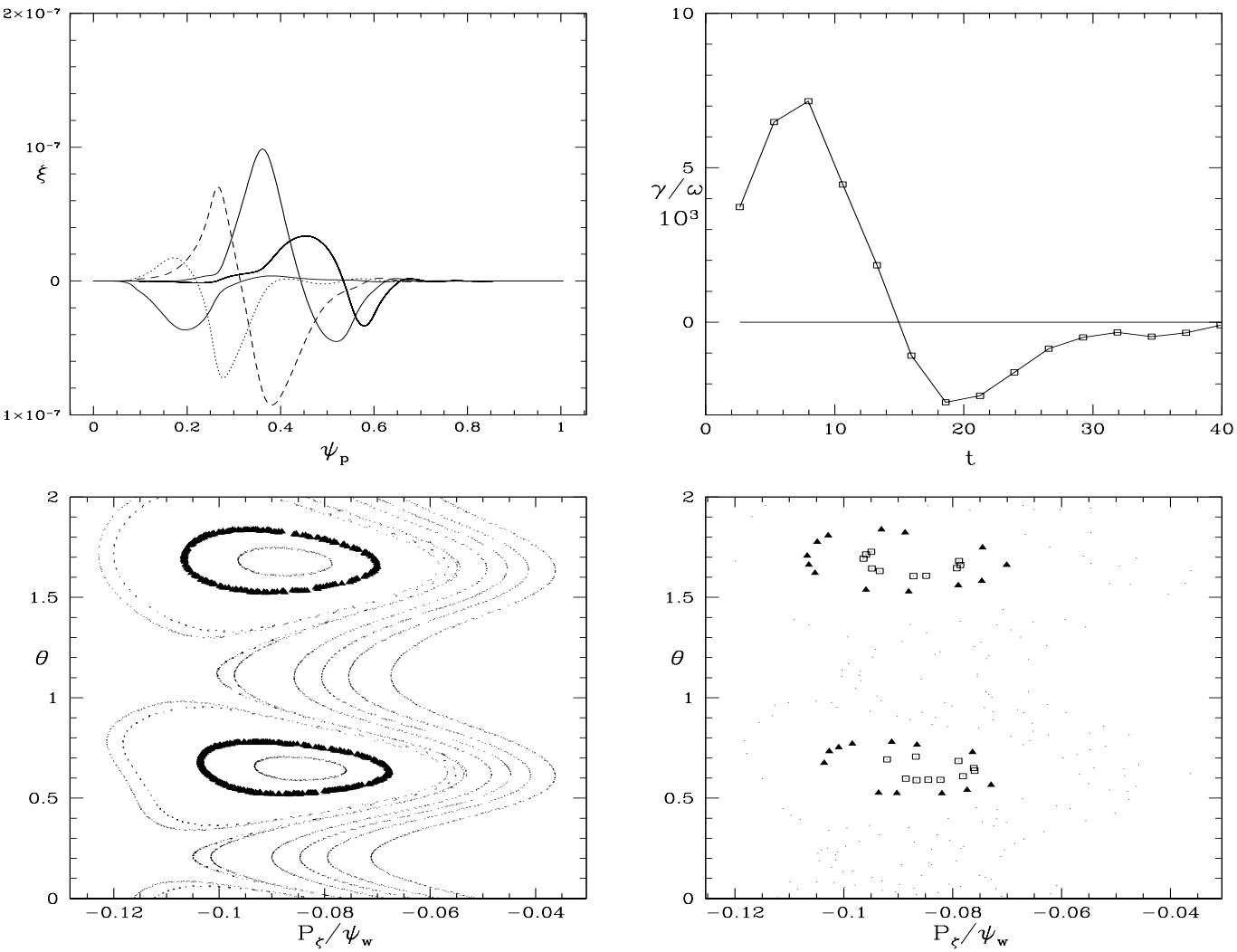
$$g(E, P_\zeta, \mu, t) = \sum_j \delta(E - E_j(t)) \delta(P_\zeta - P_{\zeta,j}(t)) \delta(\mu - \mu_j(t))$$

We then find an equation for the time evolution of the weights w_j

$$g \frac{dw_j}{dt} = -\dot{E}_j \partial_E f_0 - \dot{P}_{\zeta,j} \partial_{P_\zeta} f_0 - \dot{\mu}_j \partial_\mu f_0$$

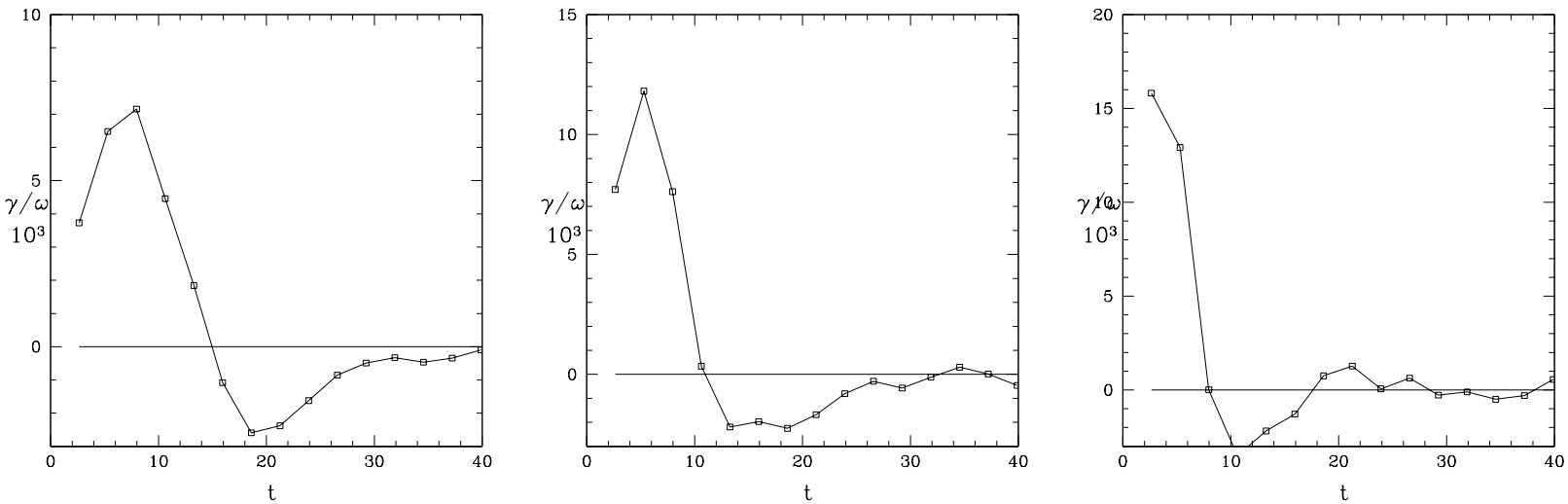
Driving terms $\partial_E f_0$, $\partial_{P_\zeta} f_0$, $\partial_\mu f_0$.

n = 5 TAE mode, $7 \leq m \leq 11$, 157 kHz, $\gamma/\omega = 1.2 \times 10^{-5}$
No collisions or source. Saturation in one bounce time
Poincaré plot of resonance with 1000 transits, and 40 transits

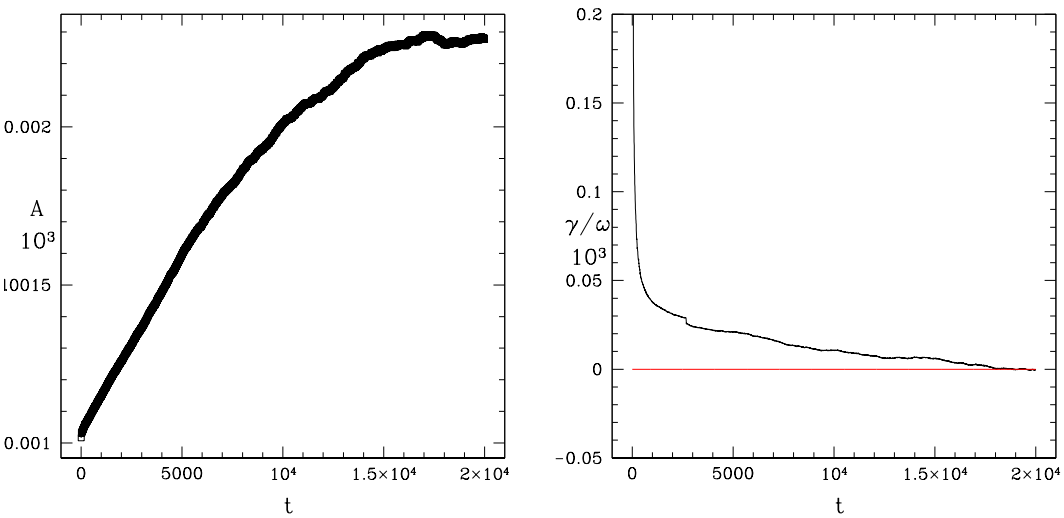


Mixing (bounce) time shortens with island size or amplitude

$A = 10^{-3}$, $A = 2 \times 10^{-3}$, $A = 4 \times 10^{-3}$

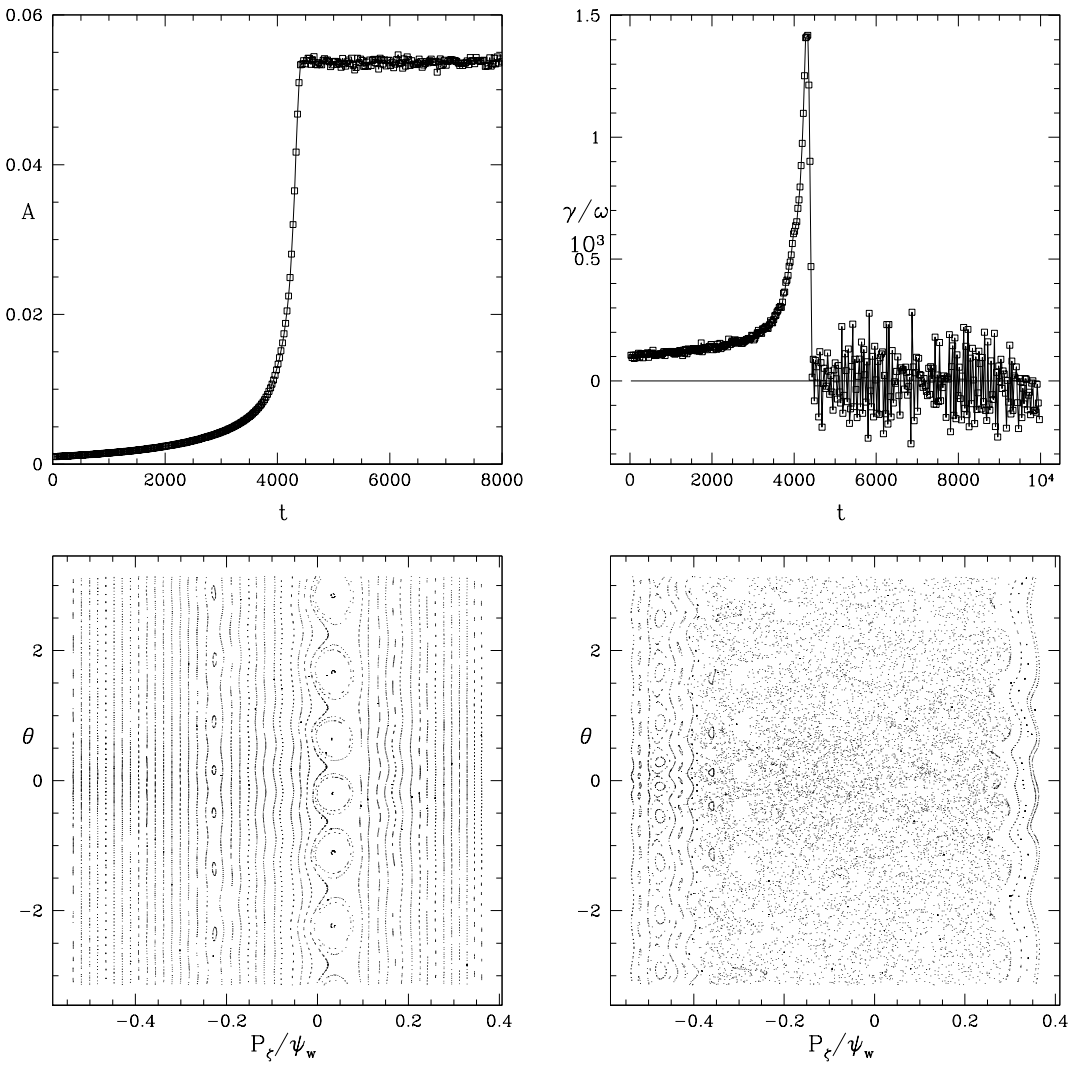


Steady state distribution, with collisions and slowing down, maintained by beam injection or alpha production, repopulates islands.
This gives a growth rate, not going to zero in one bounce time
As island increases and bounce time shortens, γ decreases
Initial transient due to particle load, lasts one mixing time
Saturation occurs when deposition rate matches island mixing time



Case with strong repopulation of islands

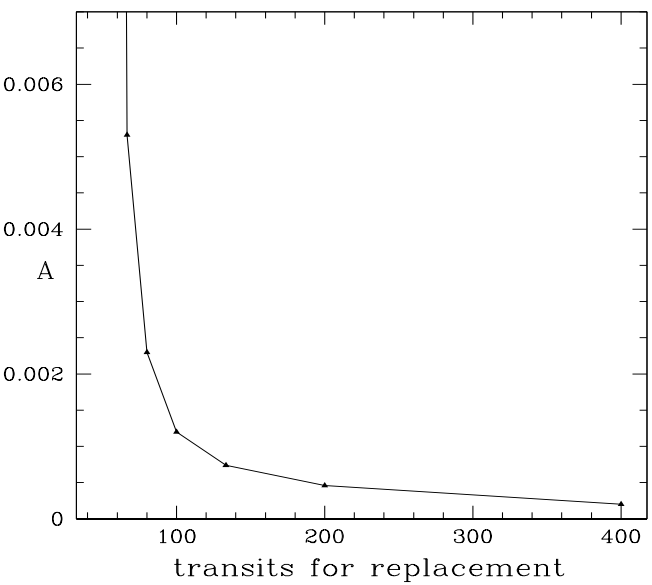
When nearby resonances overlap the mixing rate and γ strongly increase.

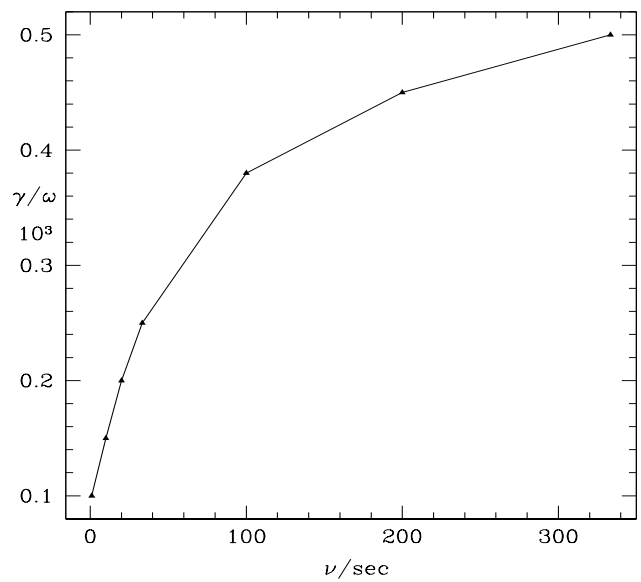


Growth rate and saturation amplitude are determined by:

1. The magnitude of the driving terms $\partial_E f_0$, $\partial_{P_\zeta} f_0$, $\partial_\mu f_0$,
2. The rate of island repopulation due to beam injection rate or alpha production rate
3. Island repopulation due to collisions.

Saturation amplitude and initial γ depend on repopulation and collisions





To Do- Realistic simulations, compare with experiment and models

Thermal Island Destabilization

and the Greenwald Density Limit

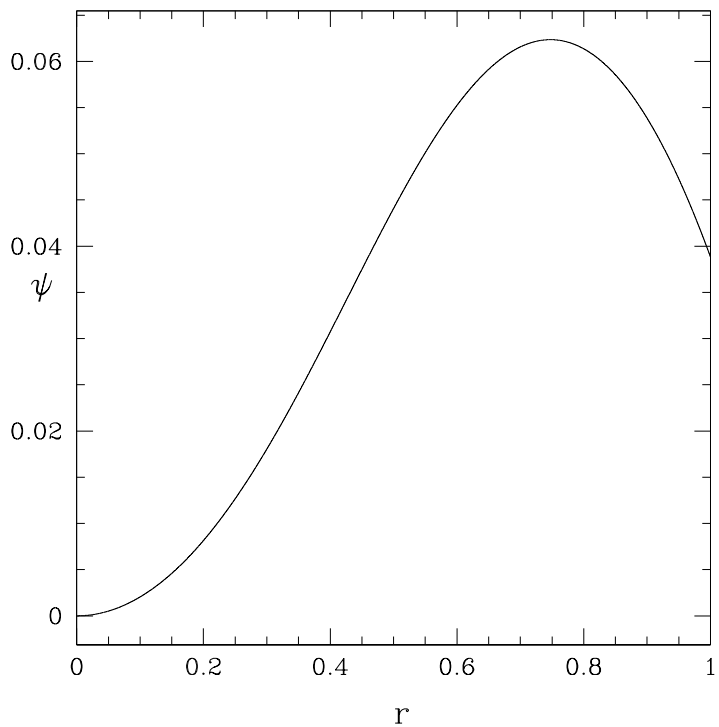
Roscoe White, David Gates, Dylan Brennan, Qian Teng

• Magnetic reconnection is ubiquitous in the magnetosphere, the solar corona, and in toroidal fusion research discharges.

• In a fusion device a magnetic island saturates
at the magnetic energy minimum of the configuration.
Further modification of the current density profile in the island interior causes additional growth or contraction of the saturated island.

• An island is thermally isolated from the outside plasma and can heat or cool depending on the balance of Ohmic heating and radiation loss, changing the resistivity and the current in the island.

• A model of island destabilization due to radiation cooling of the island is constructed. An additional destabilization effect is described, and it is shown that a small imbalance of heating can lead to exponential growth of the island.



Single helicity analysis

Equilibrium helical flux a combination of toroidal ψ_t and poloidal ψ_p fluxes

$$\psi_0(r) = \psi_t - (n/m)\psi_p/2$$

$$\psi_0'(r_s) = 0, \quad q(r_s) = m/n$$

Perturbed helical flux

$$\psi(r) = \psi_0(r) + \psi_1(r)cos(m\theta)$$

$$\nabla_\perp^2 \psi_1 = \frac{dj}{d\psi_0}\psi_1 + \delta j_1.$$

Time evolution for islands larger than the tearing layer

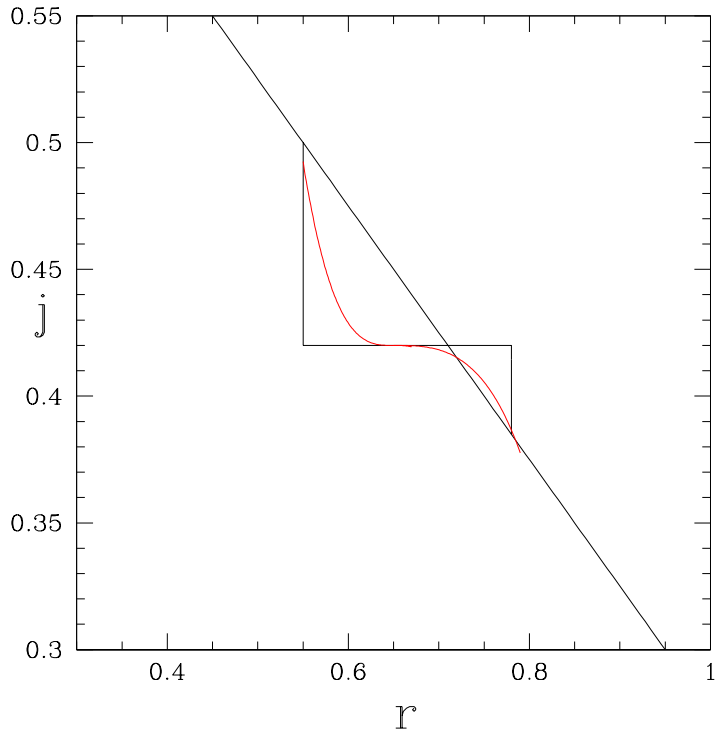
$$\frac{d\psi_1}{dt} = \frac{\eta(r_s)\Delta'(w)\psi_1}{w} - \eta(r_s)\delta j_1$$

With Δ' approximately given using the unperturbed eigenfunction

$$\Delta'(w) = \frac{\psi_1'(r_r) - \psi_1'(r_l)}{\psi_1(r_s)}, \qquad w = r_r - r_l$$

The effect of the current perturbation gives a modification of Δ' by the addition of $\Delta'_{\delta j}$ with

$$\Delta'_{\delta j}(w) = -w\frac{< \delta j_1 >}{\psi_1(r_s)},$$



Island asymmetry $A = (r_r - r_x)/(r_x - r_l) - 1$ destabilizing effect producing a δj.

$$\Delta'_A(w) = f_F \frac{\int [j(r_x) - j(r)] cos(m\theta) d\theta dr}{\psi_1(r_s)},$$

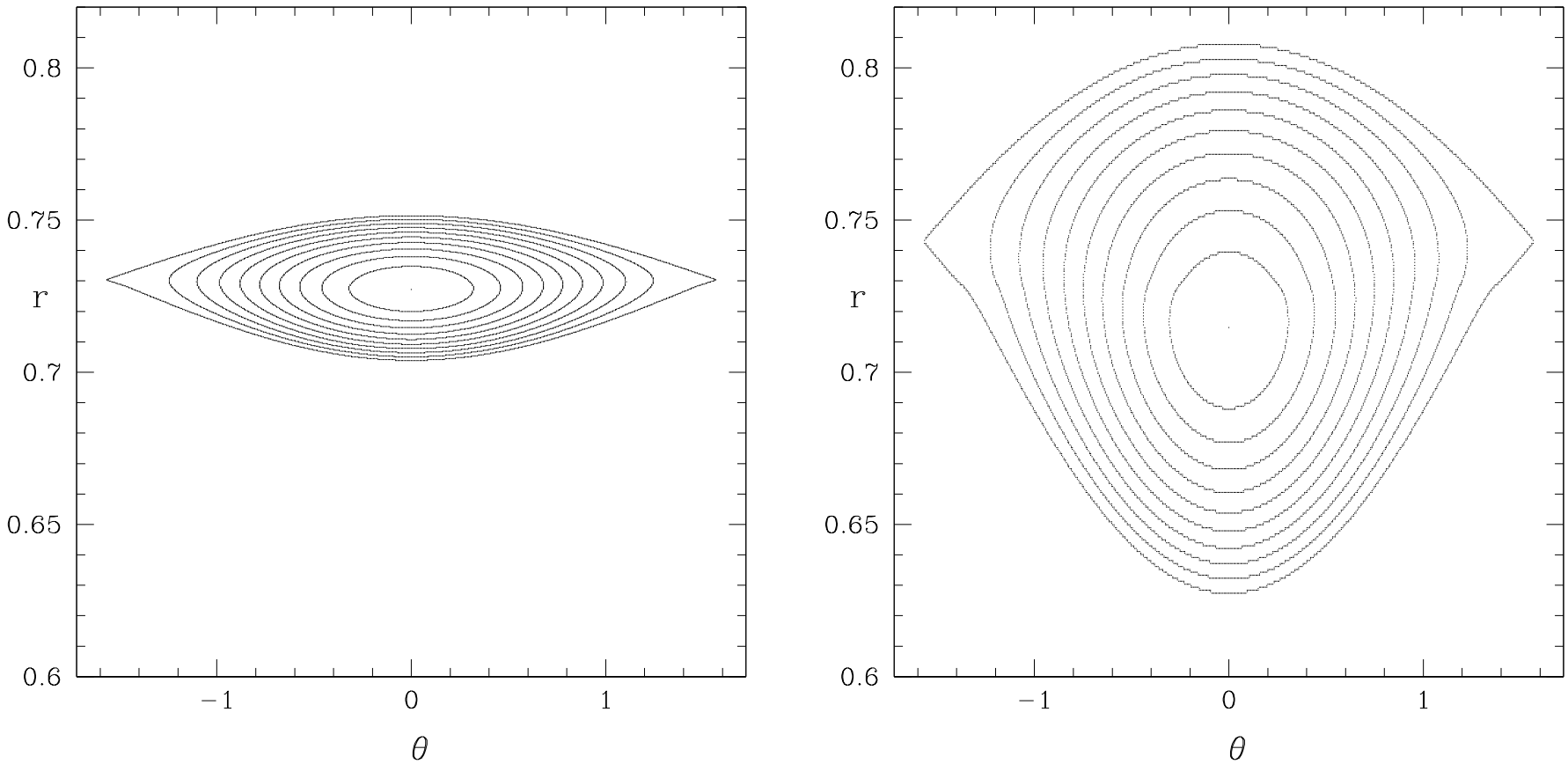
f_F Fitzpatrick, island partially flattened.
Secondly, flattening not valid for small width, due to perpendicular heat diffusion
Multiply $\Delta'_{\delta j}$, Δ'_A by $w^2/(w^2 + w_F^2)$,
$w_F = \sqrt{8}(\kappa_\perp/\kappa_\parallel)^{1/4}(Rr_s/ns)^{1/2}$,
For typical fusion parameters
$\kappa_\perp/\kappa_\parallel \sim 10^{-9}$
aspect ratio $R = 5$ $w_F \sim 0.02$.

The effective value of Δ' due to perturbed j and flattening can be found analytically using local approximations.
Including the Fitzpatrick factor for small islands we have

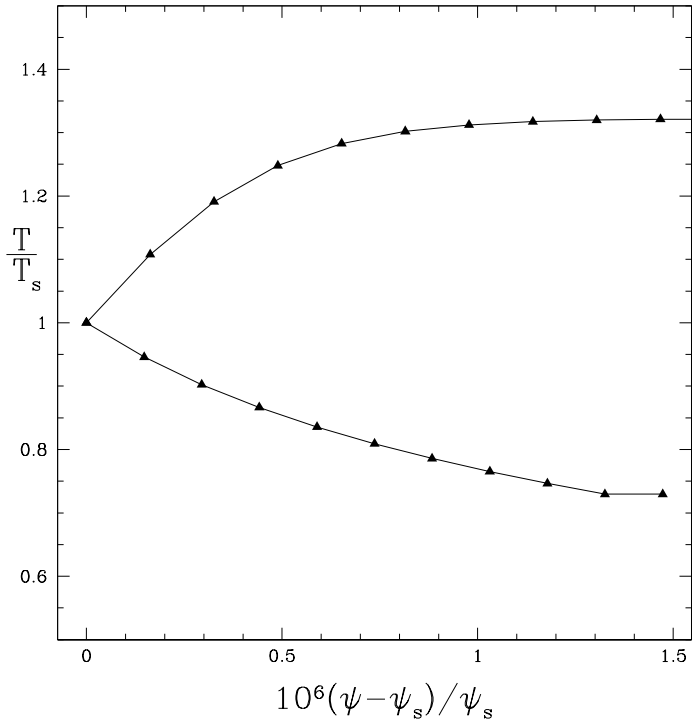
$$\Delta'_{\delta j}(w) = 16\frac{< \delta j_1 >}{\psi_0''}\frac{w}{w^2 + w_F^2}, \qquad \Delta'_A(w) = \frac{2j'(r_x)}{\pi\psi_0''}\frac{w^2}{w^2 + w_F^2} f_A,$$

where f_A takes account of asymmetry A and degree of island flattening, given by $f_A = A f_F$. Both ψ_0'', $j'(r_x)$ negative

Determination of f_A requires analysis with thermal transport
Qualitative results are independent of f_A for values between 0.5 and 1



Magnetic islands including the effect of current flattening. The small island has amplitude $\alpha = 10^{-4}$, and width $w = .05$, $A = .26$. The amplitude of the large island is $\alpha = 1.5 \times 10^{-3}$, with width $w = 0.18$, and asymmetry $A = .78$. Profile parameters are those of the disruption test case with $f_F = 1$.



Balance of Ohmic heating and radiation

$$\partial_t E = \nabla \cdot (\kappa \nabla T) + H(T) - R(T)$$

Steady state temperature profile
in the island. $\kappa_{\|} >> \kappa_{\perp}$, $T = T(\psi)$.
Average on the flux surfaces In the island
$\nabla^2 \psi \simeq \psi_0''(r_s)$
which is order one,
and $(\nabla\psi)^2 \leq (\psi_0''(r_s)w)^2/4 << 1$,
giving

$$0 = \kappa \psi_0''(r_s) \frac{dT}{d\psi} + H(T) - R(T)$$

Solve for $T(\psi)$ in the island

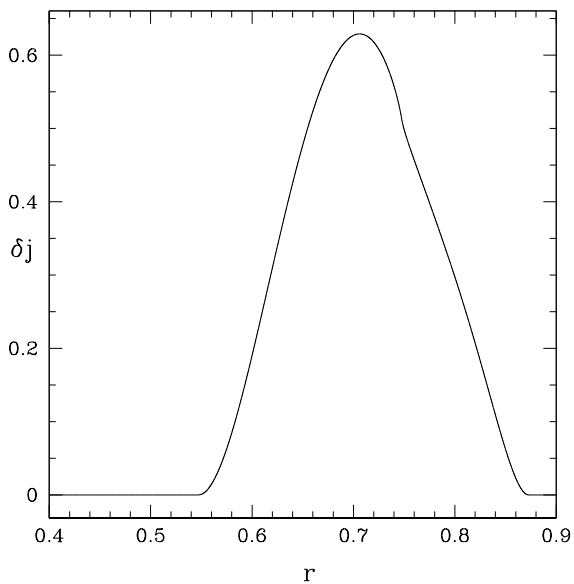
Spitzer resistivity produces a current perturbation from the temperature perturbation

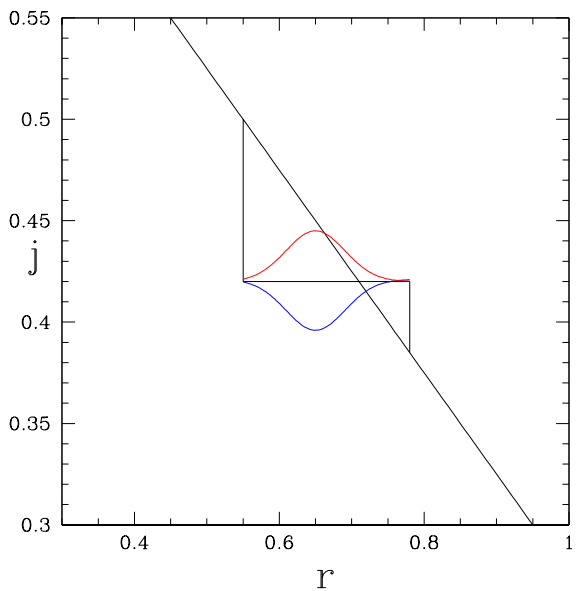
Heating gives a positive stabilizing δj

Cooling gives a negative destabilizing δj

Flattening is always destabilizing

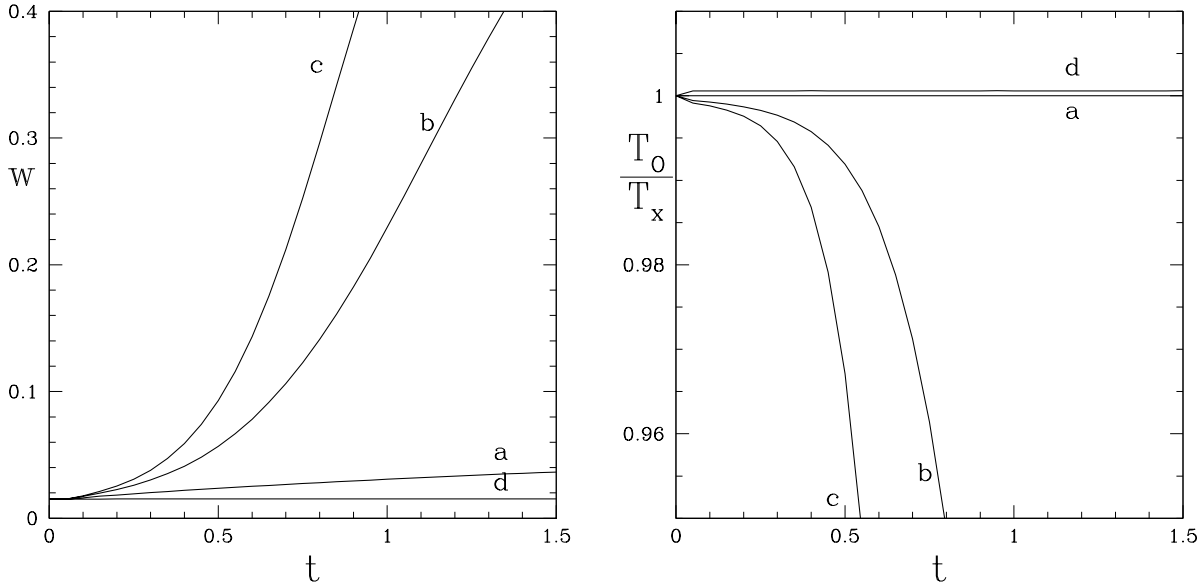
$$j_I(\psi) = j_s \frac{T^{3/2}(\psi) - T_s^{3/2}}{T_s^{3/2}},$$





Time evolution

$$\frac{dw}{dt} = r_s^2[\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

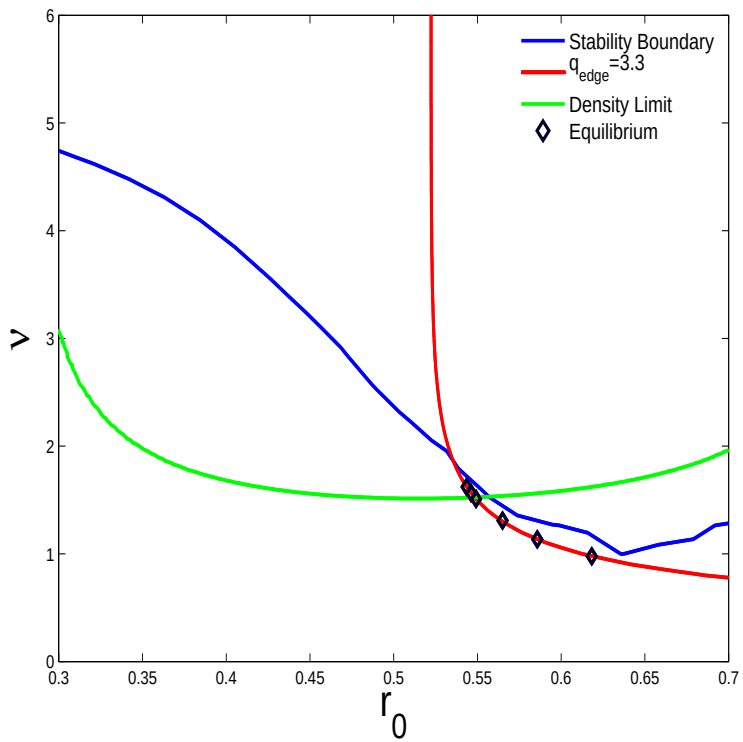


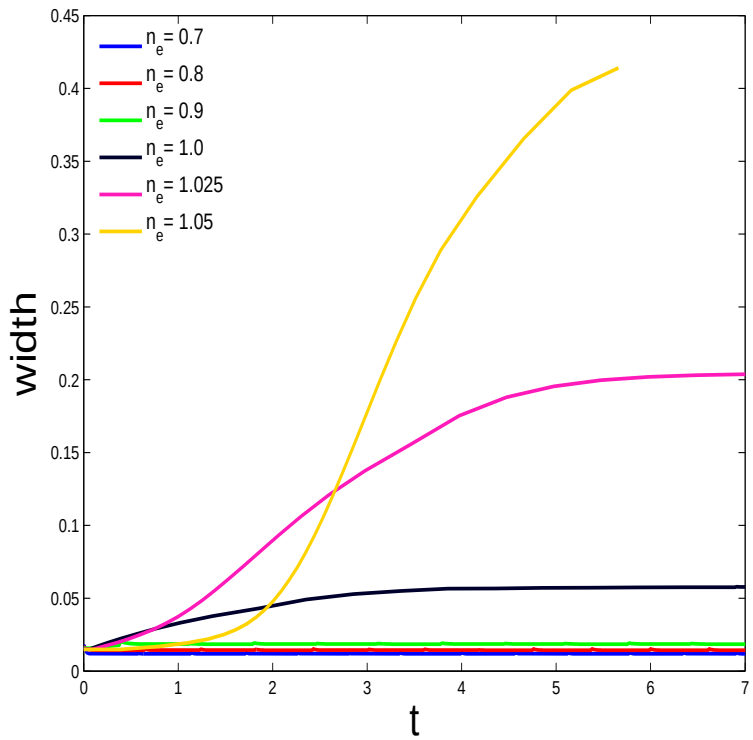
Growth of an island with a fixed temperature gradient $f_F = 1$.
a) radiation and heating balanced, b) and c) radiation dominated,
d) heating dominated. At $t = 0.2$, $T_O/T_x =$ a) 0, b) -.002, c) -.003.
In case (c) the temperature differential at t=0.5 with $w = 0.1$ was 3 percent.
In (d) the final central island temperature differential was .001.

Qian Teng - Analysis of approach to density limit

Insertion of radiation and Ohmic heating models into code.

At approach to the density limit the current channel shrinks, reaching an internal inductance limit $l_i = 0.12 q_{edge} + 0.6$ and at this point radiated power balances Ohmic heating





The relative thermic isolation of a magnetic island and the effects of Ohmic heating and radiation can lead to rapid growth or mild contraction of a saturated island due to a tearing mode.

The $m = 2$ island has long been a candidate for the onset of significant loss of plasma to the wall and violent disruption

Minor modification of the current profile in an island interior can lead to rapid change in the saturated island width, and a dominant effect can be produced by temperature changes within the island.

Small imbalance between radiation and Ohmic heating can lead to rapid island growth. Confirmed with DEBS, to do NIMROD - Dylan Brennan

With models for Ohmic heating and radiation, this mechanism is a candidate for accounting for the Greenwald density limit. - Qian Teng