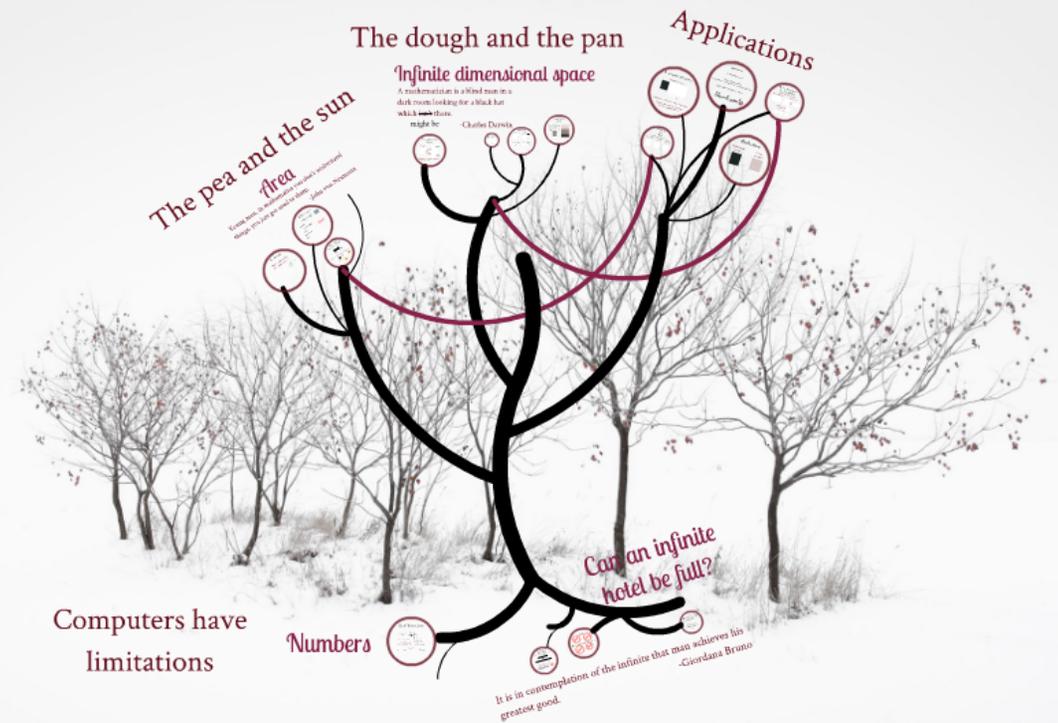


Can studying infinite dimensional space help improve health care?

Theoretical math is cool!

Three seemingly paradoxical results.

Detect infections. Predict length of stay.



But first, a word of encouragement from my cousins.

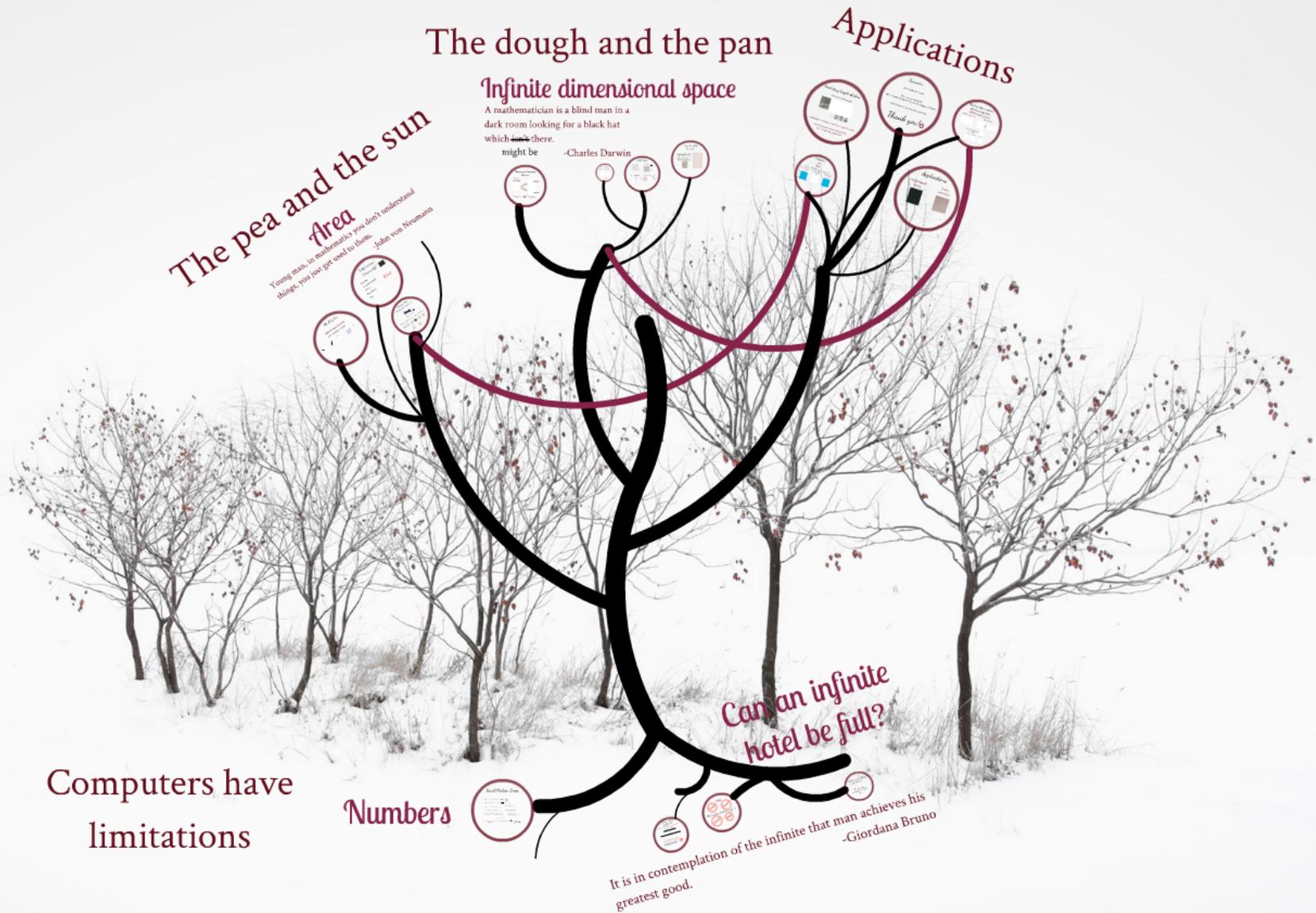


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Computers have limitations

The monk and the mountain.

On the first day, a monk wakes up at sunrise, walks up a mountain trail and sleeps at the top of the trail. On the morning of the second day, the monk wakes up at sunrise and walks down the same trail to where he came from.

Must there be a spot on the trail where he was in the exact same place at the exact same time on both days?



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ays!

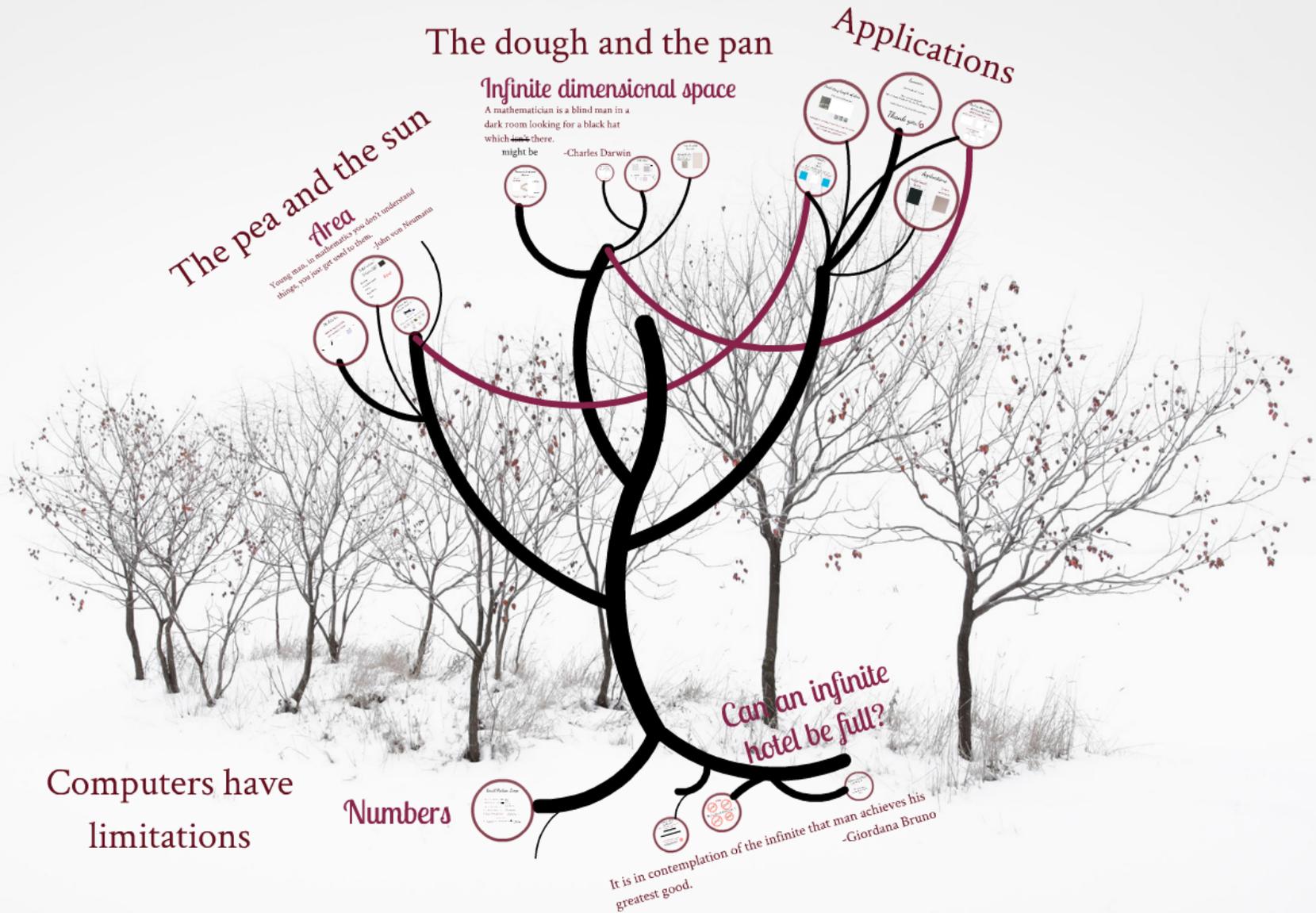


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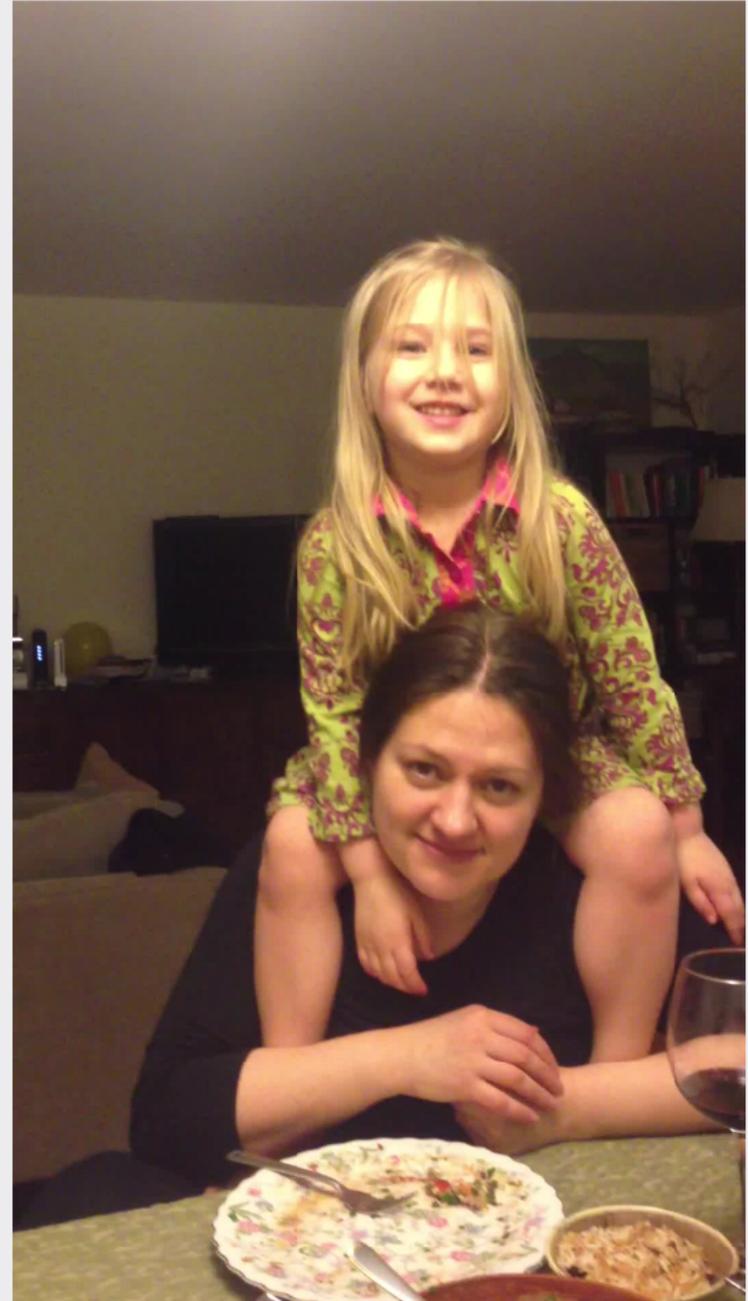
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The pea and the sun

Area
Young man, in mathematics you don't understand things, you just get used to them.
-John von Neumann

The dough and the pan

Infinite dimensional space

A mathematician is a blind man in a dark room looking for a black hat which isn't there.
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Applications

Computers have limitations

Numbers

Can an infinite hotel be full?

It is in contemplation of the infinite that man achieves his greatest good.
-Giordano Bruno

Numbers

Small Medium Large

The number of rooms in the Burj Al Arab.  202

The diameter in meters of the visible universe.  10^{26}

The number of atoms in the visible universe. 10^{80}

A googol - the number Google was named after. 10^{100}

If each atom of our universe contained a copy of our universe, the number of atoms in all those universes. $10^{80} \cdot 10^{80} = 10^{160}$

Let's see a large number!

The number of ways to arrange 202 people in 202 rooms. $202! \approx 3 \cdot 10^{400}$

Small Medium Large

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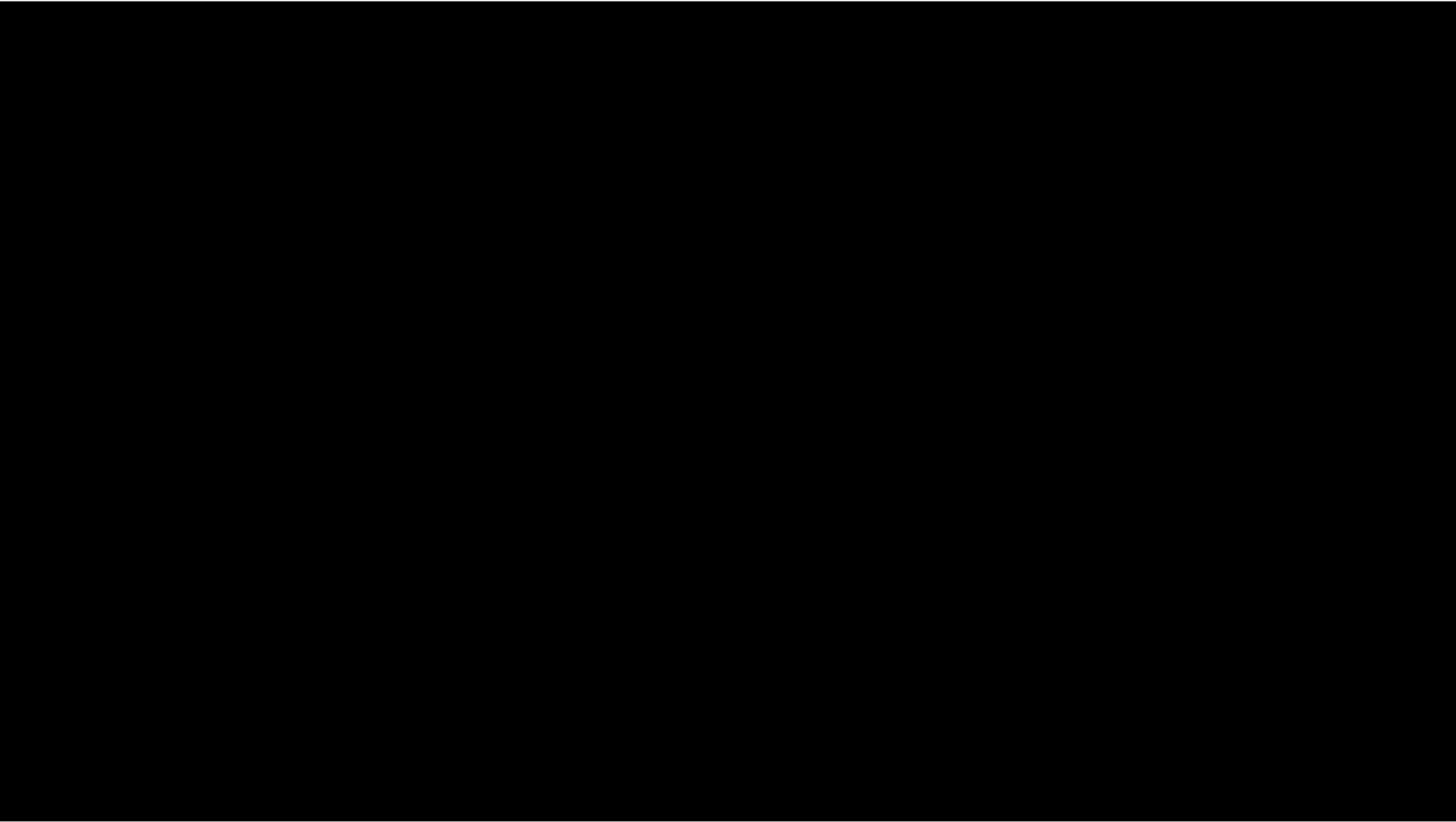
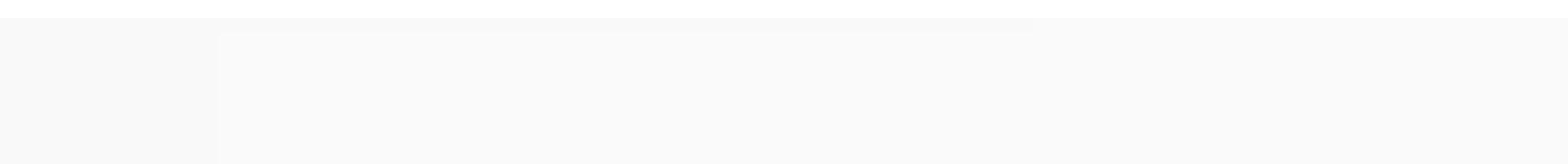
The number of ways to arrange 202 people in 202 rooms. $202 * 201 * 200 \dots 3 * 2 * 1 = 10^{363}$











Google

Googol

Google Search

I'm Feeling Lucky

=32021086730745224025233051212074577899307110966054608
8362931363347874823874141025292501945394545996997574031
67264415175786120812569431551739724344826334800863068818
93090753220647279254510113913712218047148279425971626745
609604362763277403929023231561936352369318134164674249
734737312349757960375826884245660008970430162564120838
144000
0000000

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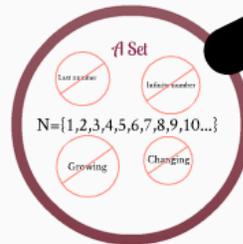
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A Set

~~Last number~~

~~Infinite number~~

$N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$

~~Growing~~

~~Changing~~

A Set

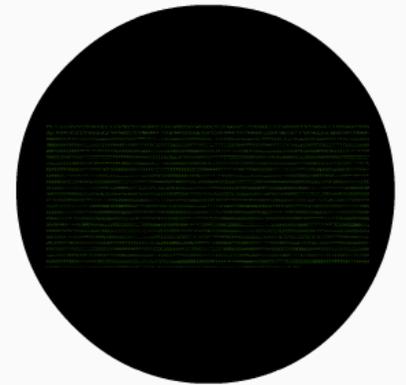
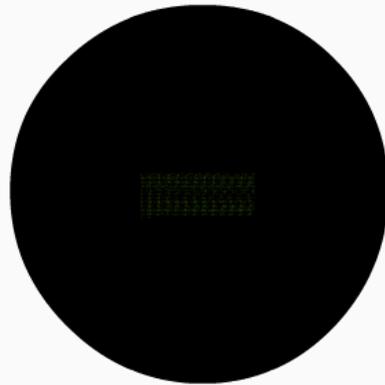
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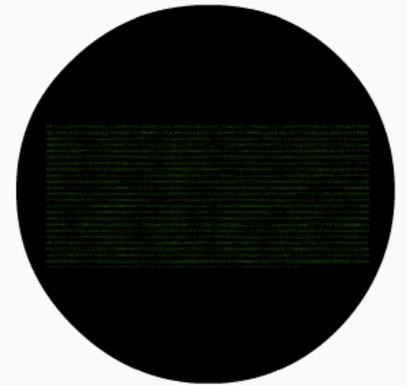
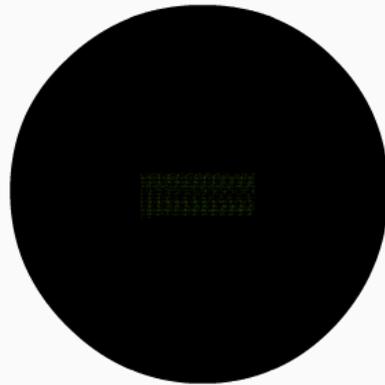
~~Changing~~



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N=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,267,268,269,270,271,272,273,274,275,276,277,278,279,280,281,282,283,284,285,286,287,288,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,326,327,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,374,375,376,377,378,379,380,381,382,383,384,385,386,387,388,389,390,391,392,393,394,395,396,397,398,399,400,401,402,403,404,405,406,407,408,409,410,411,412,413,414,415,416,417,418,419,420,421,422,423,424,425,426,427,428,429,430,431,432,433,434,435,436,437,438,439,440,441,442,443,444,445,446,447,448,449,450,451,452,453,454,455,456,457,458,459,460,461,462,463,464,465,466,467,468,469,470,471,472,473,474,475,476,477,478,479,480,481,482,483,484,485,486,487,488,489,490,491,492,493,494,495,496,497,498,499,500,501,502,503,504,505,506,507,508,509,510,511,512,513,514,515,516,517,518,519,520,521,522,523,524,525,526,527,528,529,530,531,532,533,534,535,536,537,538,539,540,541,542,543,544,545,546,547,548,549,550,551,552,553,554,555,556,557,558,559,560,561,562,563,564,565,566,567,568,569,570,571,572,573,574,575,576,577,578,579,580,581,582,583,584,585,586,587,588,589,590,591,592,593,594,595,596,597,598,599,600,601,602,603,604,605,606,607,608,609,610,611,612,613,614,615,616,617,618,619,620,621,622,623,624,625,626,627,628,629,630,631,632,633,634,635,636,637,638,639,640,641,642,643,644,645,646,647,648,649,650,651,652,653,654,655,656,657,658,659,660,661,662,663,664,665,666,667,668,669,670,671,672,673,674,675,676,677,678,679,680,681,682,683,684,685,686,687,688,689,690,691,692,693,694,695,696,697,698,699,700,701,702,703,704,705,706,707,708,709,710,711,712,713,714,715,716,717,718,719,720,721,722,723,724,725,726,727,728,729,730,731,732,733,734,735,736,737,738,739,740,741,742,743,744,745,746,747,748,749,750,751,752,753,754,755,756,757,758,759,760,761,762,763,764,765,766,767,768,769,770,771,772,773,774,775,776,777,778,779,780,781,782,783,784,785,786,787,788,789,790,791,792,793,794,795,796,797,798,799,800,801,802,803,804,805,806,807,808,809,810,811,812,813,814,815,816,817,818,819,820,821,822,823,824,825,826,827,828,829,830,831,832,833,834,835,836,837,838,839,840,841,842,843,844,845,846,847,848,849,850,851,852,853,854,855,856,857,858,859,860,861,862,863,864,865,866,867,868,869,870,871,872,873,874,875,876,877,878,879,880,881,882,883,884,885,886,887,888,889,890,891,892,893,894,895,896,897,898,899,900,901,902,903,904,905,906,907,908,909,910,911,912,913,914,915,916,917,918,919,920,921,922,923,924,925,926,927,928,929,930,931,932,933,934,935,936,937,938,939,940,941,942,943,944,945,946,947,948,949,950,951,952,953,954,955,956,957,958,959,960,961,962,963,964,965,966,967,968,969,970,971,972,973,974,975,976,977,978,979,980,981,982,983,984,985,986,987,988,989,990,991,992,993,994,995,996,997,998,999,1000...]



A Set

~~Last number~~

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$N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$

~~Growing~~

~~Changing~~

Can an infinite hotel be full?

A hotel is full when there is a guest in each room.

Does the size matter?

~~Imagine an infinite hotel.~~



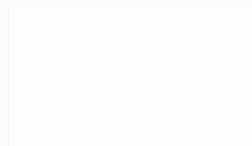
~~Imagine a hotel with infinitely many rooms.~~



Imagine a hotel with one room for each natural number.

It is **full** if there is a guest in each room.

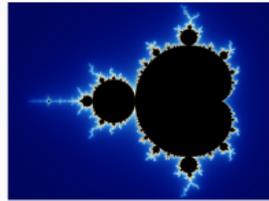
What happens when a new guest shows up?



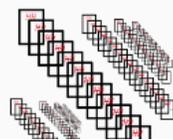
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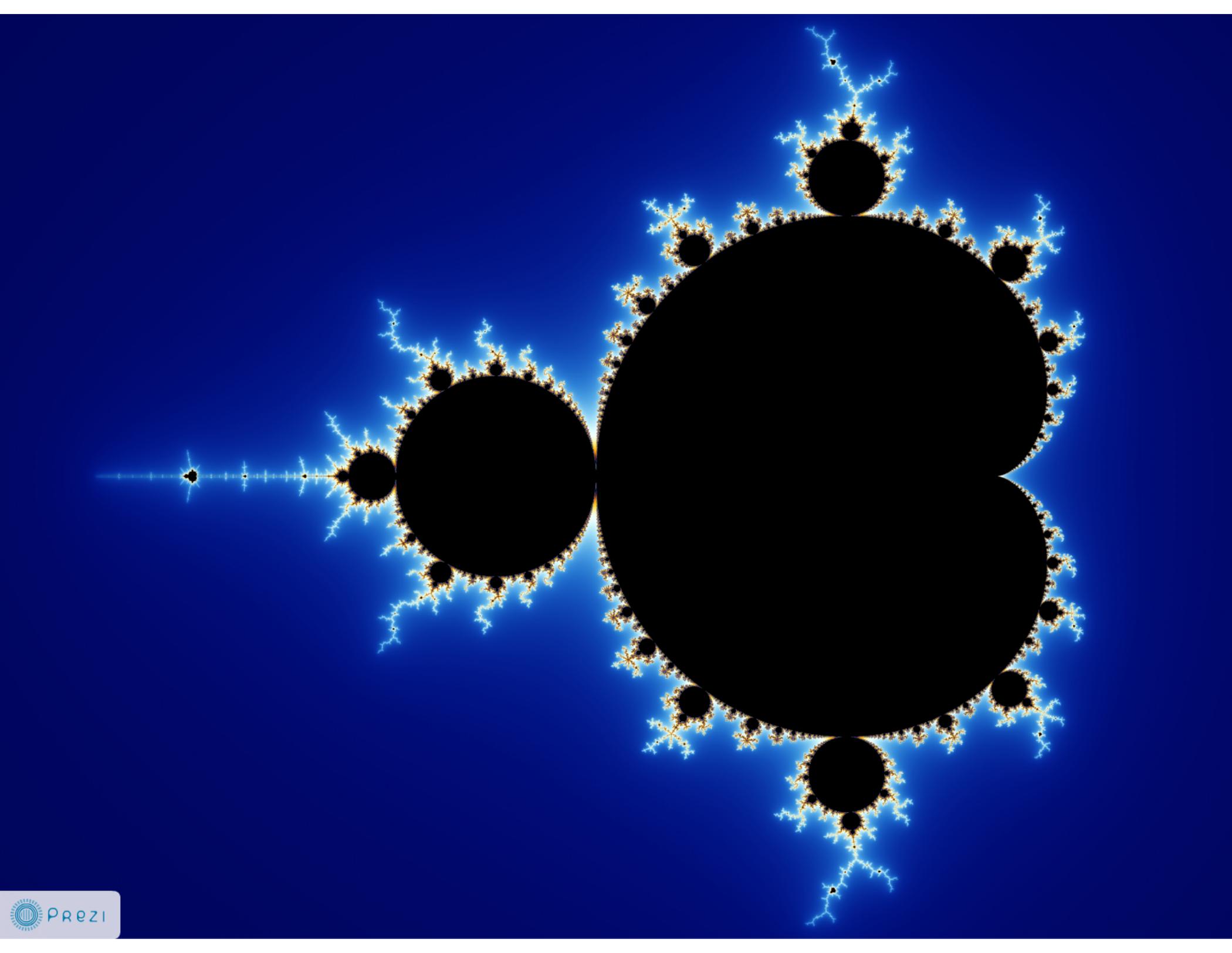
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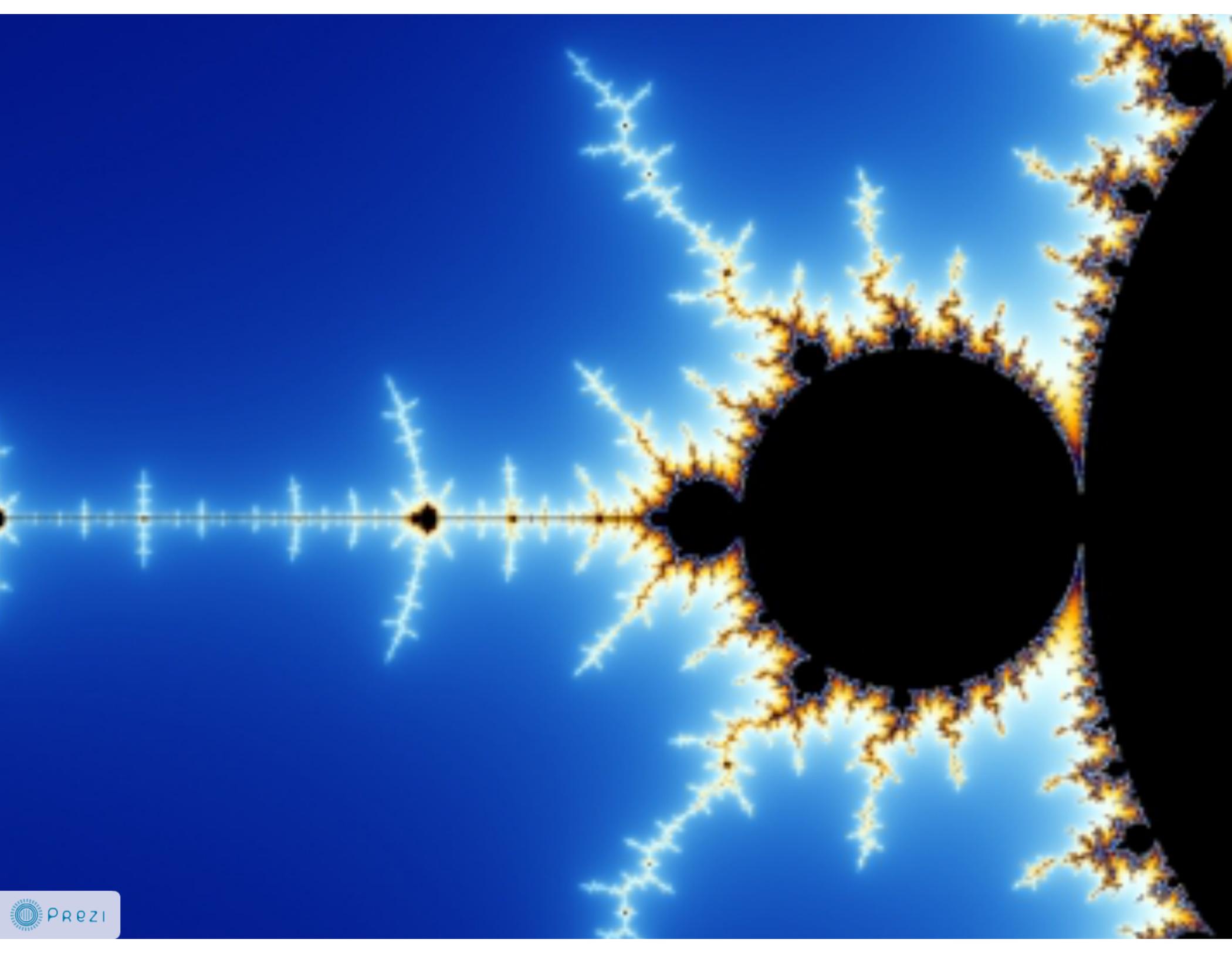
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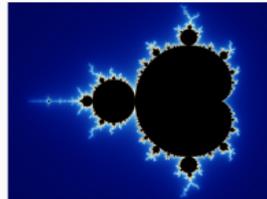




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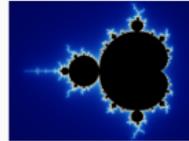


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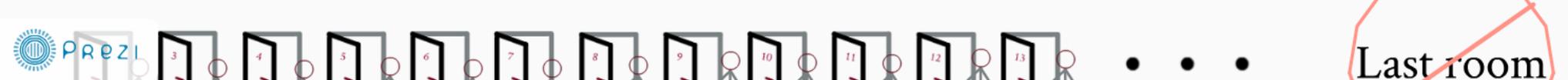
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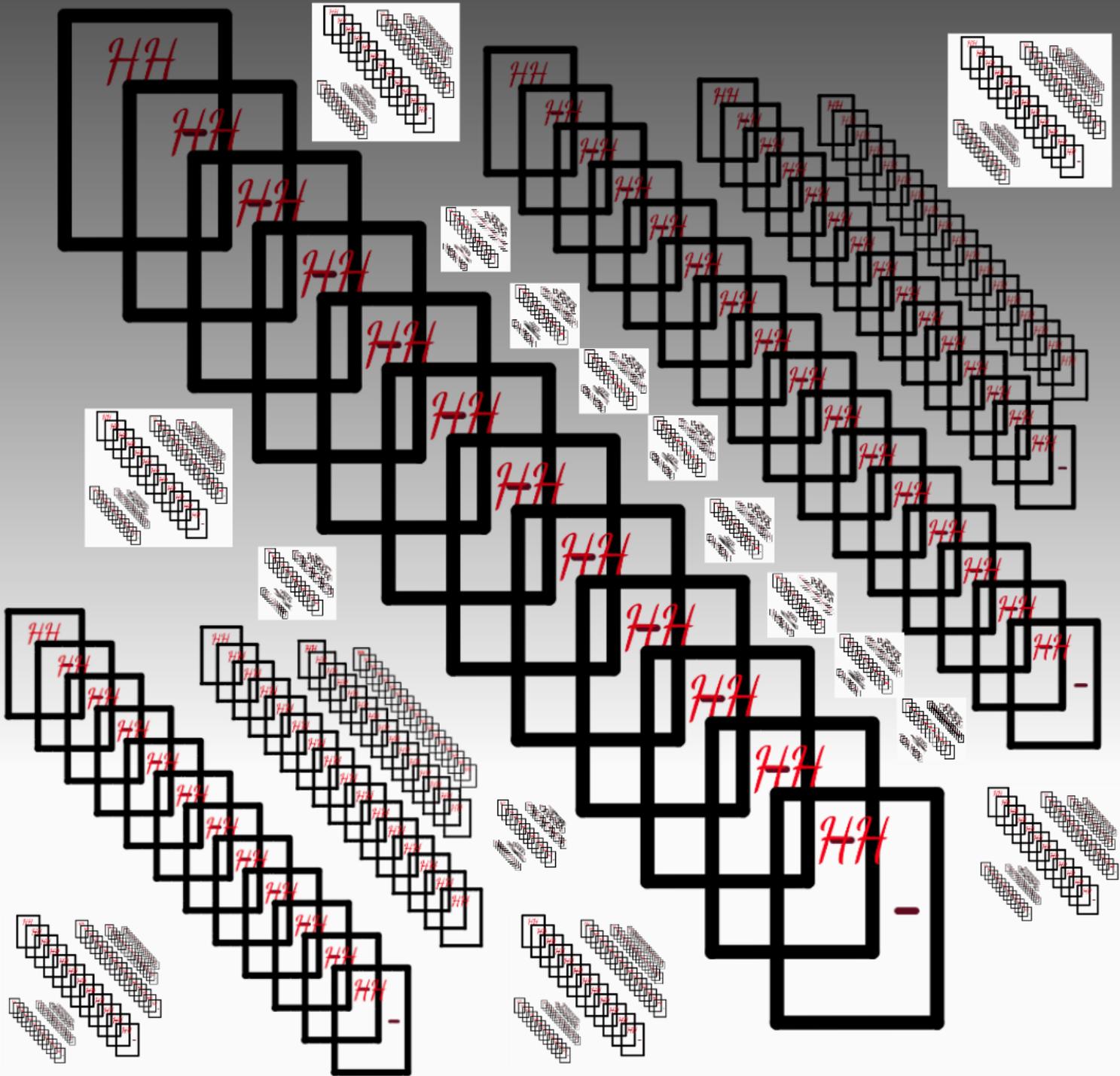


Imagine a hotel with one room for each natural number.

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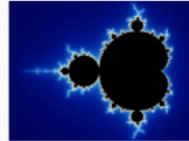
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What happens when a new guest shows up?



*The Rationals are Dense
in the Reals*

*The Cantor set is
uncountable and nowhere
dense.*

is cool!

paradoxical results.

Predict length of stay.

The pea and the sun

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The Calculus

How to measure the area under a curve?



What is area?

What is life?



Breathes
Consumes energy
Grows
Reproduces
Dies

Fire?

What is Area?

What properties should it have?



The problem is what area does this shape have?

What if you had a shape that was not a square or a circle?

The Banach Tarski paradox - The Pea and The Sun



Did they break math? No!



The Calculus

How to measure the
area under a curve?

Area under a half circle



$$\text{area} = \frac{\pi r^2}{2} = \frac{\pi}{2}$$



For more complex
curves



One can measure the area
with more advanced math



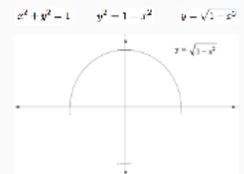
What about even
stranger curves?



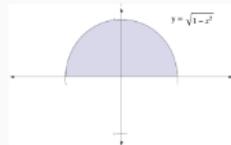
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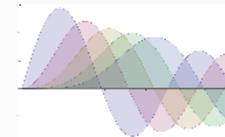
$$\text{area} = \frac{\pi r^2}{2} = \frac{\pi}{2}$$



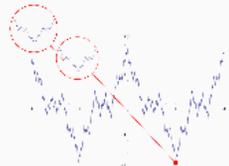
For more complex curves



One can measure the area with more advanced math



What about even stranger curves? $f(x) = \sum_{n=0}^{\infty} a^n \cos^n(x)$

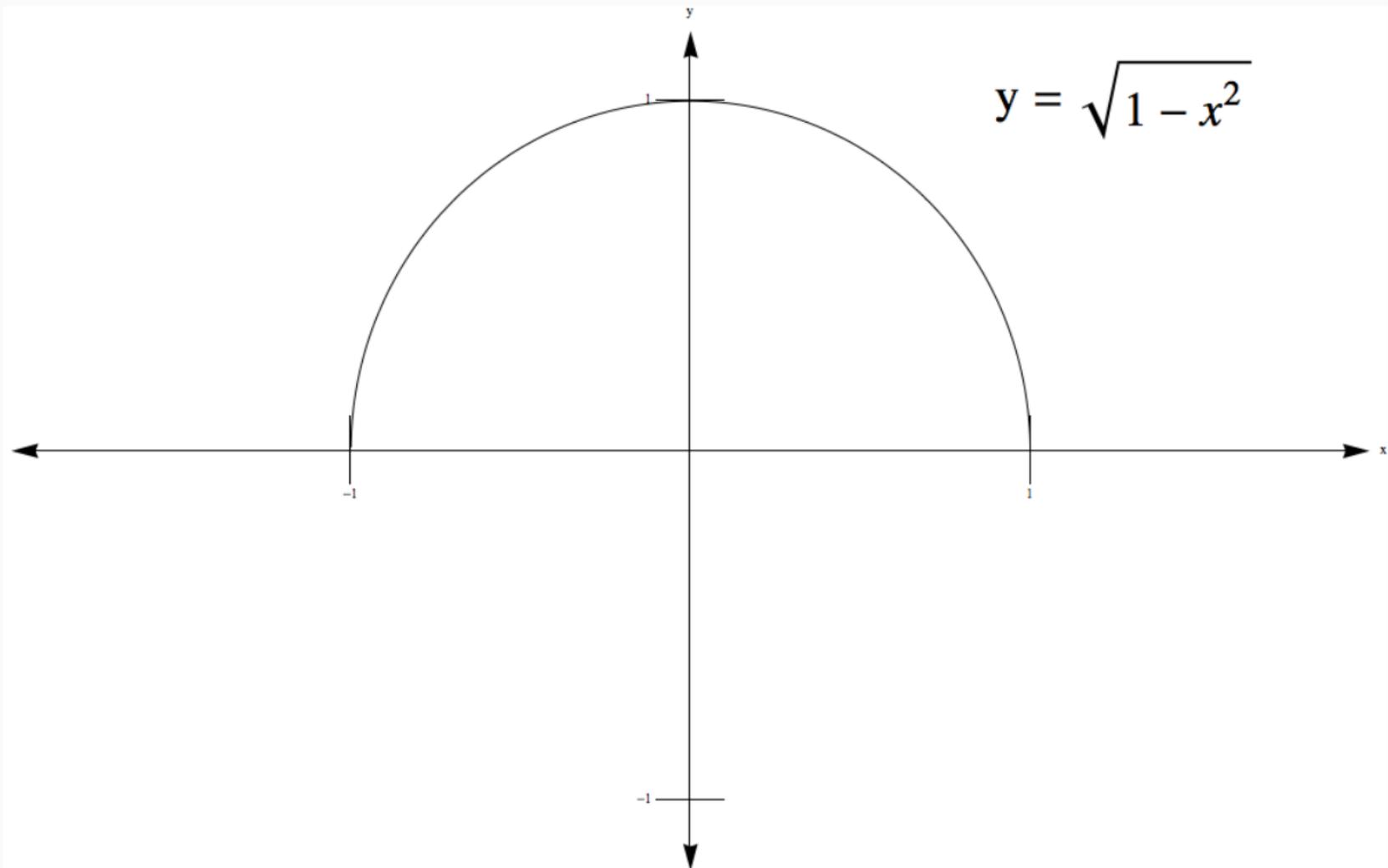


Area under a half circle

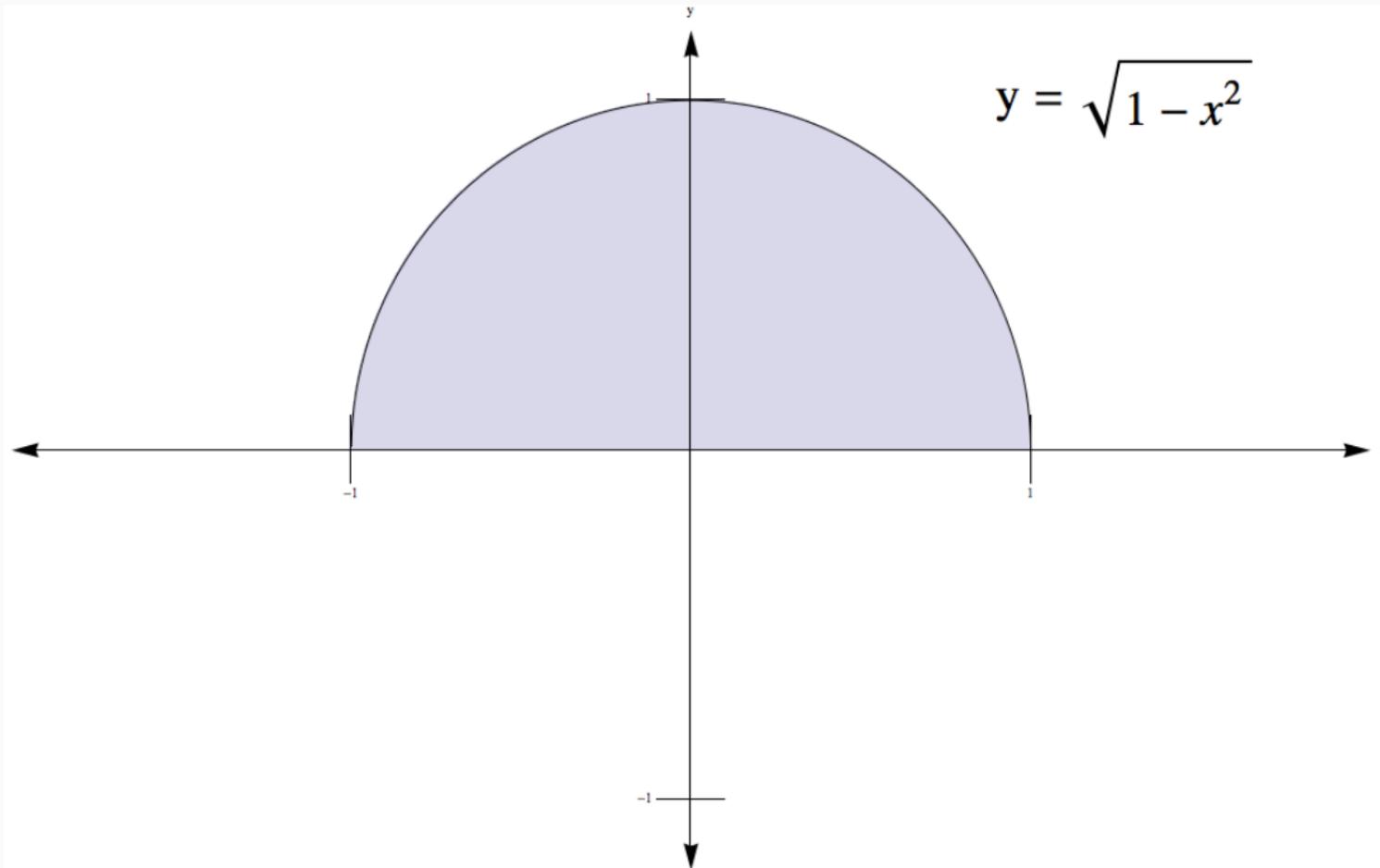
$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

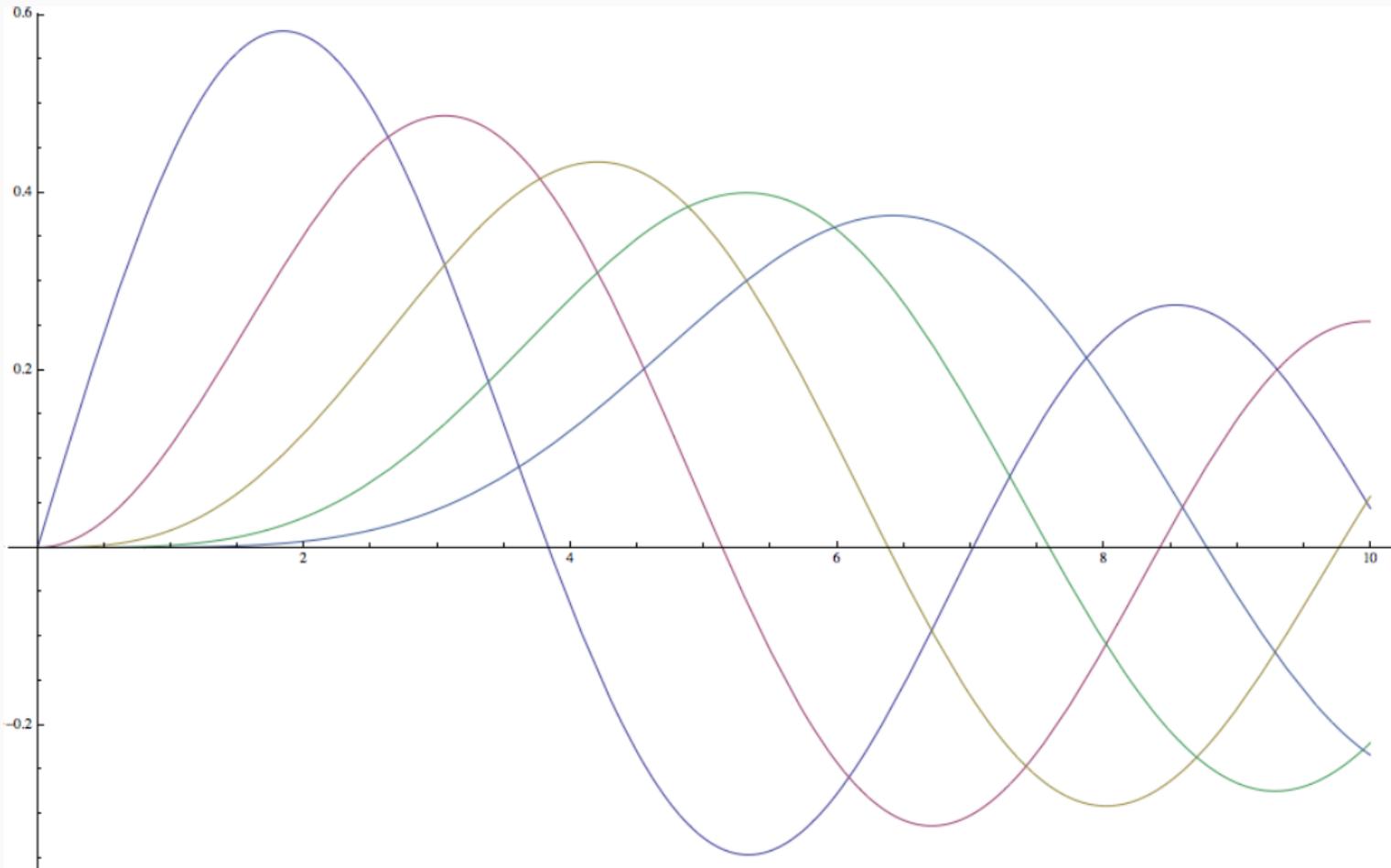
$$y = \sqrt{1 - x^2}$$



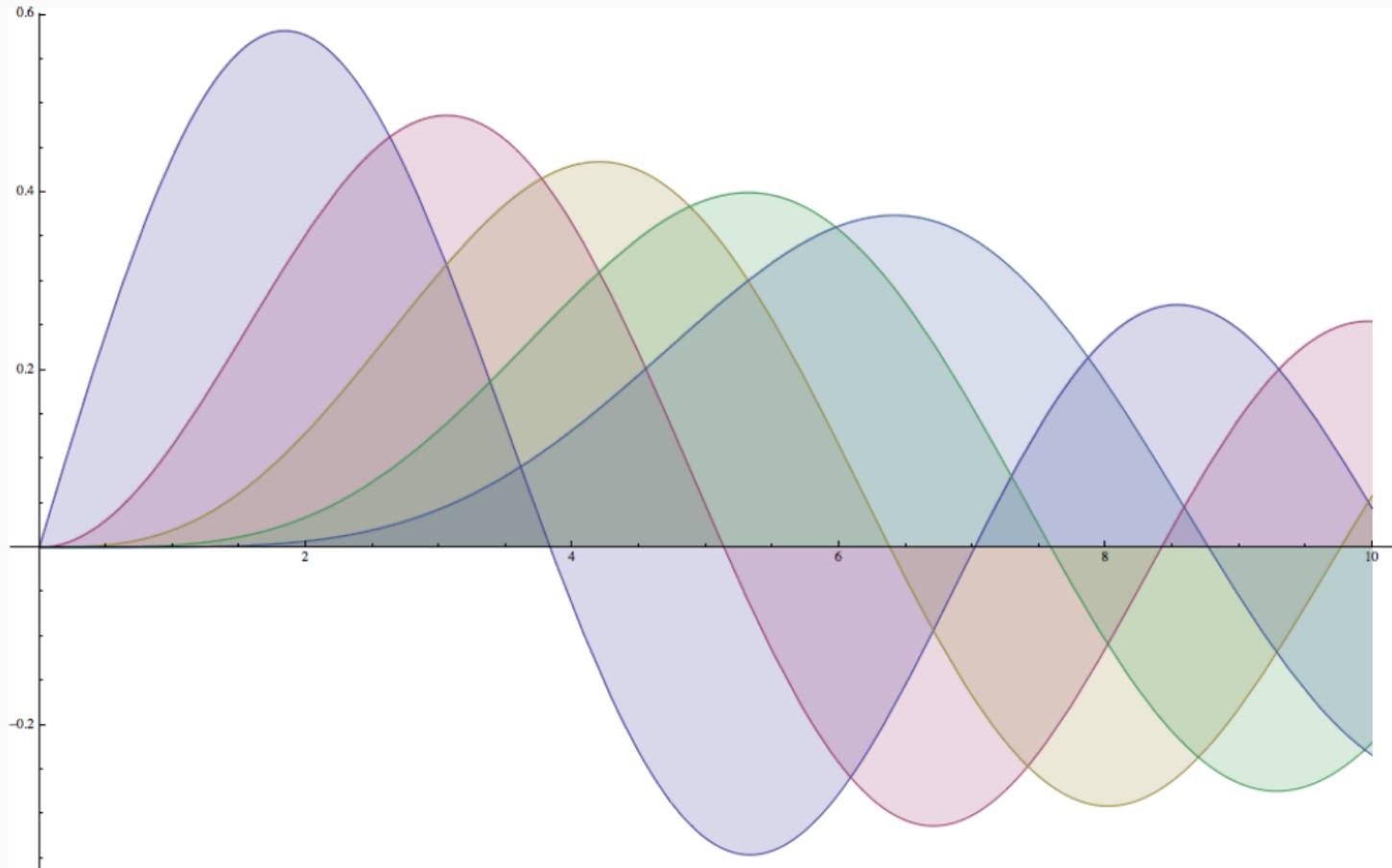
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For more complex curves

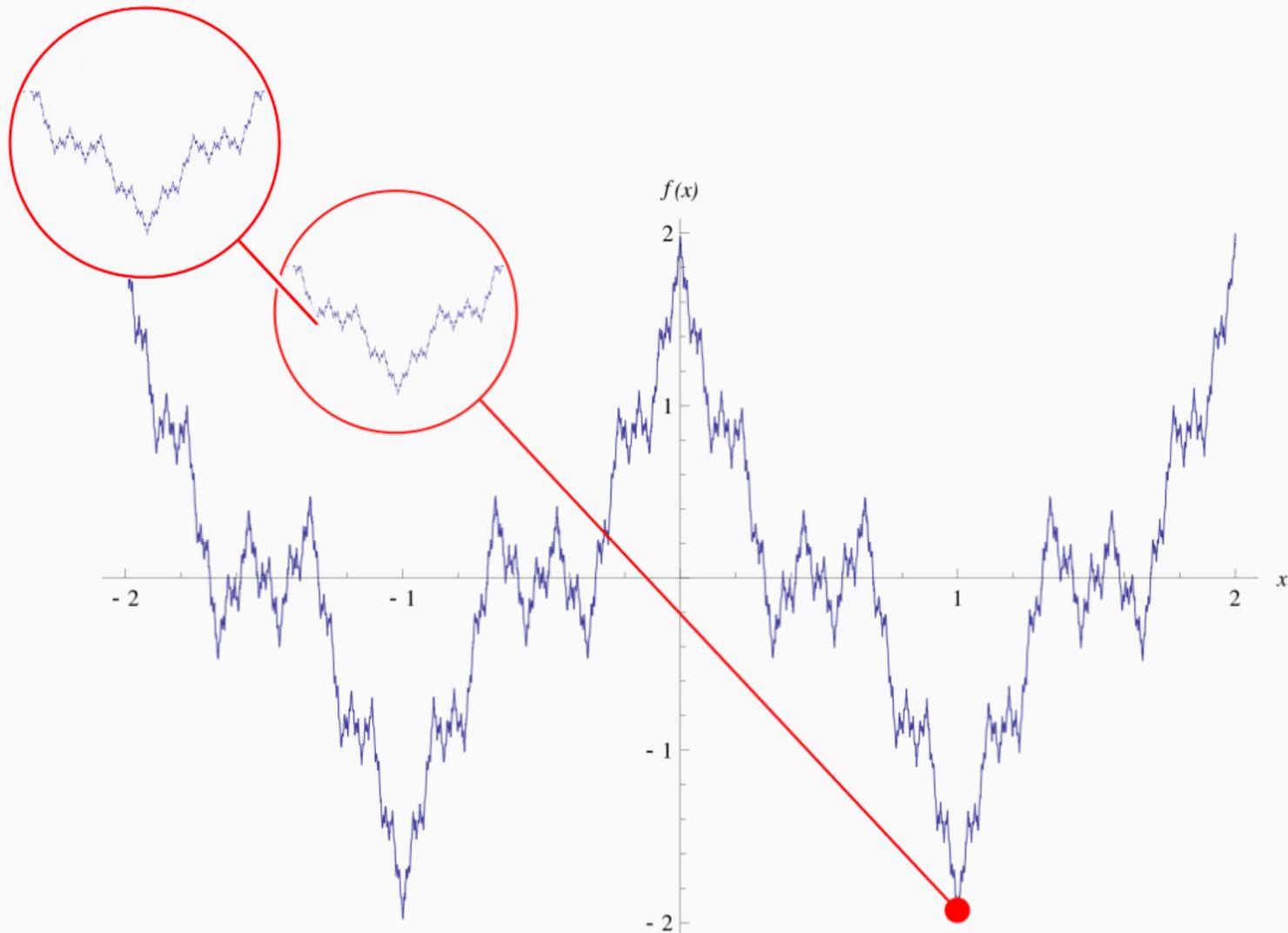


One can measure the area with more advanced math



What about even stranger curves?

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$



The Calculus

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$$\text{area} = \frac{\pi r^2}{2} = \frac{\pi}{2}$$



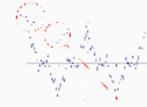
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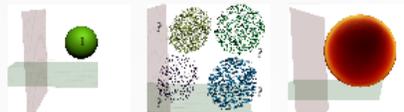
What properties should it have?

- ~~Any set of points should have an area.~~ 
- The cutting an object into pieces should not changes its area. 
- Moving or rotating an object should not change its area. 

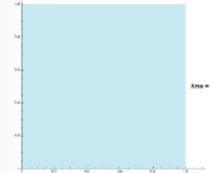
The Banach Tarski paradox - The Pea and The Sun



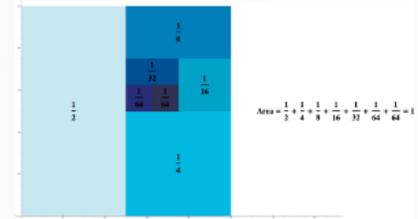
Did they break math? No!



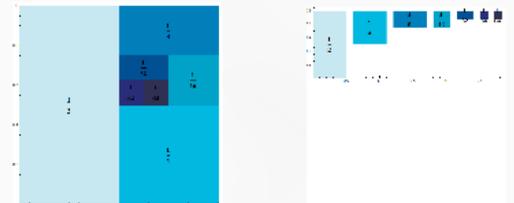
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The cutting an object into pieces should not change its area.



Moving or rotating an object should not change its area.

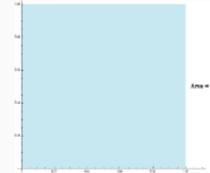


1.0
0.8
0.6
0.4
0.2

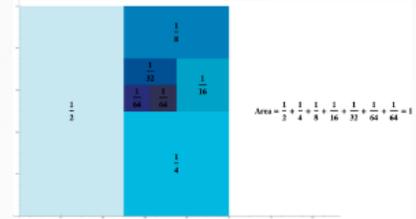


Area = 1

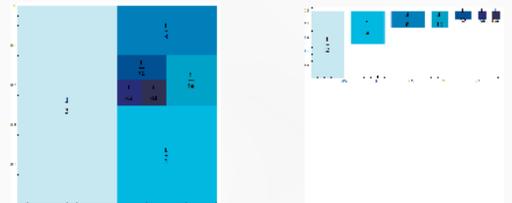
~~Any set of points should have an area.~~

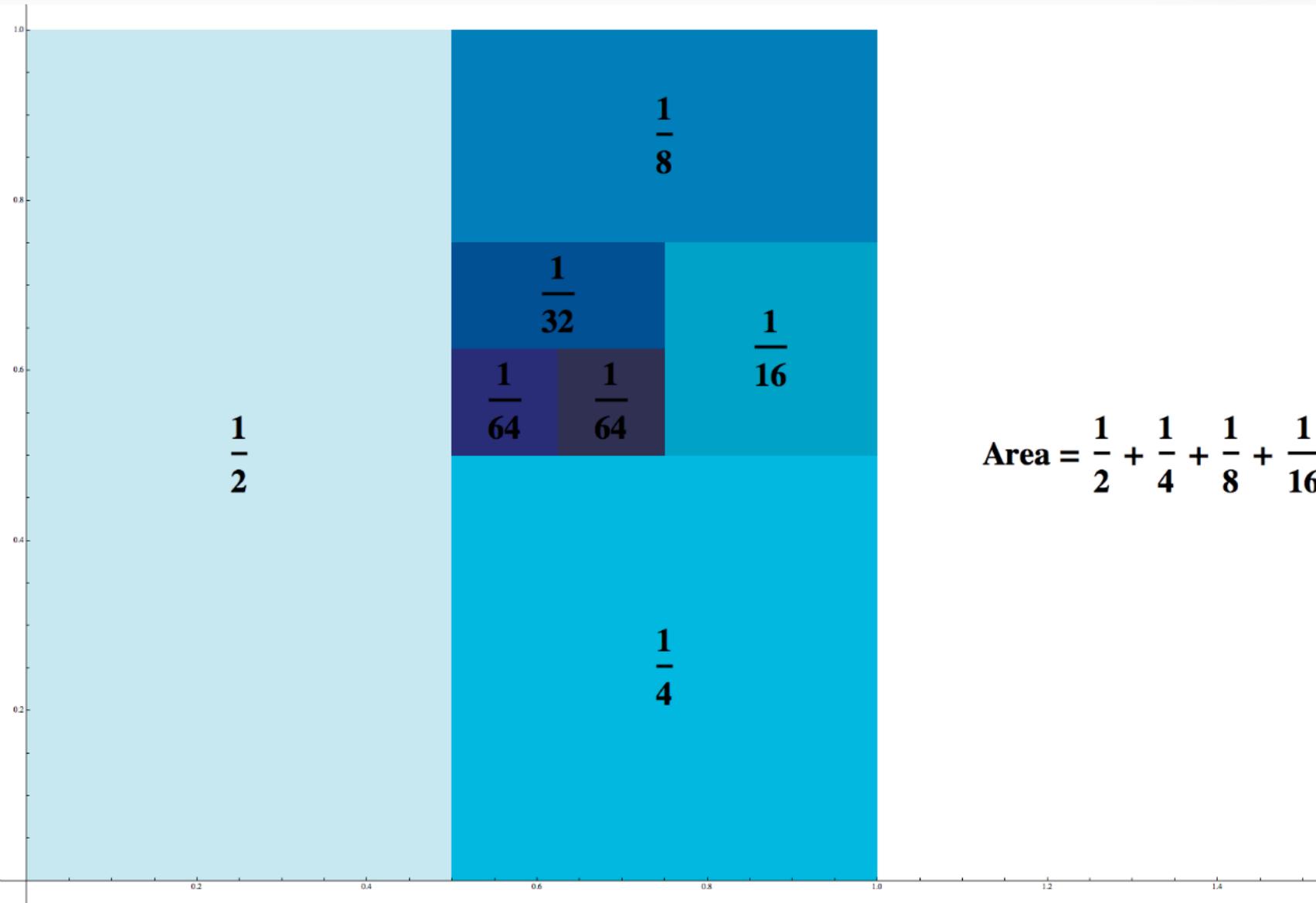


The cutting an object into pieces should not changes its area.



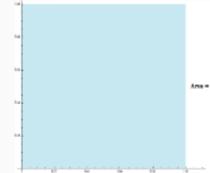
Moving or rotating an object should not change its area.



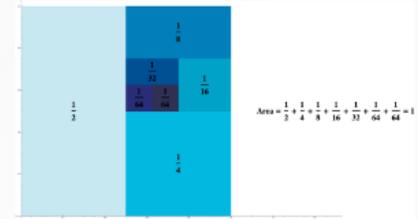


$$\text{Area} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{64} = 1$$

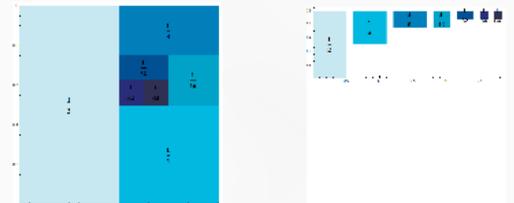
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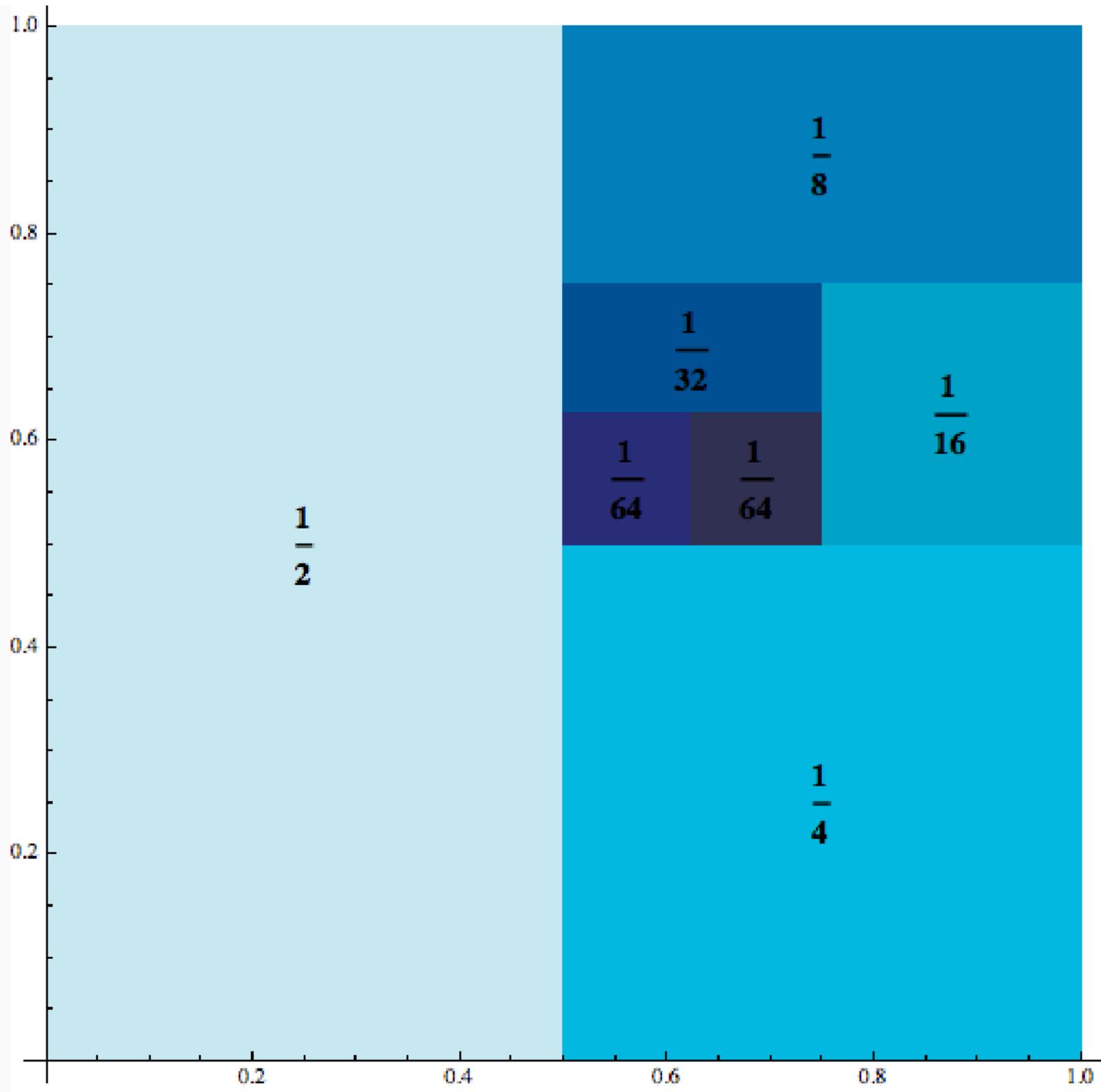


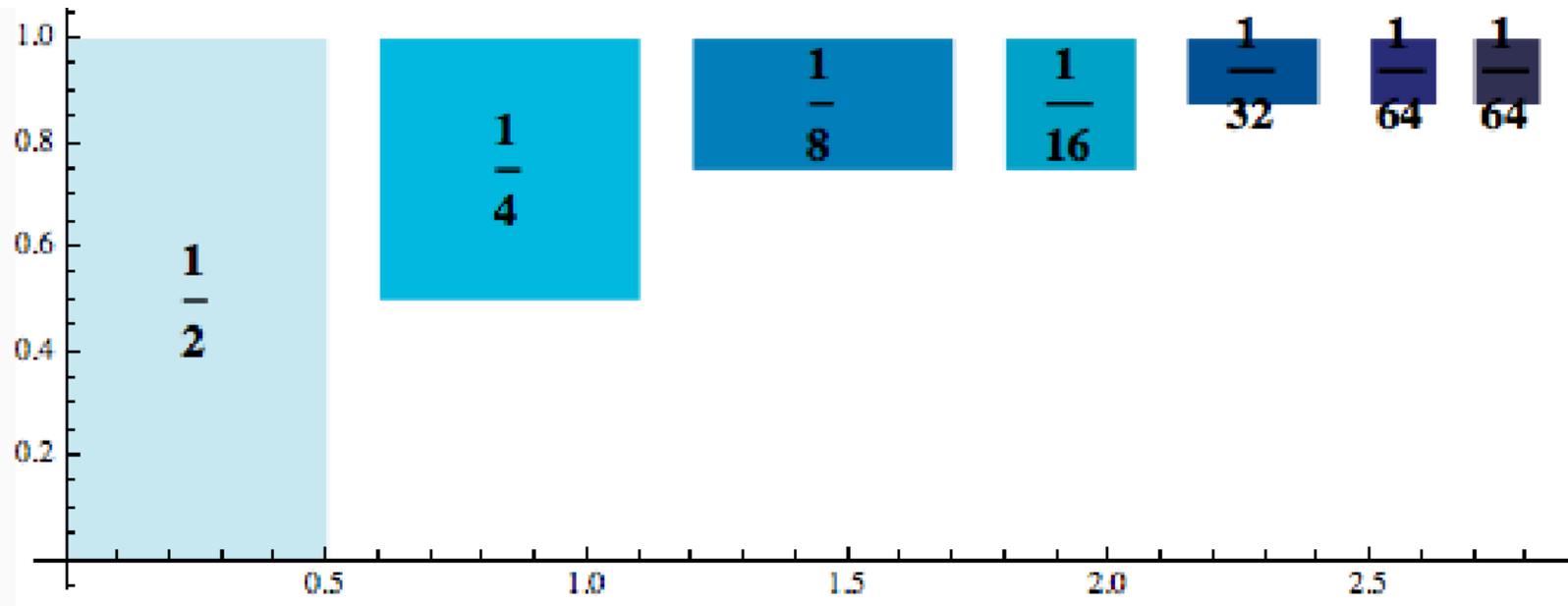
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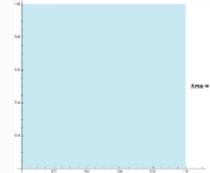
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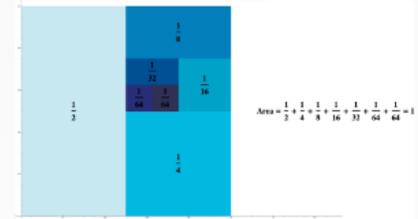




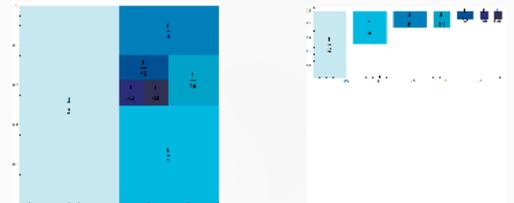
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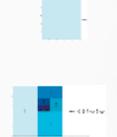
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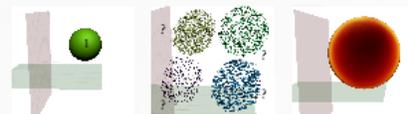


The Banach Tarski paradox - The Pea and The Sun

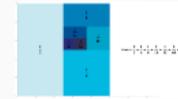


Did they break math?

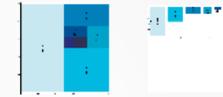
No!



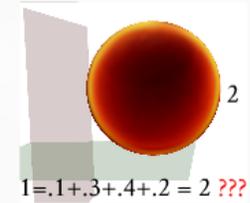
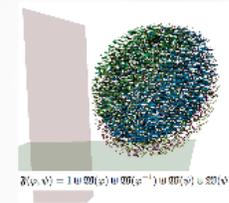
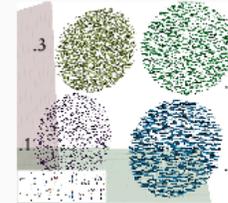
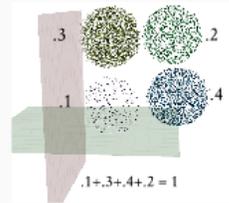
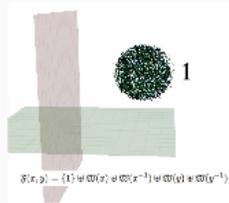
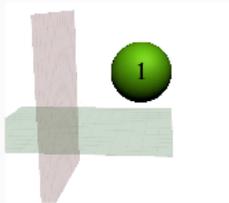
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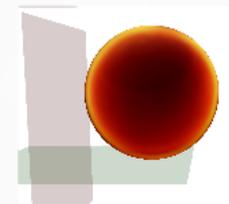
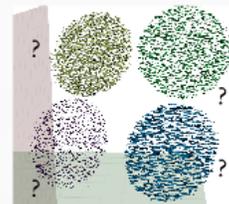
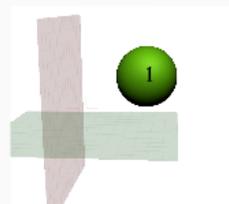


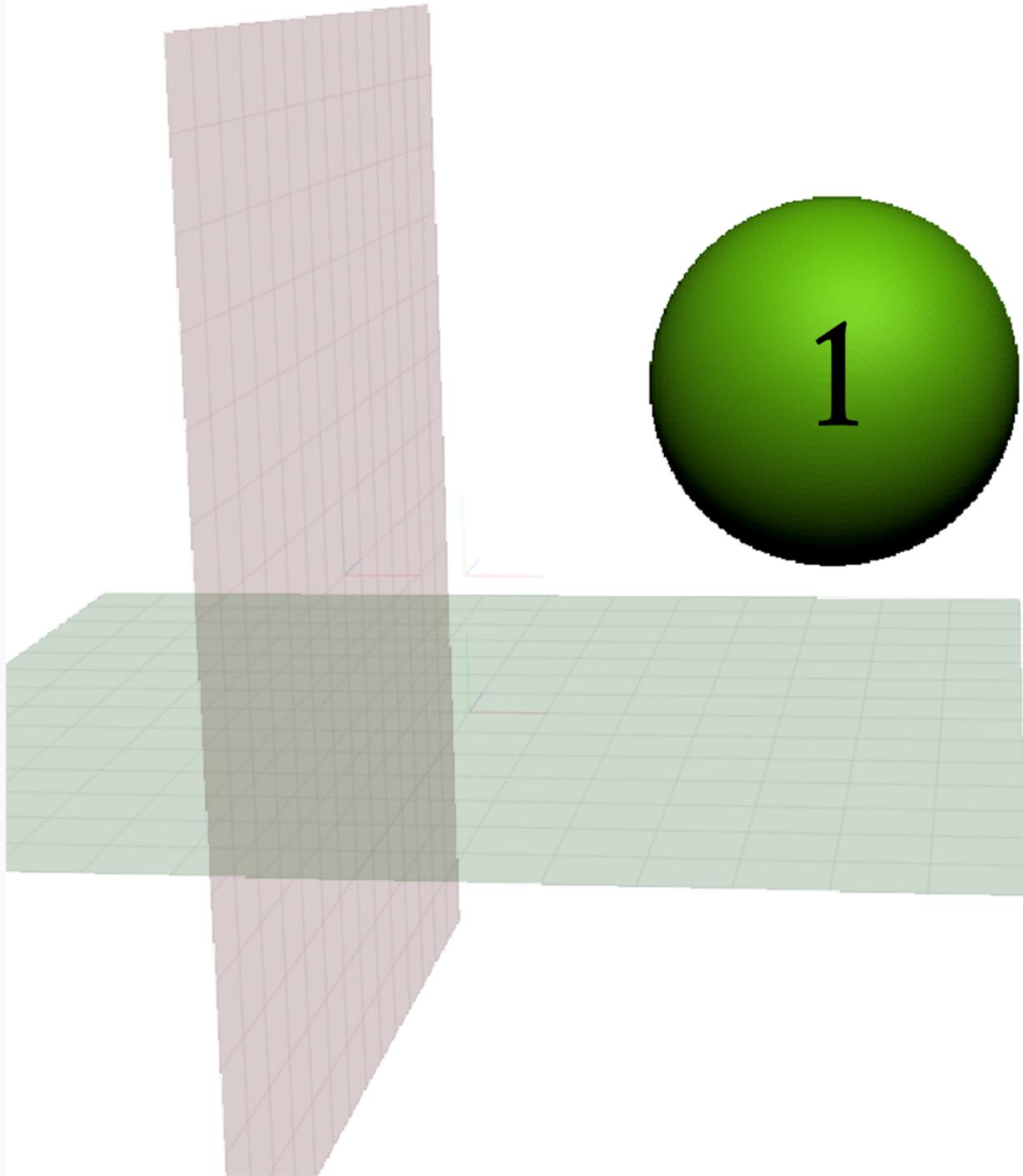
The Banach Tarski paradox - The Pea and The Sun

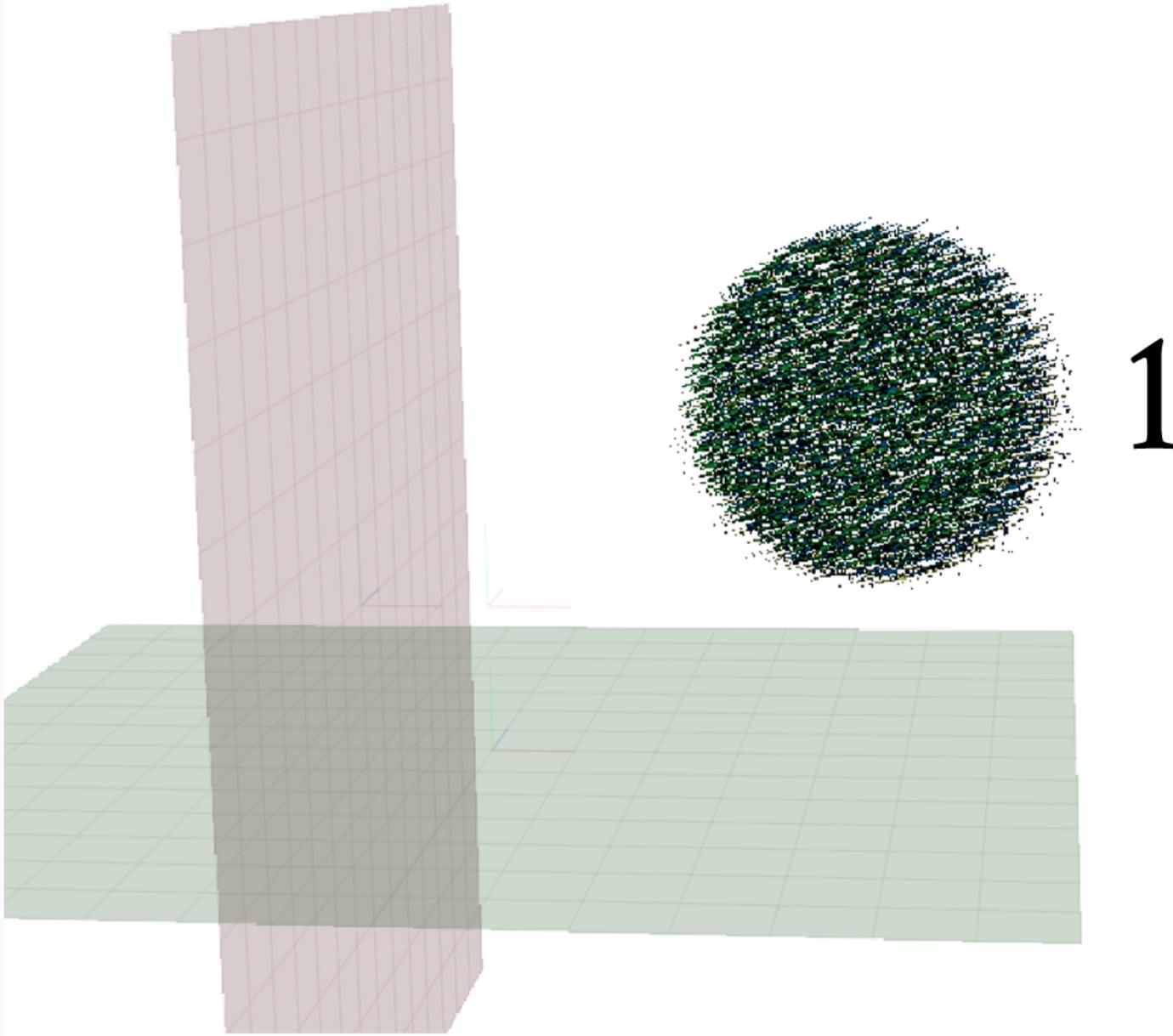


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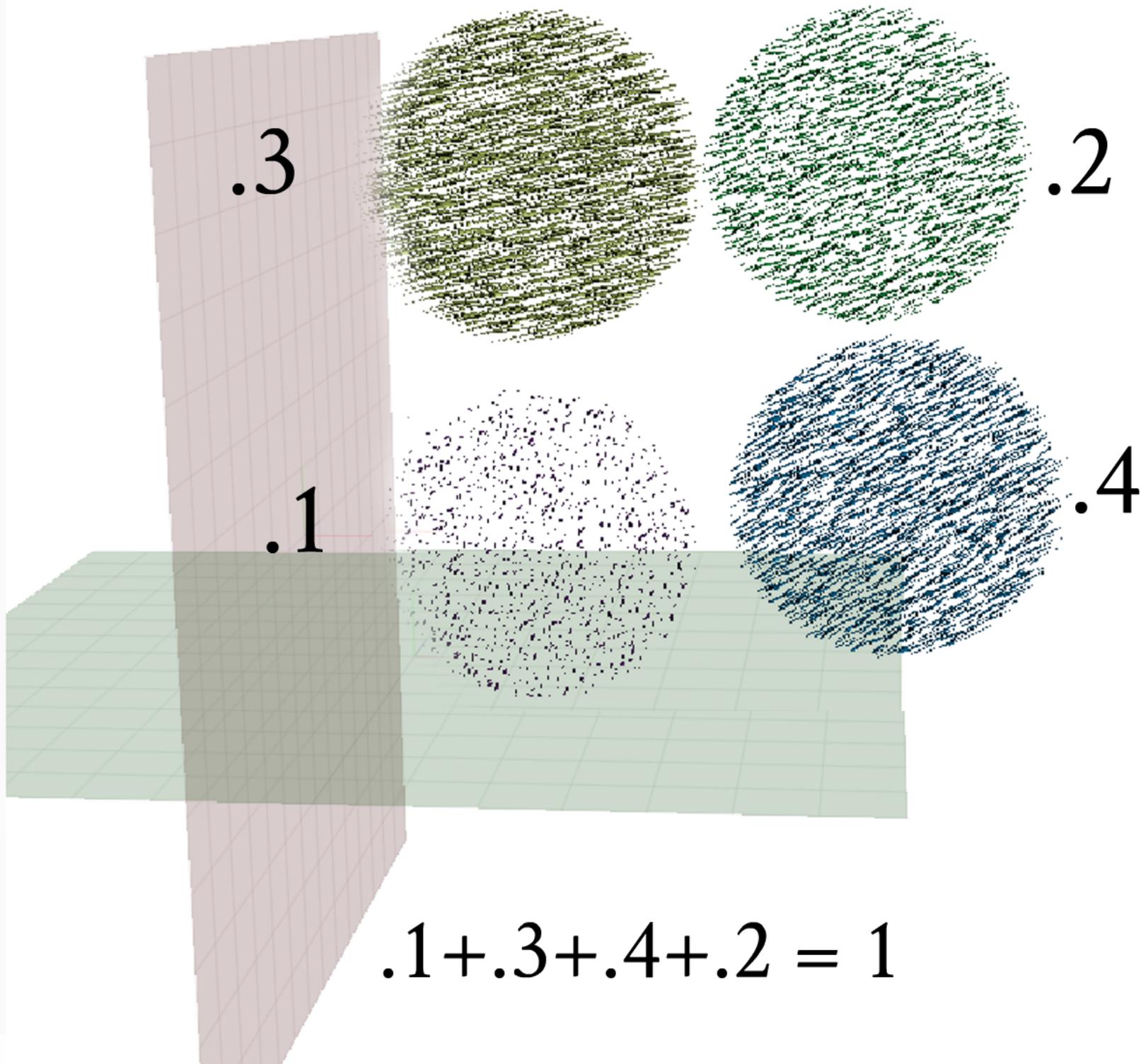
No!



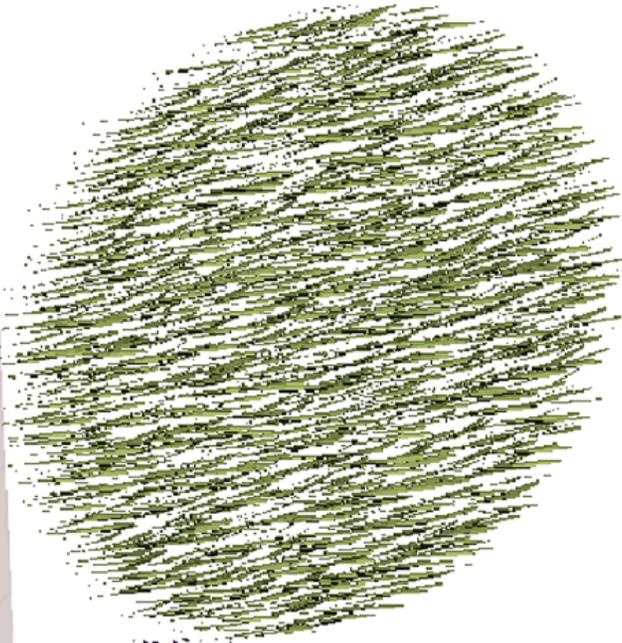




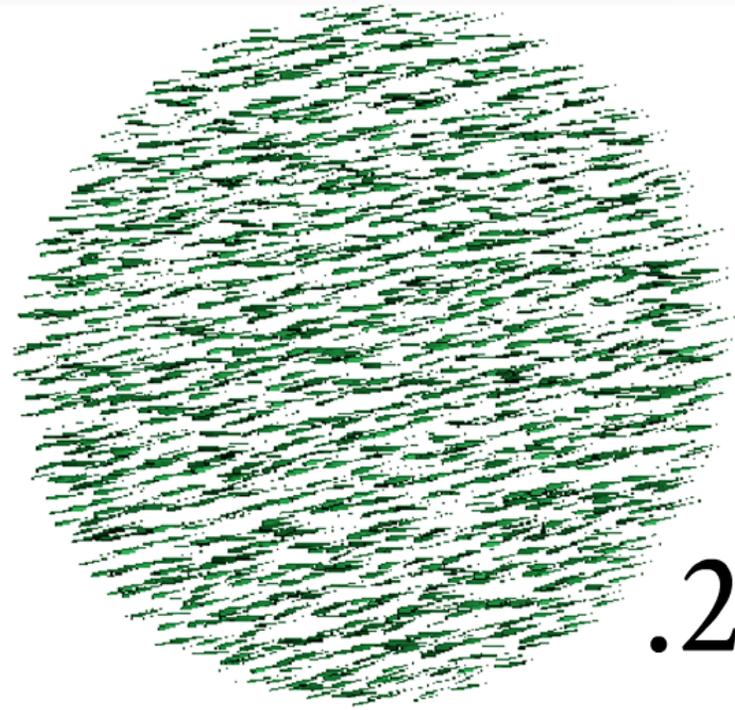
$$\mathfrak{F}(x, y) = \{1\} \uplus \mathfrak{W}(x) \uplus \mathfrak{W}(x^{-1}) \uplus \mathfrak{W}(y) \uplus \mathfrak{W}(y^{-1})$$



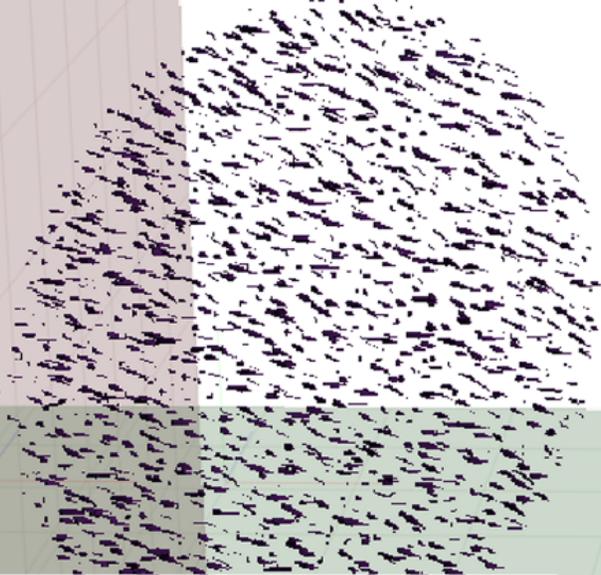
.3



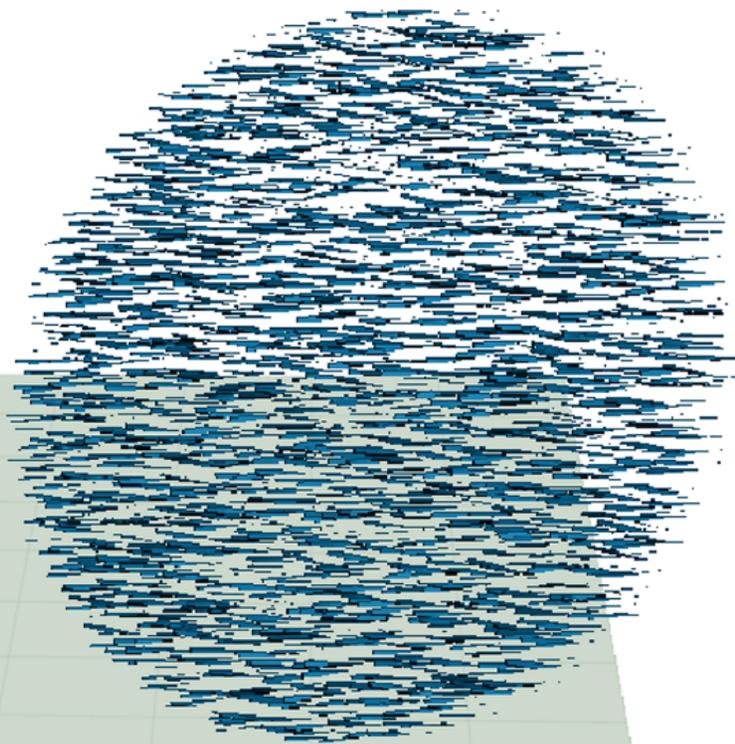
.2



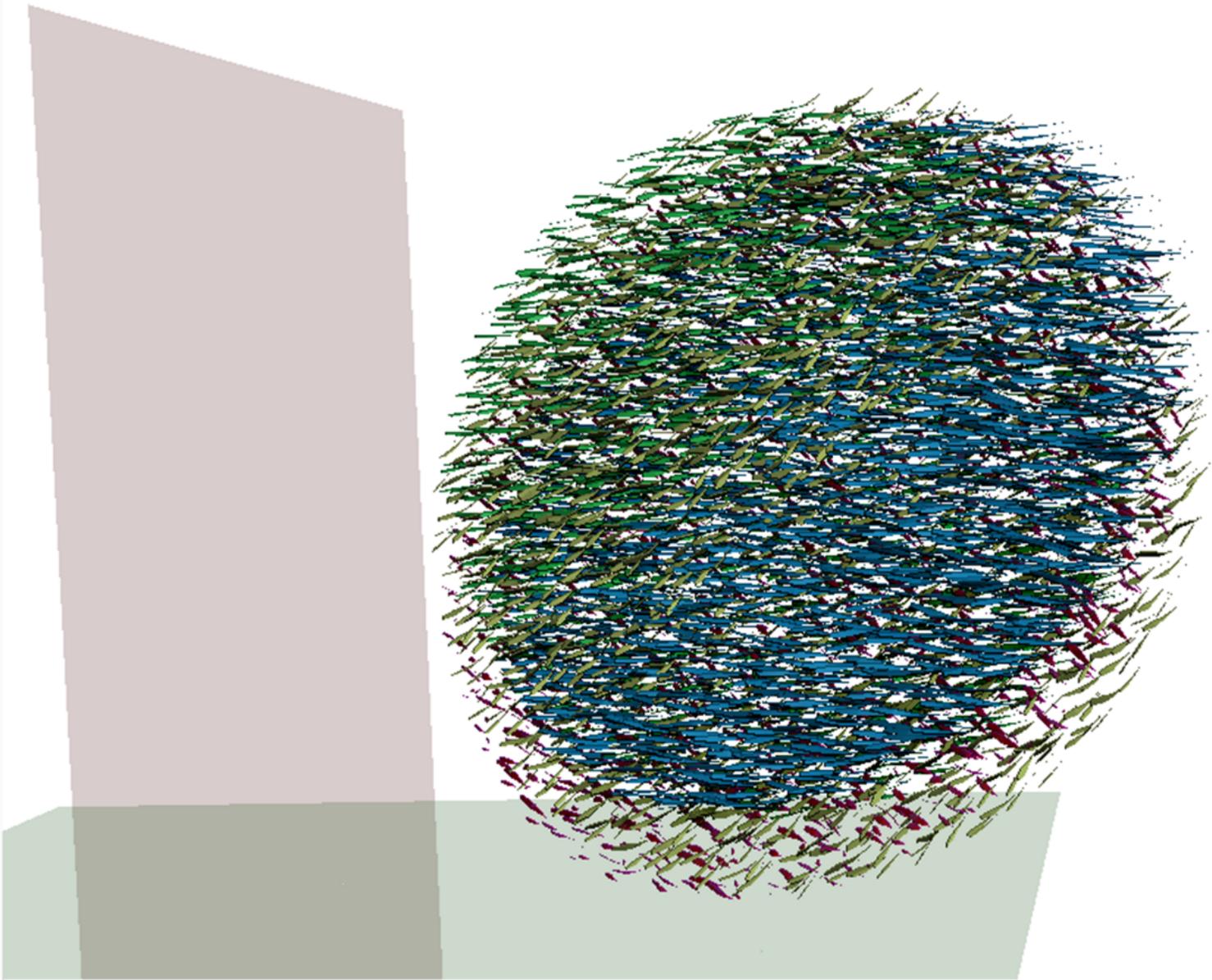
.1



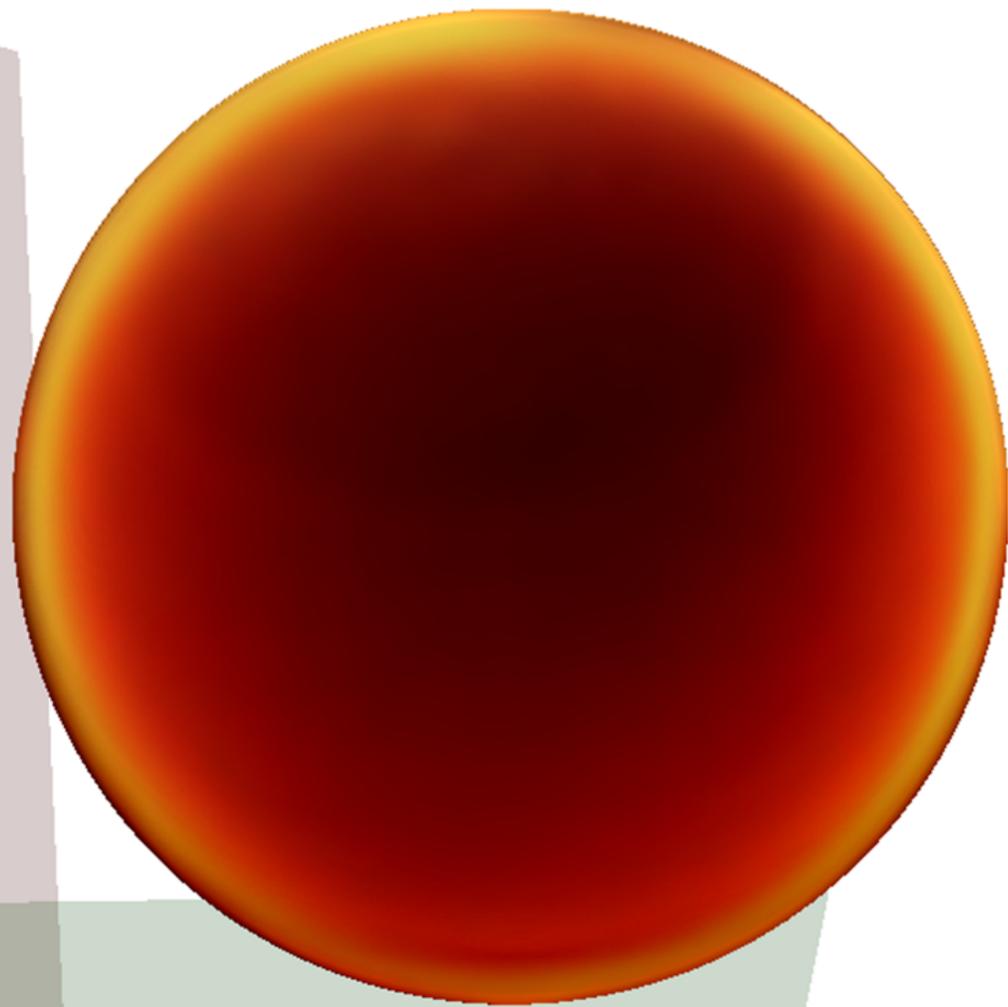
.4



$$\phi^{\pm 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & \mp \frac{2\sqrt{2}}{3} \\ 0 & \pm \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & \mp 2\sqrt{2} \\ 0 & \pm 2\sqrt{2} & 1 \end{pmatrix}$$
$$\psi^{\pm 1} = \begin{pmatrix} \frac{1}{3} & \mp \frac{2\sqrt{2}}{3} & 0 \\ \pm \frac{2\sqrt{2}}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & \mp 2\sqrt{2} & 0 \\ \pm 2\sqrt{2} & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$



$$\mathfrak{F}(\varphi, \psi) = 1 \uplus \mathfrak{W}(\varphi) \uplus \mathfrak{W}(\varphi^{-1}) \uplus \mathfrak{W}(\psi) \uplus \mathfrak{W}(\psi^{-1})$$



2

$$1 = .1 + .3 + .4 + .2 = 2 \text{ ???}$$

$\mathfrak{H}(y^{-1})$

$.1+.3+.4+.2 = 1$

$.4$

$$y^{0.1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

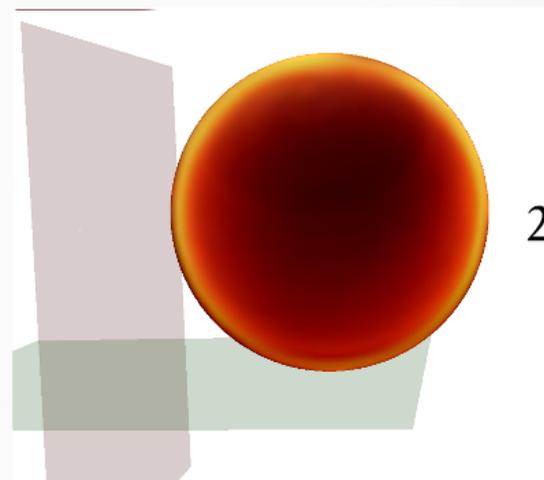
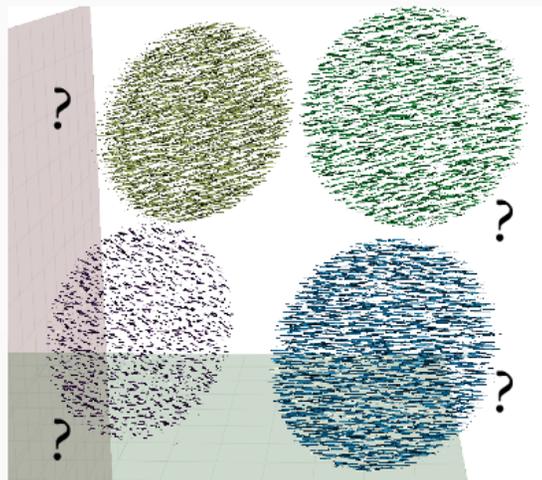
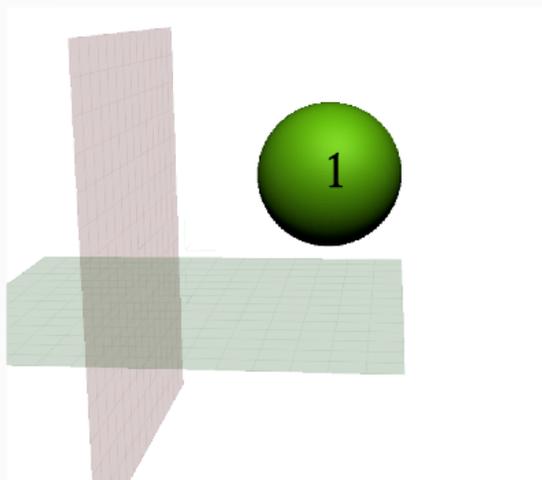
$$y^{0.3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$y^{0.4} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

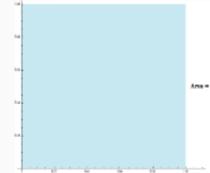
$\mathfrak{F}(\varphi, \psi) = 1 \oplus$

Did they break math?

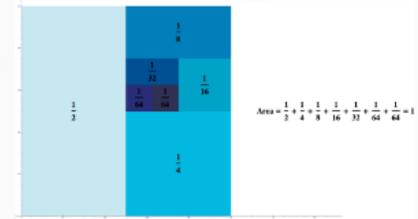
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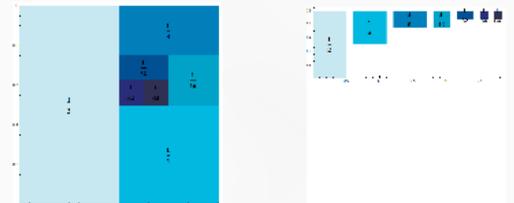
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The cutting an object into pieces should not change its area.



Moving or rotating an object should not change its area.



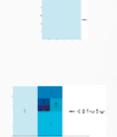
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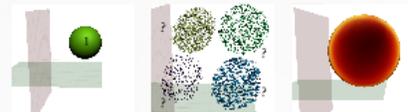


The Banach Tarski paradox - The Pea and The Sun



Did they break math?

No!



The pea and the sun

Area

Young man, in mathematics you don't understand things, you just get used to them.

-John von Neumann

The Calculus

How to measure the area under a curve?

What is area?

What is life? ■

Breathes
Consumes energy
Grows
Reproduces
Dies

Fire?

is cool!

paradoxical results.

Predict length of stay.

The pea and the sun

Area
Young man, in mathematics you don't understand things, you just get used to them.
-John von Neumann

The dough and the pan

Infinite dimensional space

A mathematician is a blind man in a dark room looking for a black hat which isn't there.
might be -Charles Darwin

Applications

Computers have limitations

Numbers

Can an infinite hotel be full?

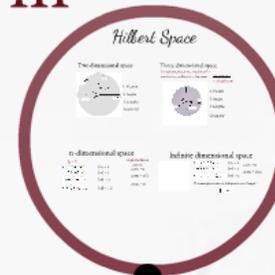
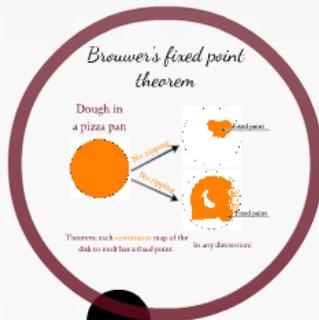
It is in contemplation of the infinite that man achieves his greatest good.
-Giordano Bruno

Infinite dimensional space

A mathematician is a blind man in a dark room looking for a black hat which ~~isn't~~ there.

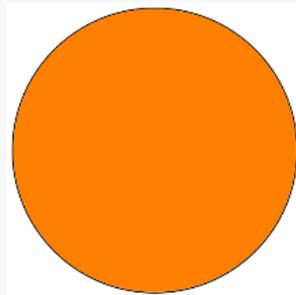
might be

-Charles Darwin



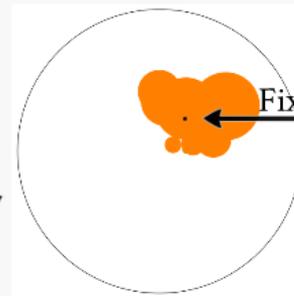
Brouwer's fixed point theorem

Dough in a pizza pan

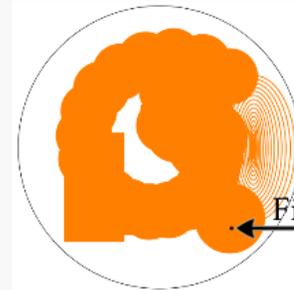


No ripping

No ripping



Fixed point

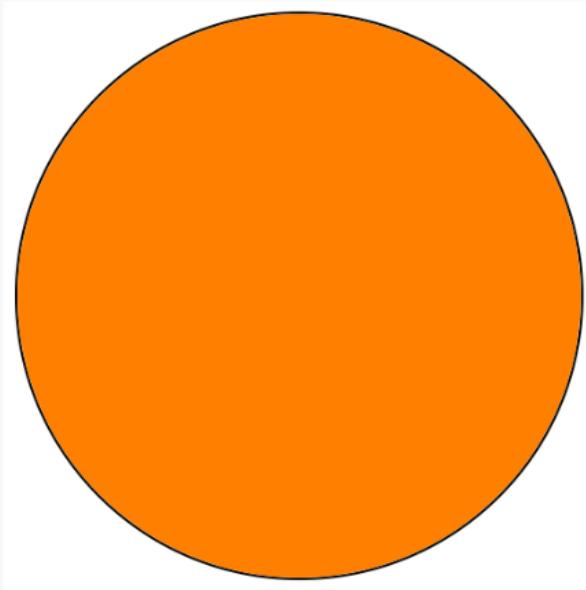


Fixed point

Theorem: each **continuous** map of the disk to itself has a fixed point.

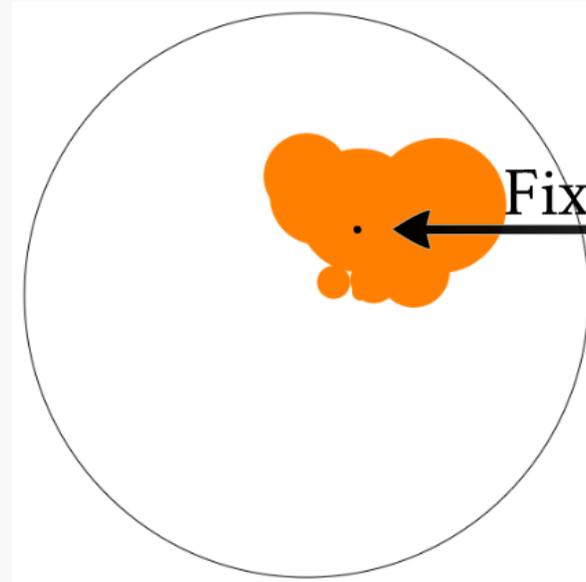
In any dimension!

Dough in a pizza pan

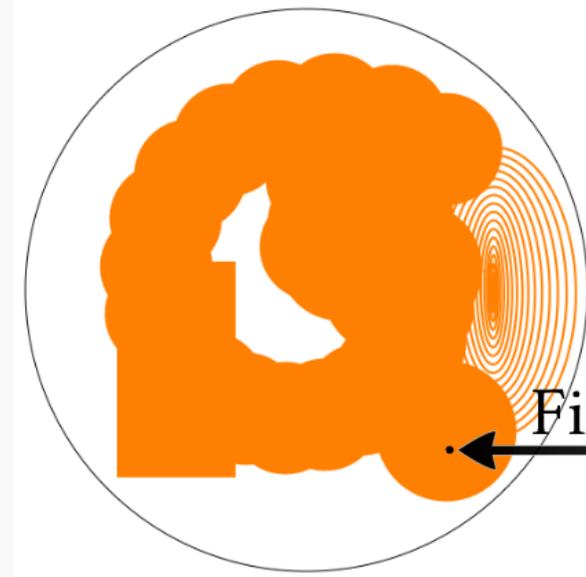


No ripping

No ripping



Fixed point



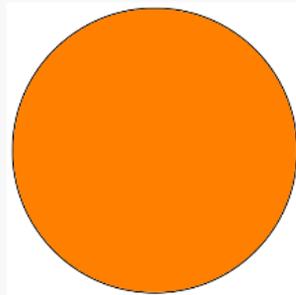
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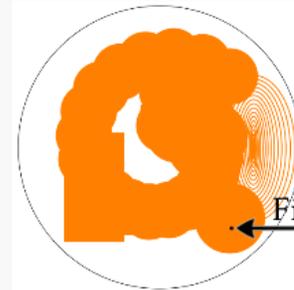
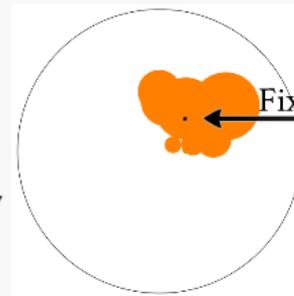
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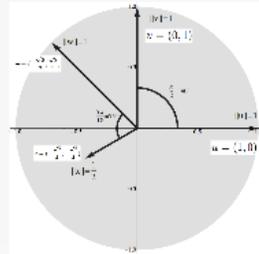


Theorem: each **continuous** map of the disk to itself has a fixed point.

In any dimension!

Hilbert Space

Two dimensional space

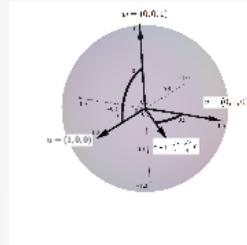


1. Vectors
 2. Angles
 3. Lengths
- Geometry

Three dimensional space

A mathematician is a machine for converting coffee into theorems.

~~Paul Erdos~~
-Alfred Renyi



1. Vectors
 2. Angles
 3. Lengths
- Geometry

n-dimensional space

(n = 7)

$$u = (1, 0, 0, 0, 0, 0, 0)$$

$$\|u\| = 1$$

$$v = (0, 0, 0, 0, 0, 1, 0)$$

$$\|v\| = 1$$

$$w = (0, 0, \frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}, 0)$$

$$\|w\| = 1$$

$$x = (-\frac{\sqrt{2}}{4}, 0, 0, 0, 0, 0, \frac{1}{4})$$

$$\|x\| = 1/2$$

Angles between

vectors

$$\langle u, v \rangle = 0$$

$$\langle v, w \rangle = 1/2$$

$$\langle w, x \rangle = 0$$

Infinite dimensional space



$$u = (1, 0, 0, 0, 0, \dots)$$

$$\|u\| = 1$$

$$\langle u, v \rangle = 0$$

$$v = (0, 0, 0, 1, 0, \dots)$$

$$\|v\| = 1$$

$$\langle v, w \rangle = 1/16$$

$$w = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{16}}, \frac{1}{\sqrt{32}}, \frac{1}{\sqrt{64}}, \dots)$$

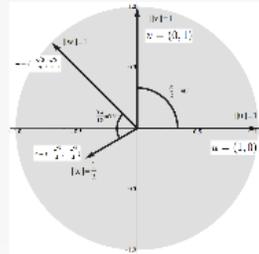
$$\|w\| = 1$$

The unit sphere is the set of all vectors x with length 1:

$$\|x\| = 1$$

Hilbert Space

Two dimensional space

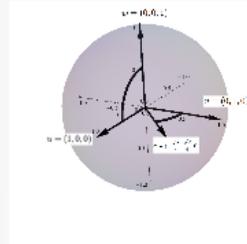


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n-dimensional space

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Angles between

vectors

$$\langle u, v \rangle = 0$$

$$\langle v, w \rangle = 1/2$$

$$\langle w, x \rangle = 0$$

Infinite dimensional space



$$u = (1, 0, 0, 0, 0, \dots)$$

$$\|u\| = 1$$

$$\langle u, v \rangle = 0$$

$$v = (0, 0, 0, 1, 0, \dots)$$

$$\|v\| = 1$$

$$\langle v, w \rangle = 1/16$$

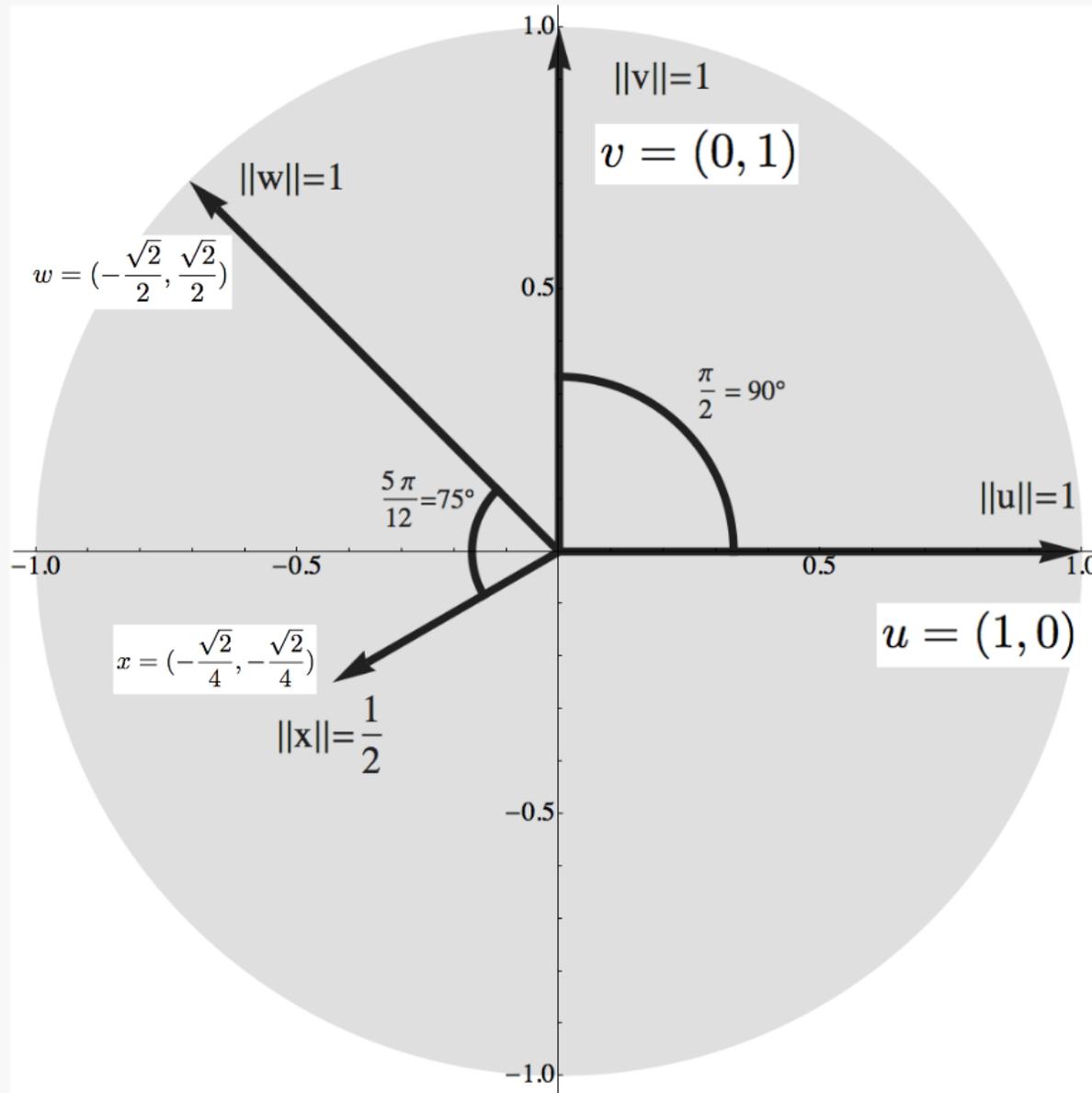
$$w = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{16}}, \frac{1}{\sqrt{32}}, \frac{1}{\sqrt{64}}, \dots)$$

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The unit sphere is the set of all vectors x with length 1:

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Two dimensional space



1. Vectors

2. Angles

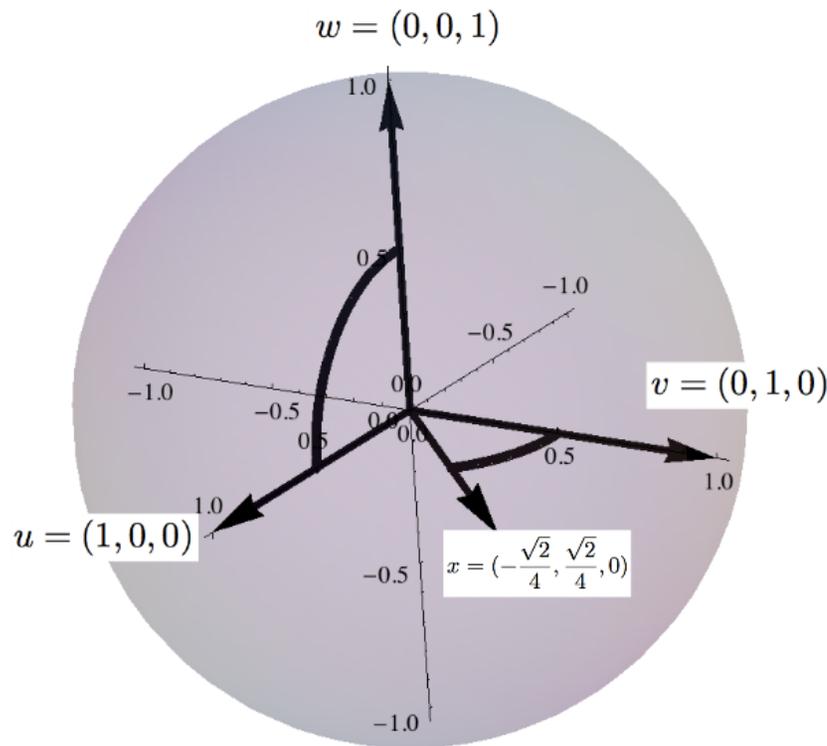
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Geometry

n-dimensional space

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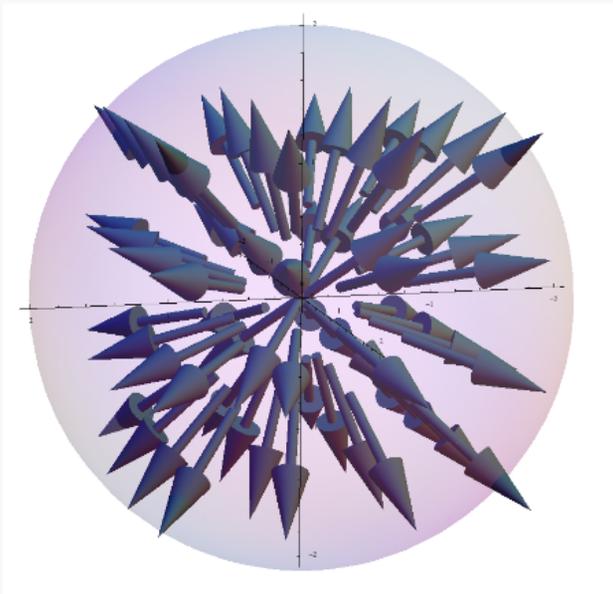
Angles between
vectors

$$\langle u, v \rangle = 0$$

$$\langle v, w \rangle = 1/2$$

$$\langle w, x \rangle = 0$$

Infinite dimensional space



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$$\langle u, v \rangle = 0$$

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$$w = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{16}}, \frac{1}{\sqrt{32}}, \frac{1}{\sqrt{64}}, \dots\right)$$

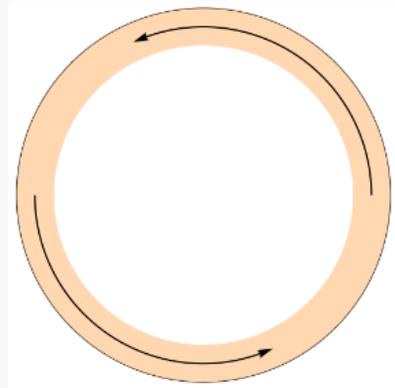
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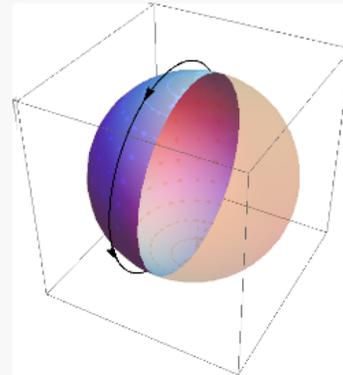
$$\|x\| = 1$$

Brouwer's Theorem in infinite dimensional space

If we could break the rules



In infinite dimensional space we don't have to



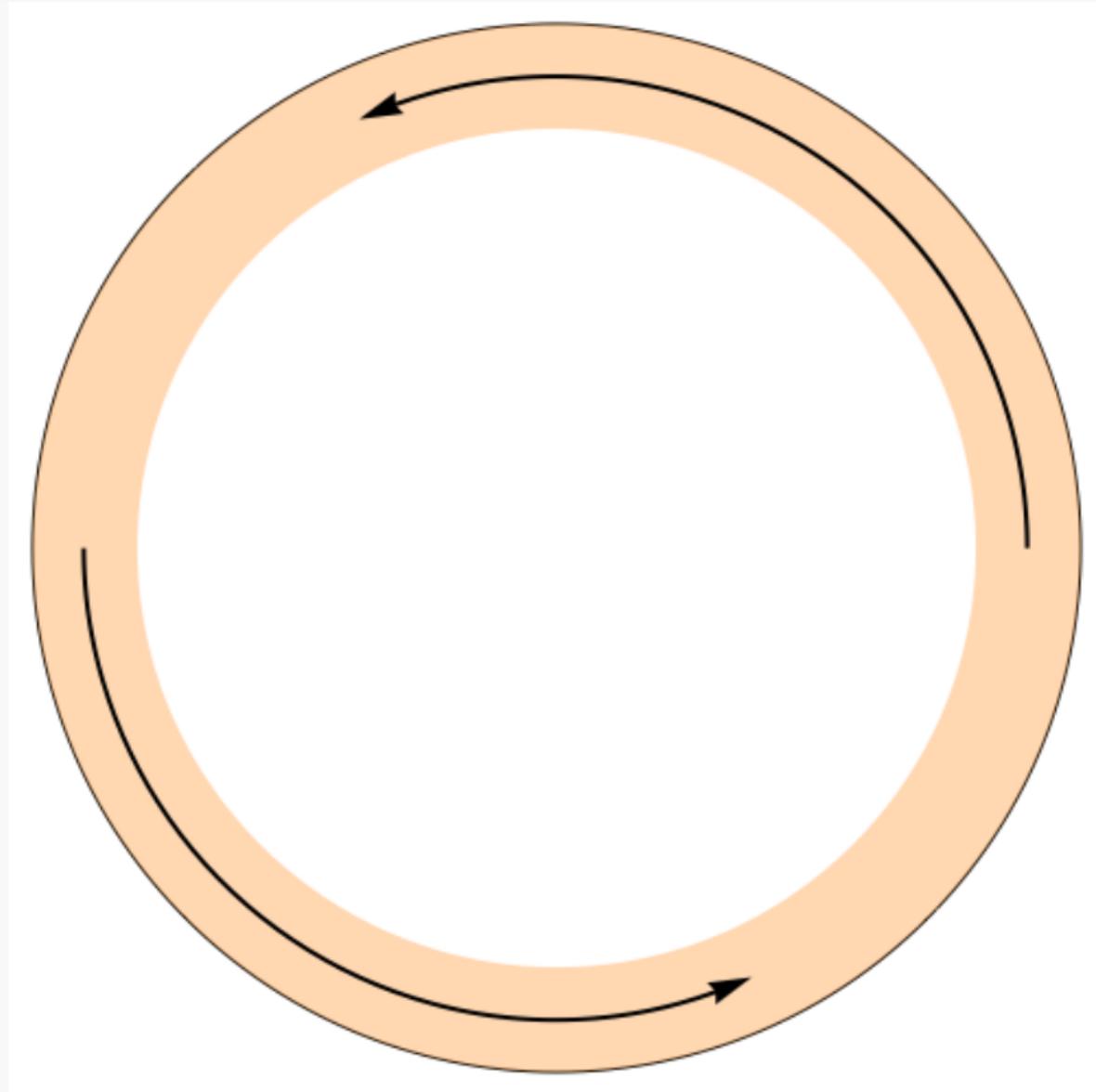
How?

$$x = (x_1, x_2, x_3, x_4, x_5, \dots) \rightarrow (\sqrt{1 - \|x\|^2}, x_1, x_2, x_3, x_4, \dots)$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

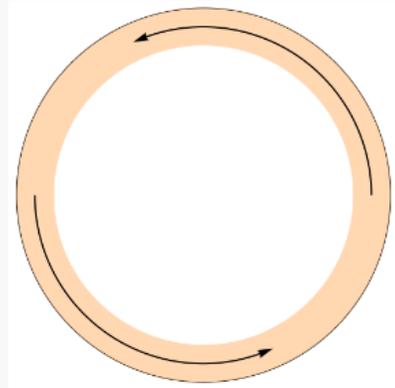
Familiar?

If we could break the rules

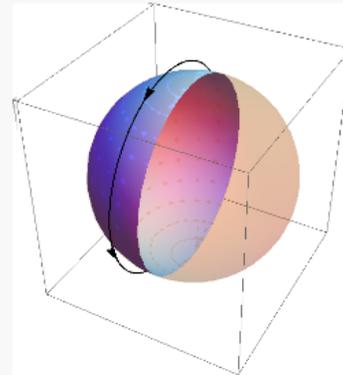


Brouwer's Theorem in infinite dimensional space

If we could break the rules



In infinite dimensional space we don't have to



How?

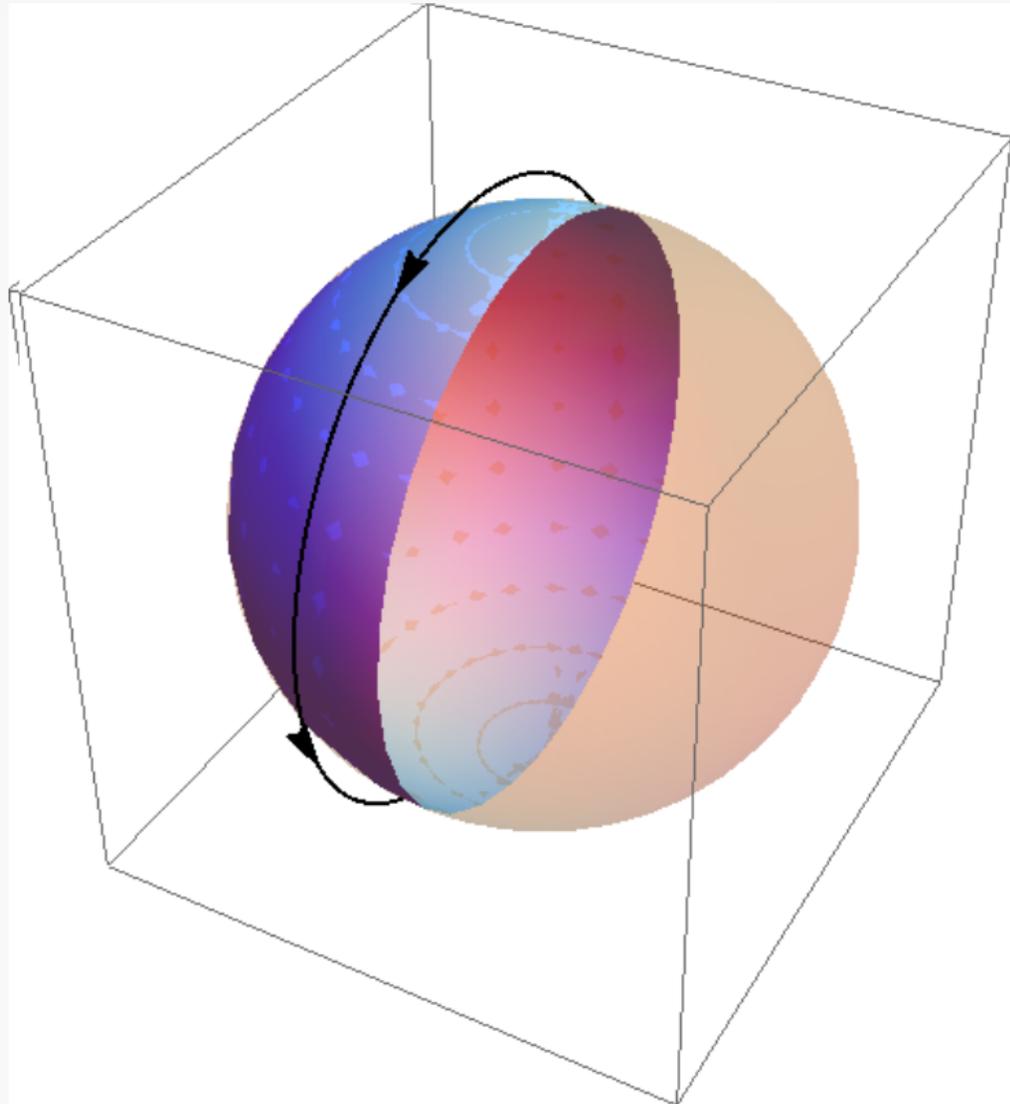
$$x = (x_1, x_2, x_3, x_4, x_5, \dots) \rightarrow (\sqrt{1 - \|x\|^2}, x_1, x_2, x_3, x_4, \dots)$$

x_1 x_2 x_3 x_4 x_5

Familiar?

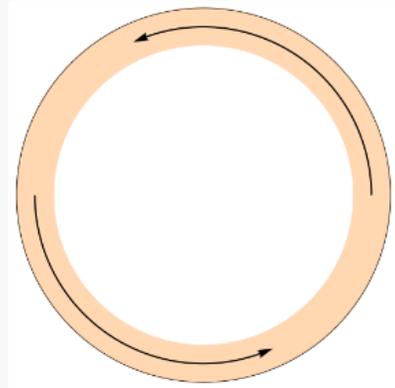
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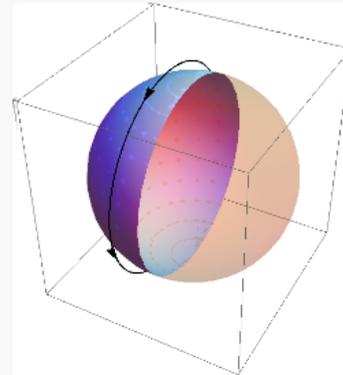


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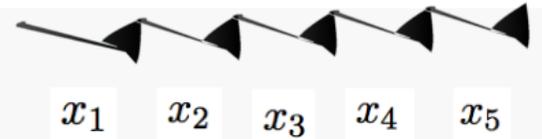
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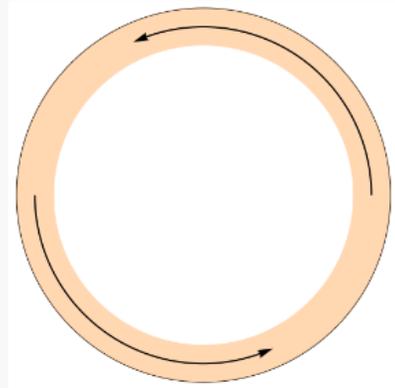
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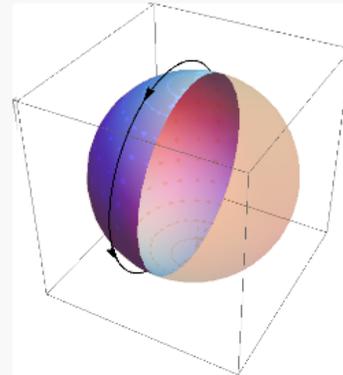
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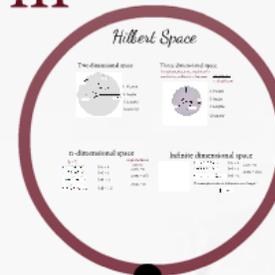
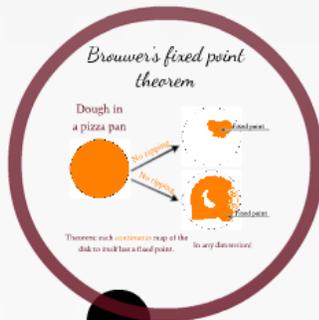
Familiar?

Infinite dimensional space

A mathematician is a blind man in a dark room looking for a black hat which ~~isn't~~ there.

might be

-Charles Darwin



is cool!

paradoxical results.

Predict length of stay.

The pea and the sun

Area
Young man in mathematics you don't understand things, you just get used to them.
-John von Neumann

The dough and the pan

Infinite dimensional space

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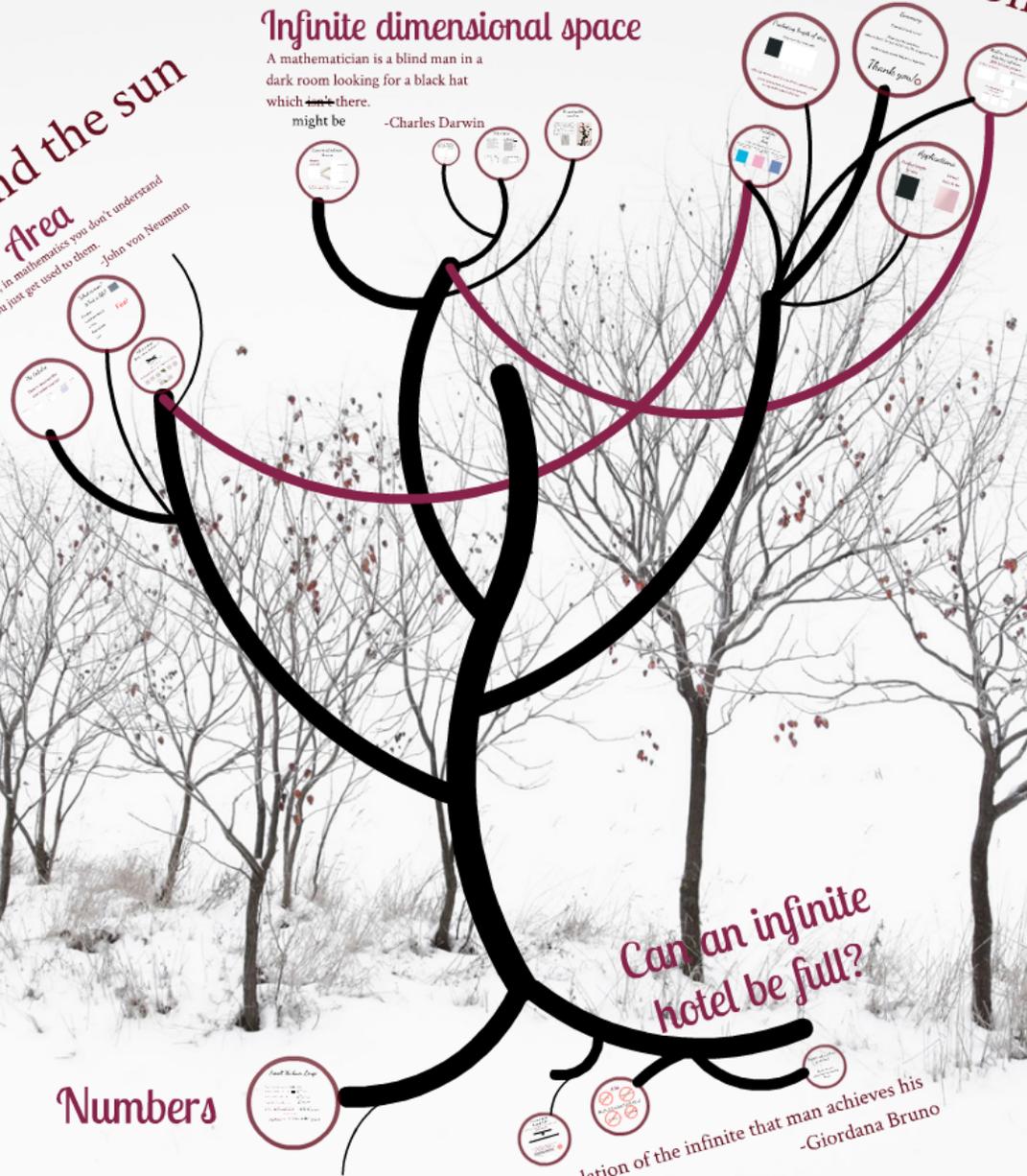
Applications

Computers have limitations

Numbers

Can an infinite hotel be full?

It is in contemplation of the infinite that man achieves his greatest good.
-Giordano Bruno



Summary

Theoretical math is cool!

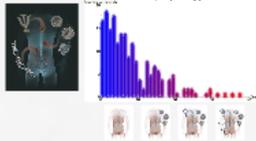
Three apparent paradoxes:
Hilbert's Hotel, The pea and the sun, The dough and the pan.

Predict length of stay. Help detect infections.

Thank you!

Predicting length of stay

Back surgery for chronic pain



Although we can never be sure about a specific patient.

Given information about many patients,
we can make some predictions.

Probability and Area

1654 letter from Blaise Pascal to Pierre de Fermat



Machine learning and detecting infection IBM Artemis project

Prediction/Classification Higher dimensions



Applications

Predict length
of stay

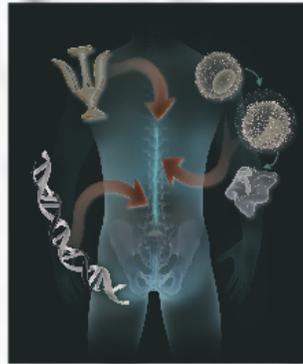


Detect
infections

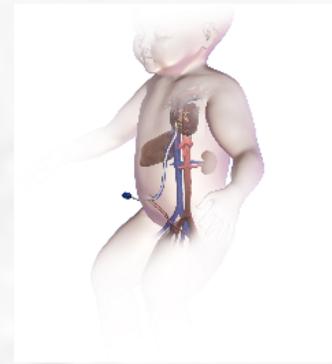


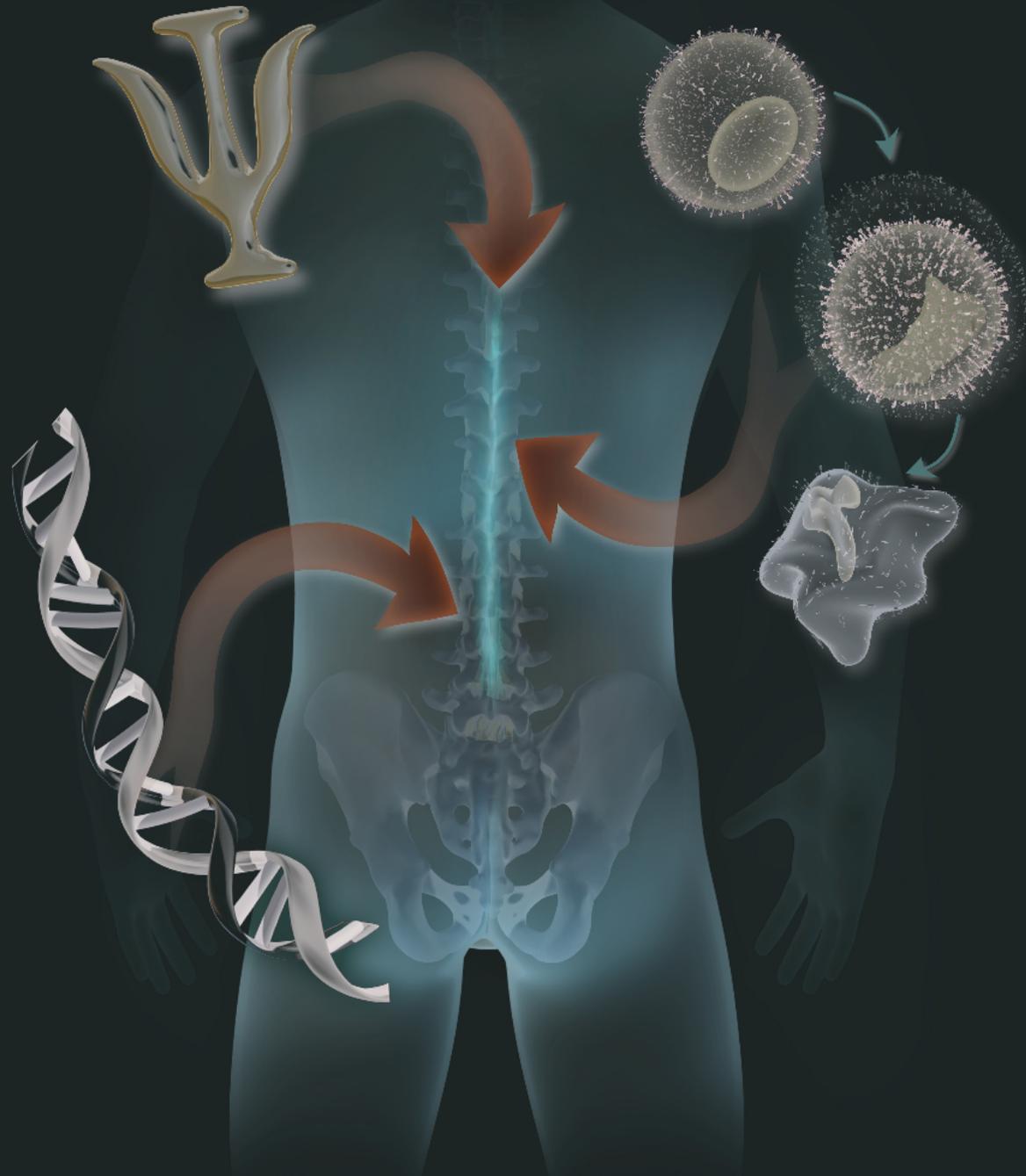
Applications

Predict length
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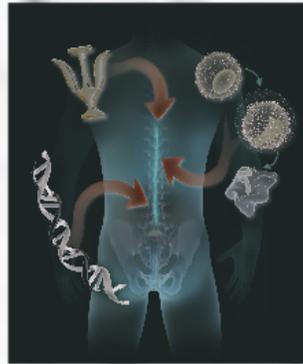
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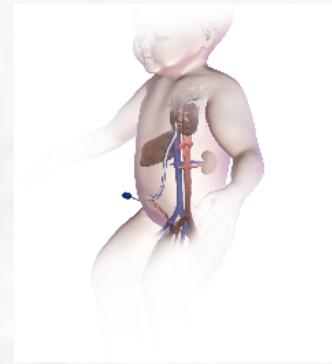


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Predict length
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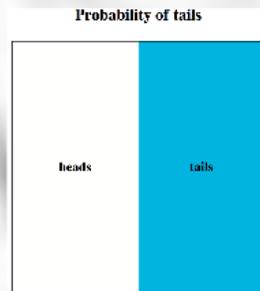


Probability and Area

1654 letter from Blaise Pascal to Pierre de Fermat

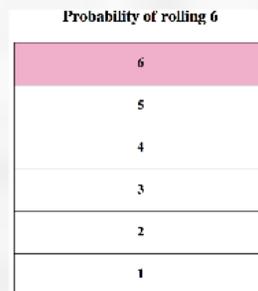
What is the probability that a fair coin will land tails?

$$1/2$$



What is the probability that a fair die will land on 6?

$$1/6$$



What is the probability of both tails and a 6?

$$1/2 * 1/6 = 1/12$$



Area

1654 letter from Blaise Pascal to Pierre de Fermat

What is the probability that a fair coin will land tails?

$$1/2$$

heads	tails
-------	-------

What is the probability that a fair die will land on 6?

$$1/6$$

6
5
4
3
2
1

What is the probability of both tails and a 6?

$$1/2 * 1/6 = 1/12$$

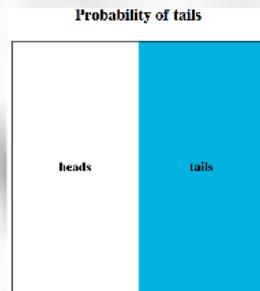
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Probability and Area

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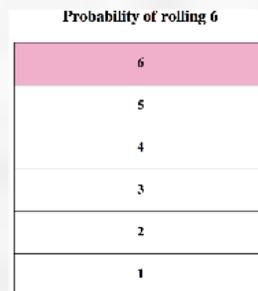
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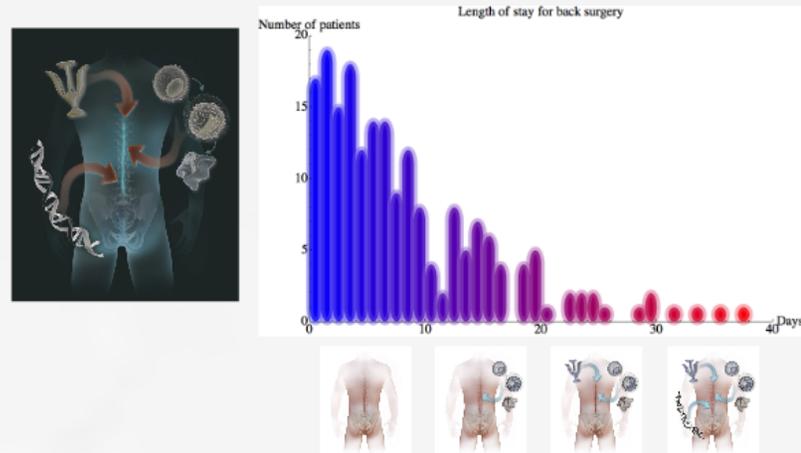
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Predicting length of stay

Back surgery for chronic pain

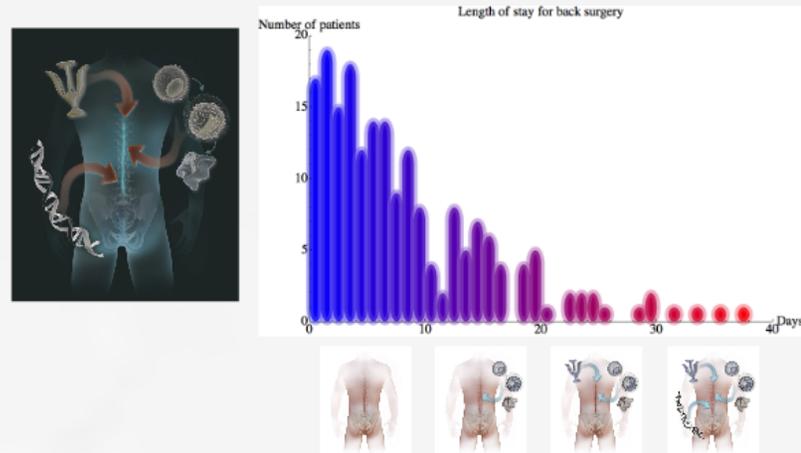


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Given information about many patients,
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Predicting length of stay

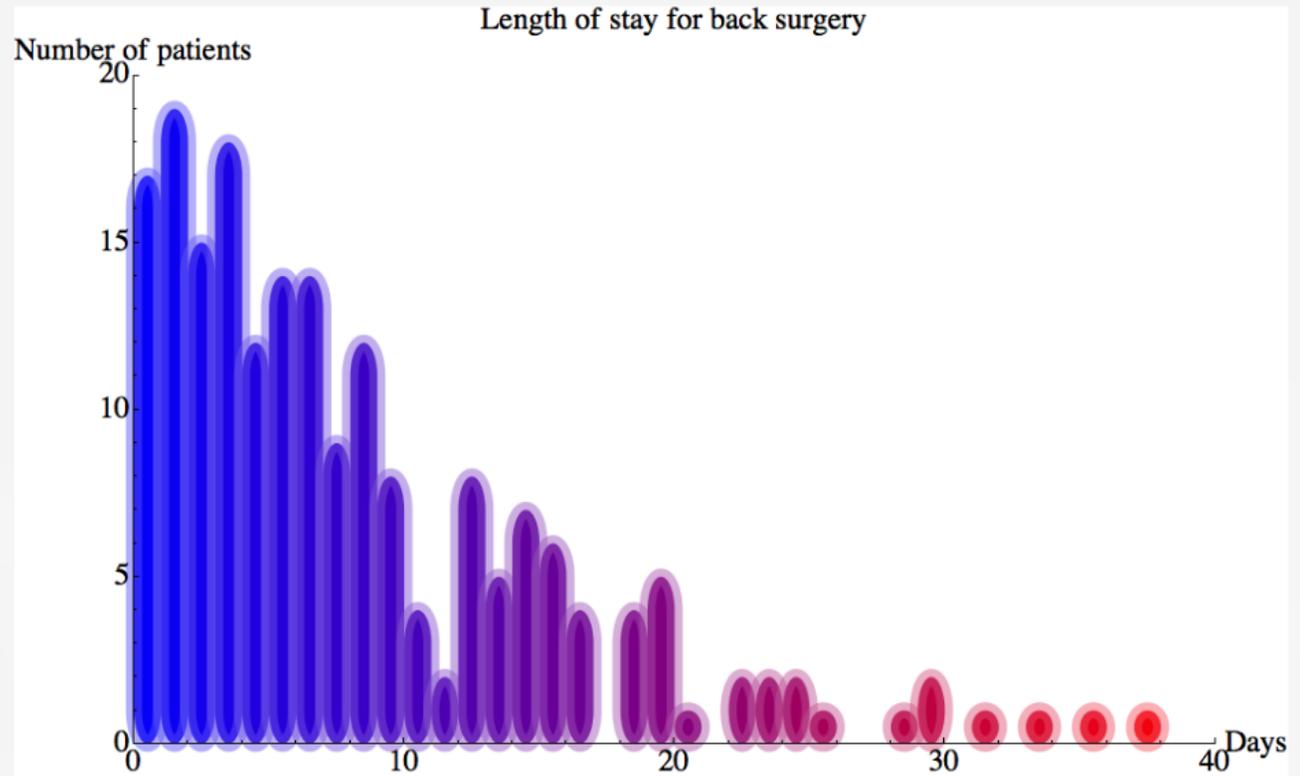
Back surgery for chronic pain



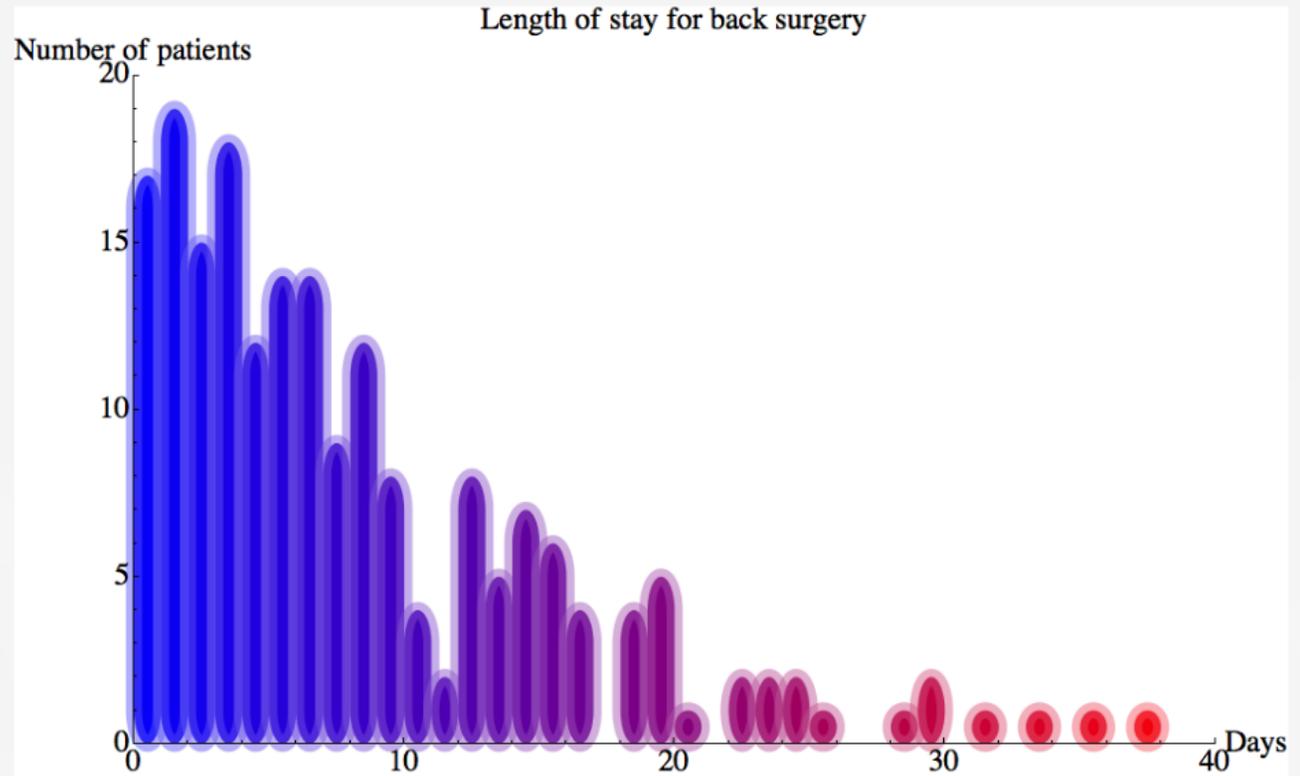
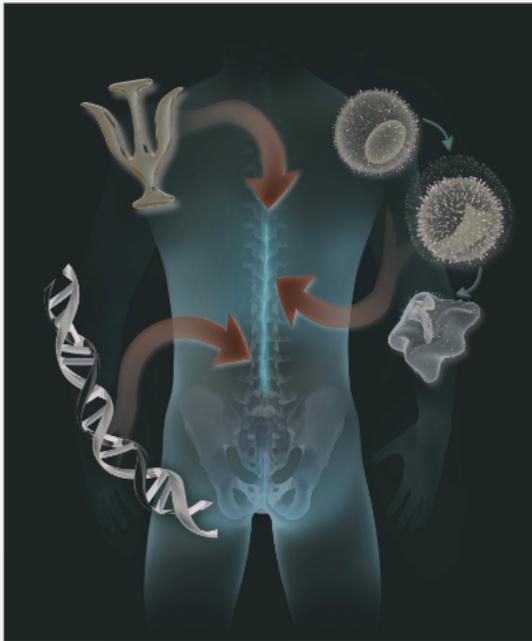
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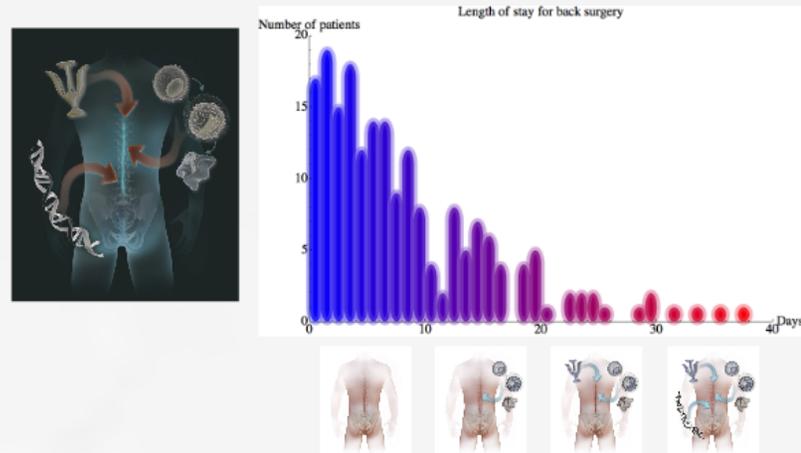


Back surgery for chronic pain



Predicting length of stay

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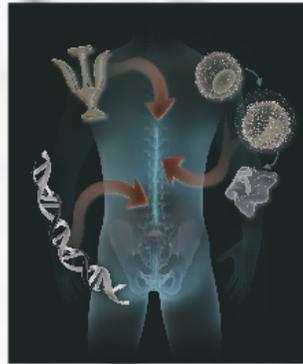


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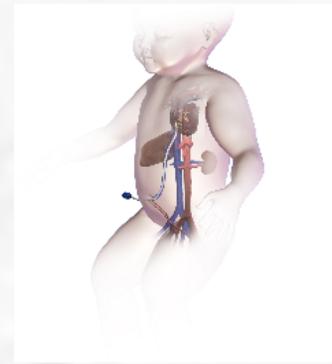
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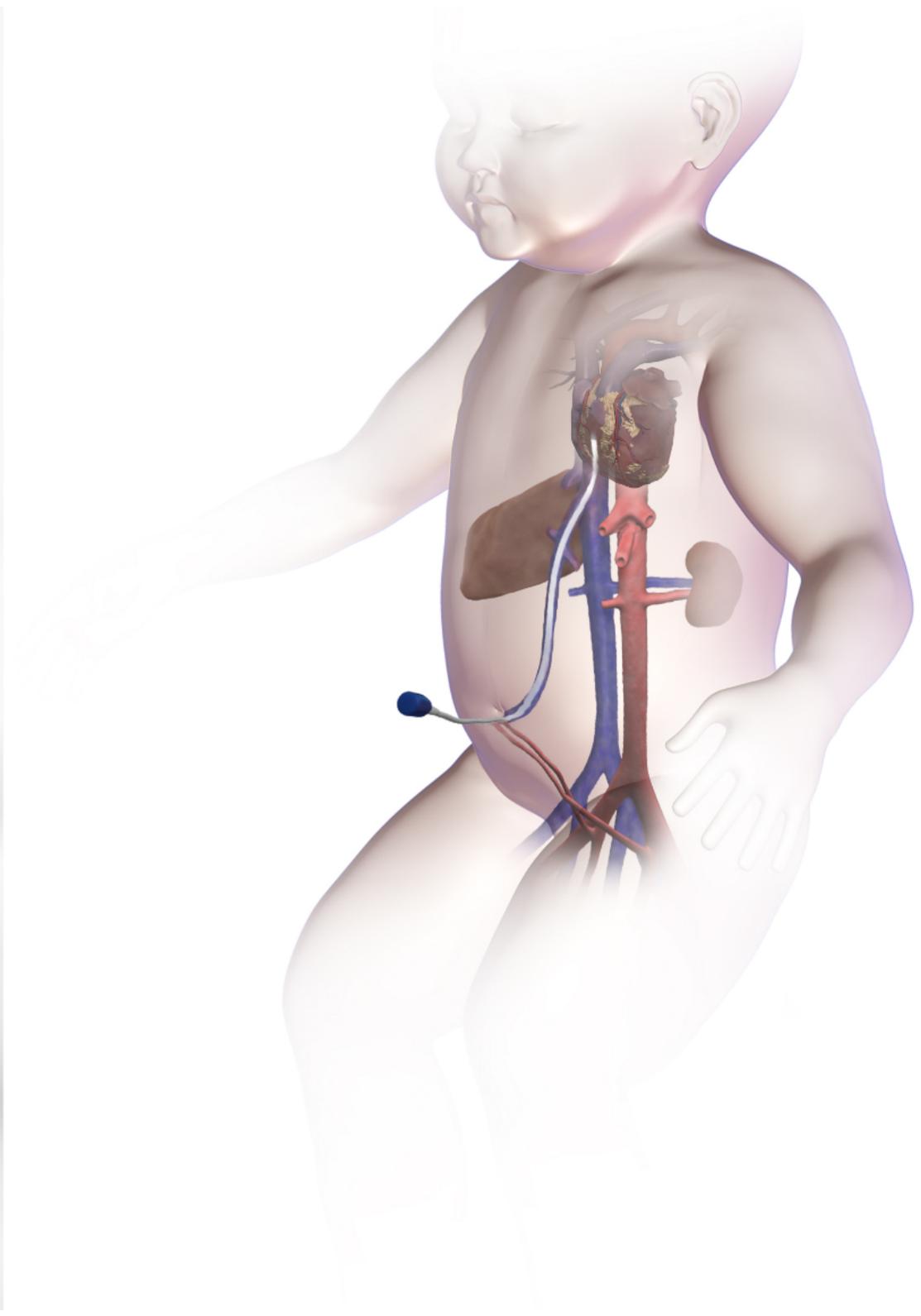
Applications

Predict length
of stay



Detect
infections

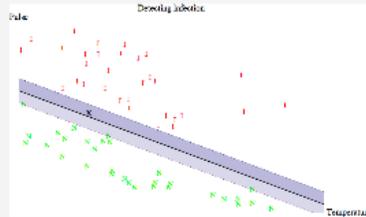




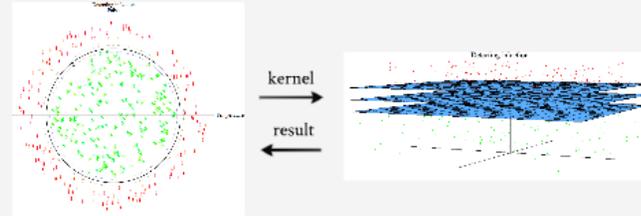
Machine learning and detecting infection

IBM Artemis project

Prediction/Classification

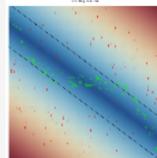


Higher dimensions

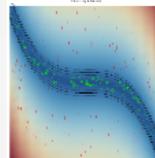


How many dimensions
are enough?

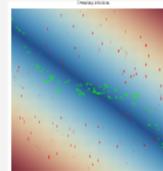
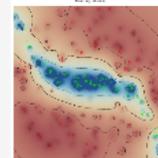
1,000,000,000?



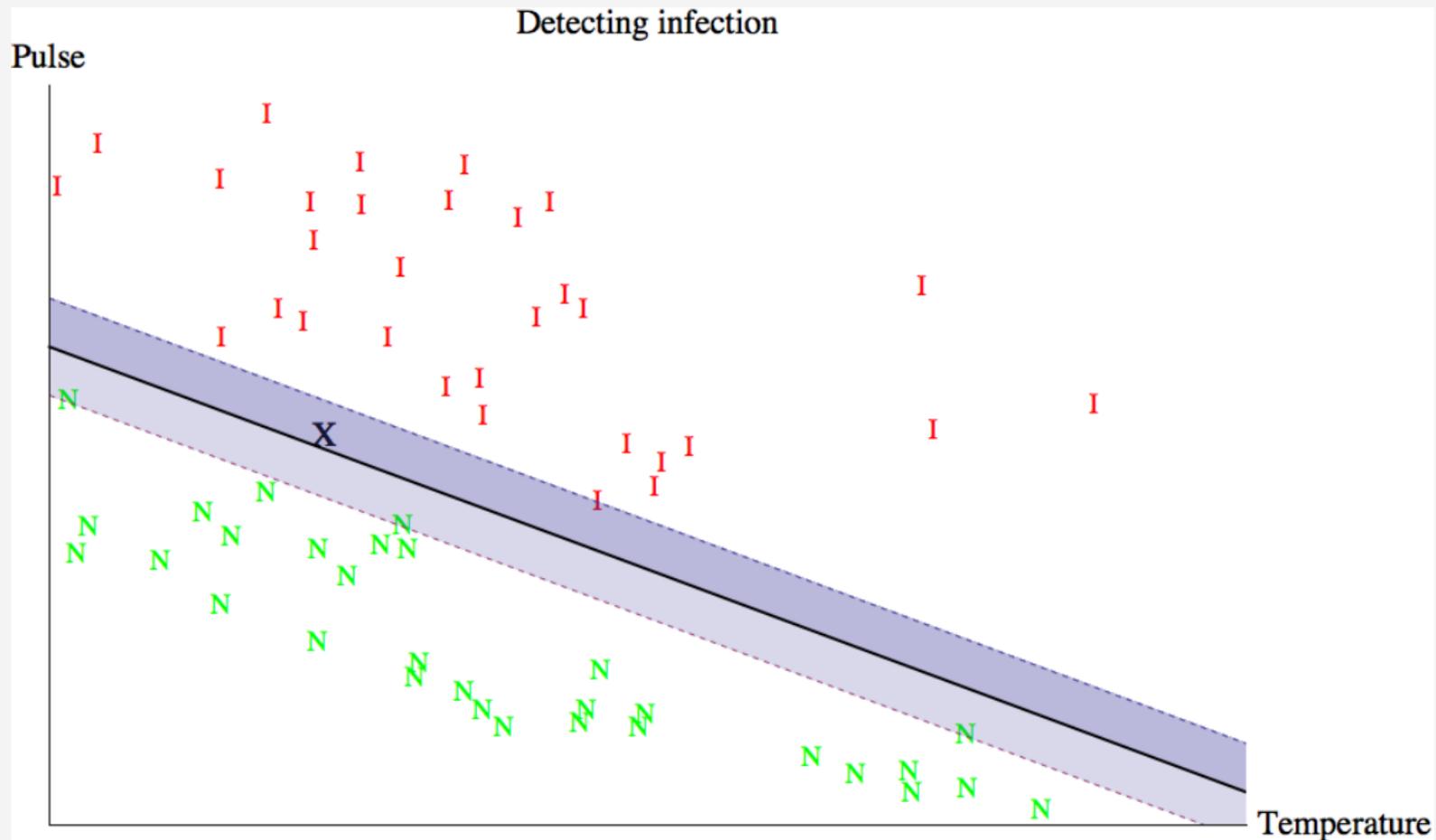
Infinitely many!



Too many?



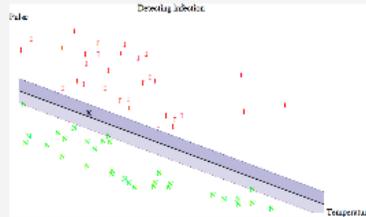
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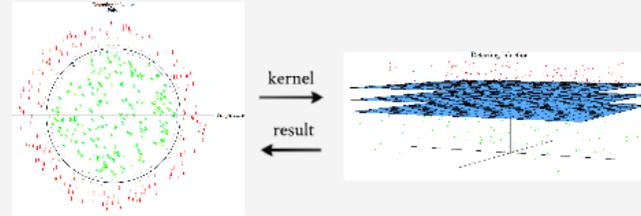
Machine learning and detecting infection

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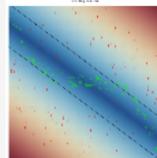


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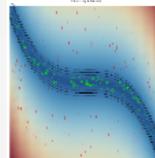


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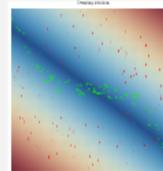
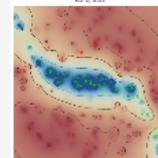
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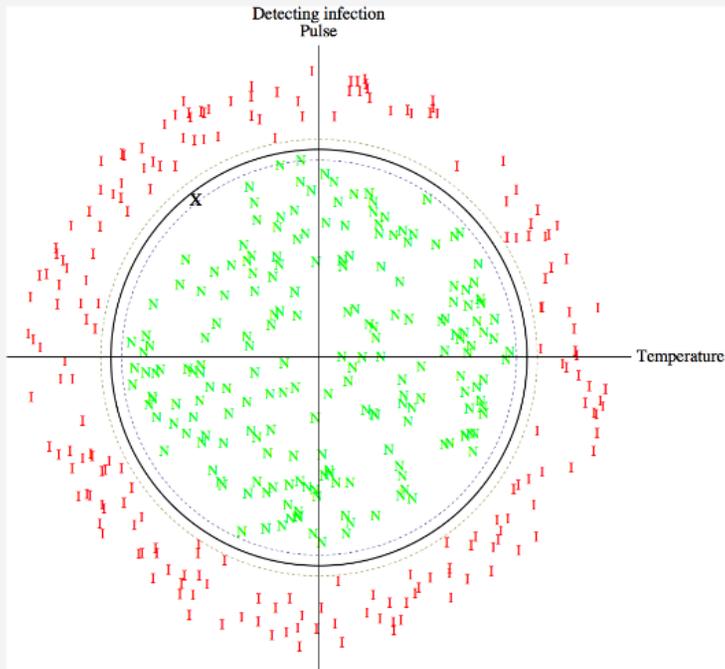


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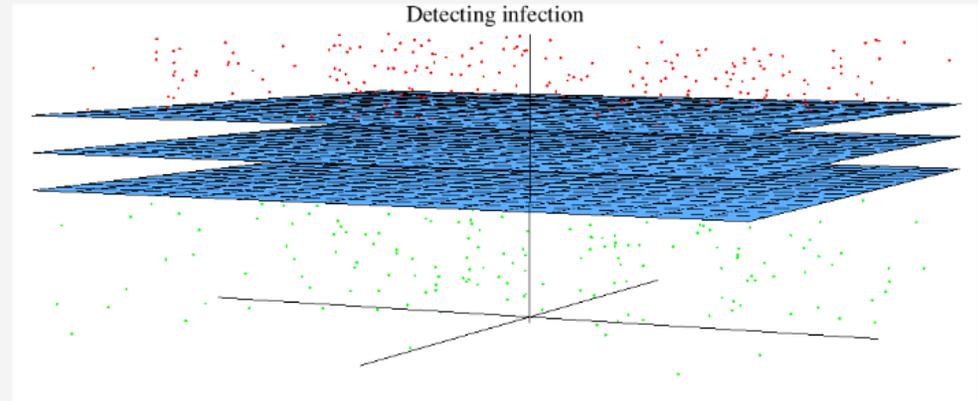


emmis project

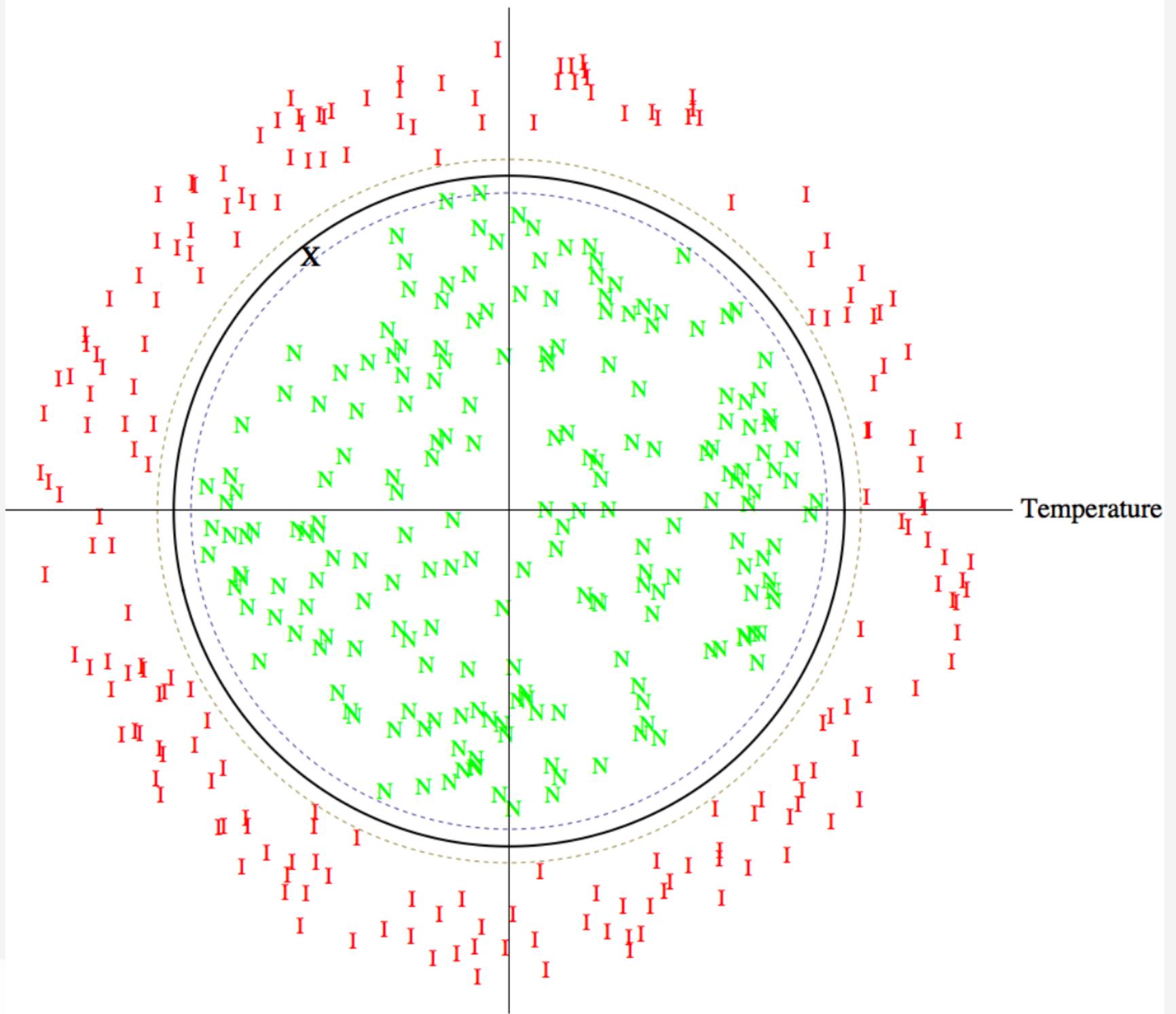
Higher dimensions



kernel
→
result
←



Detecting infection
Pulse



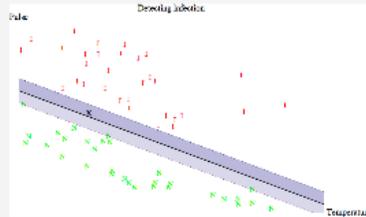
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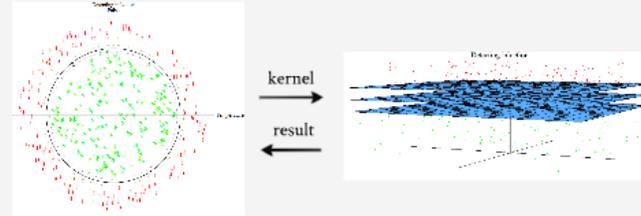
Machine learning and detecting infection

IBM Artemis project

Prediction/Classification

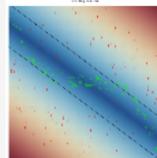


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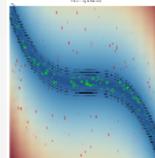


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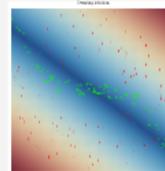
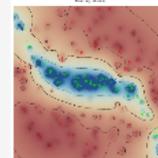
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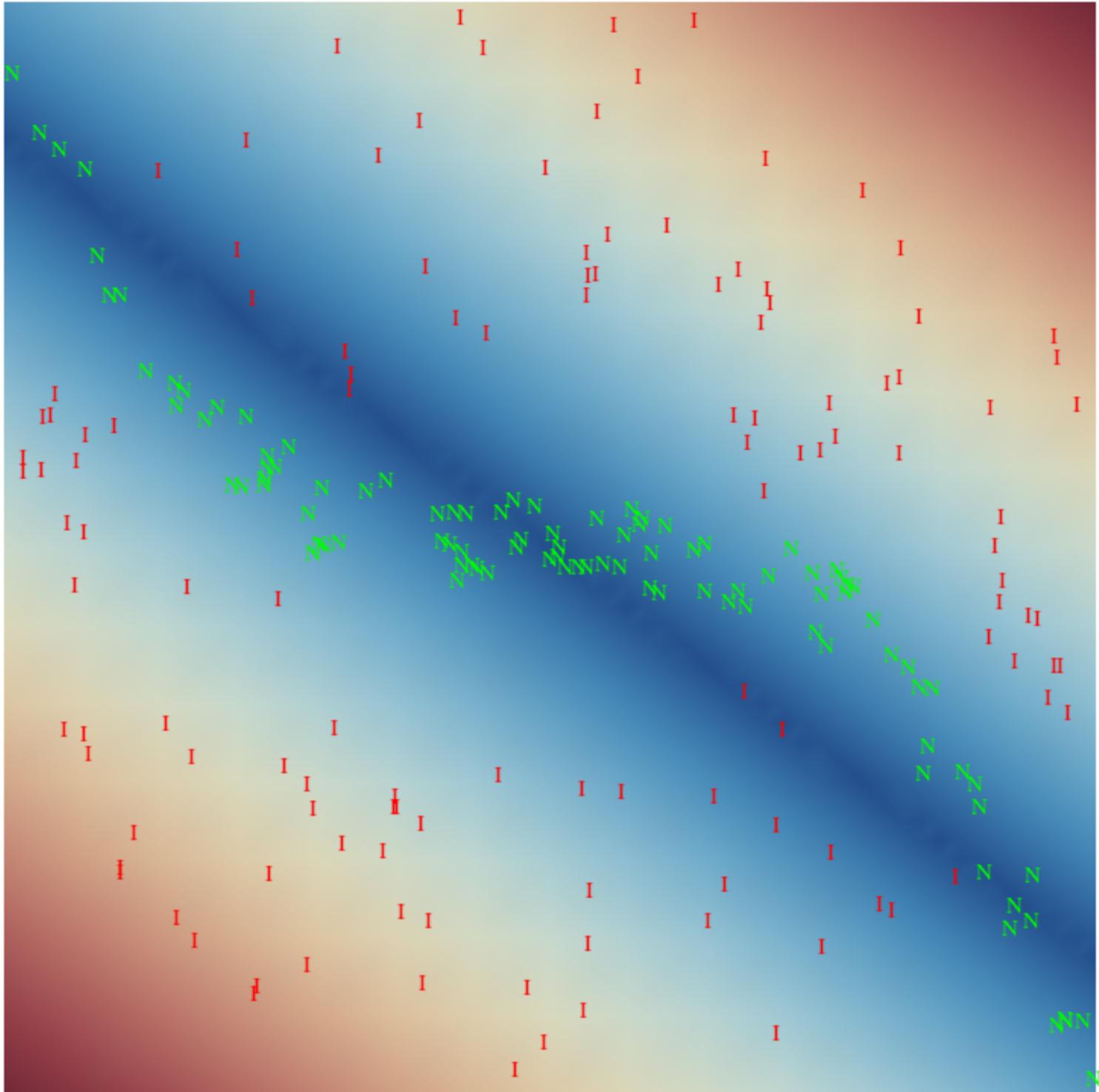
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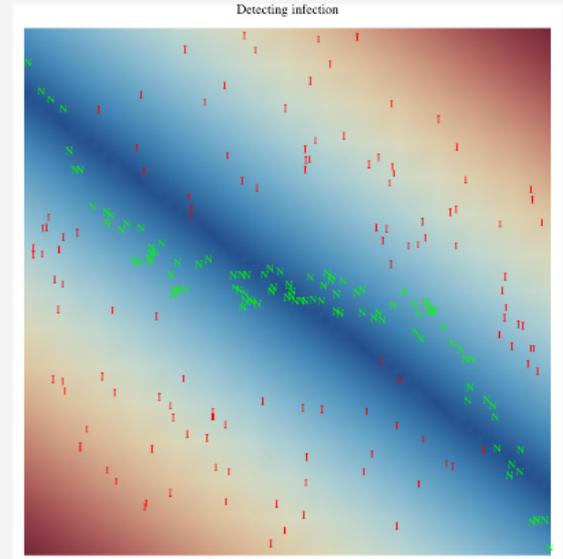


Detecting infection

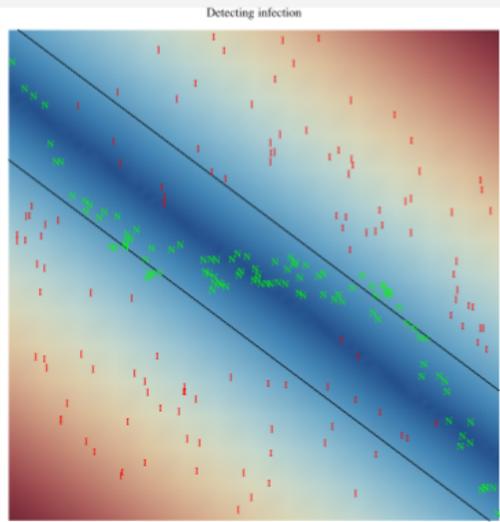


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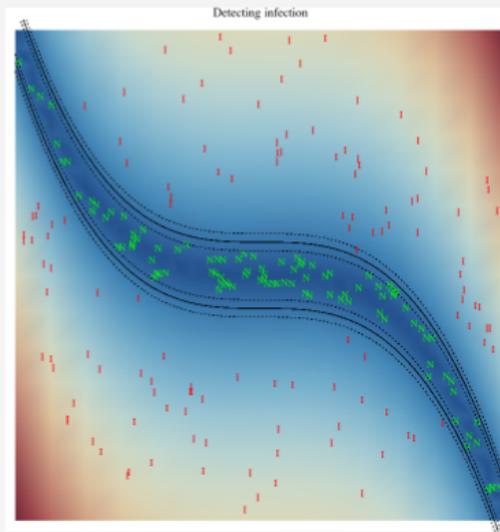
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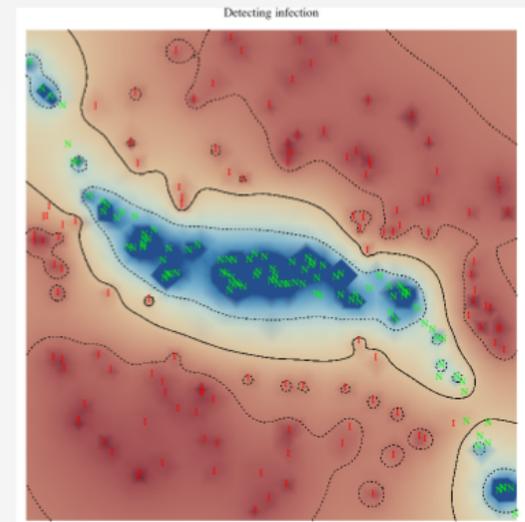
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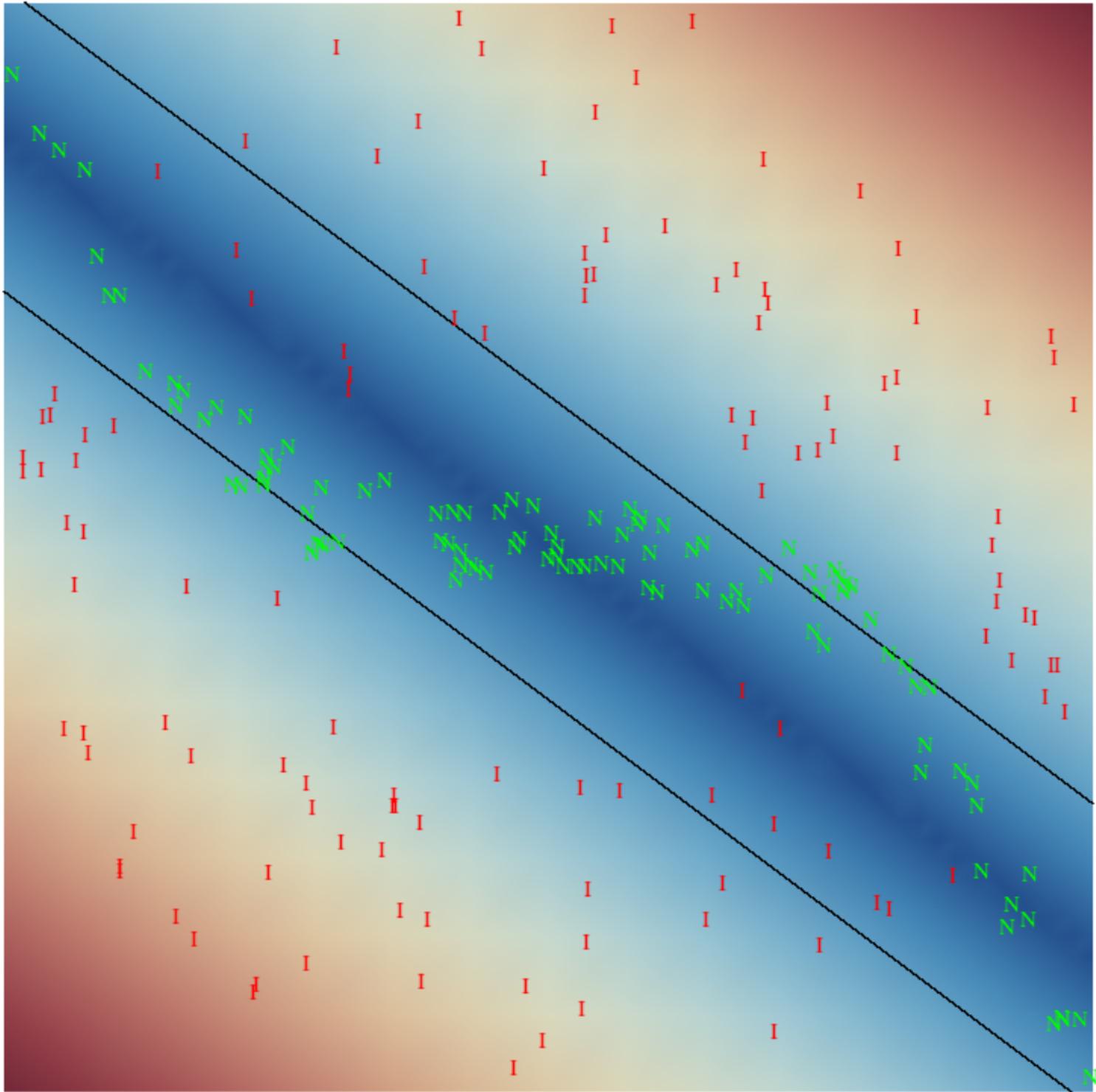
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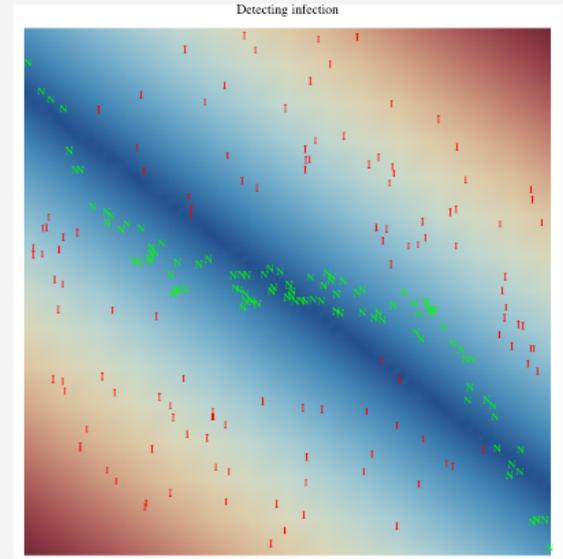
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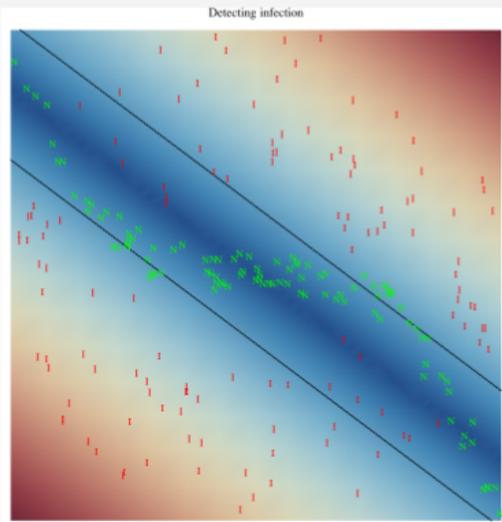
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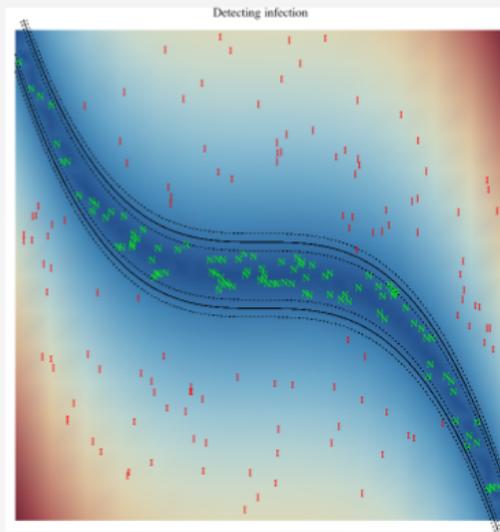
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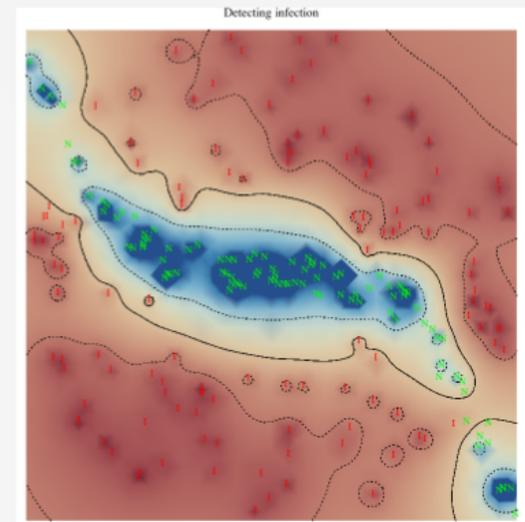
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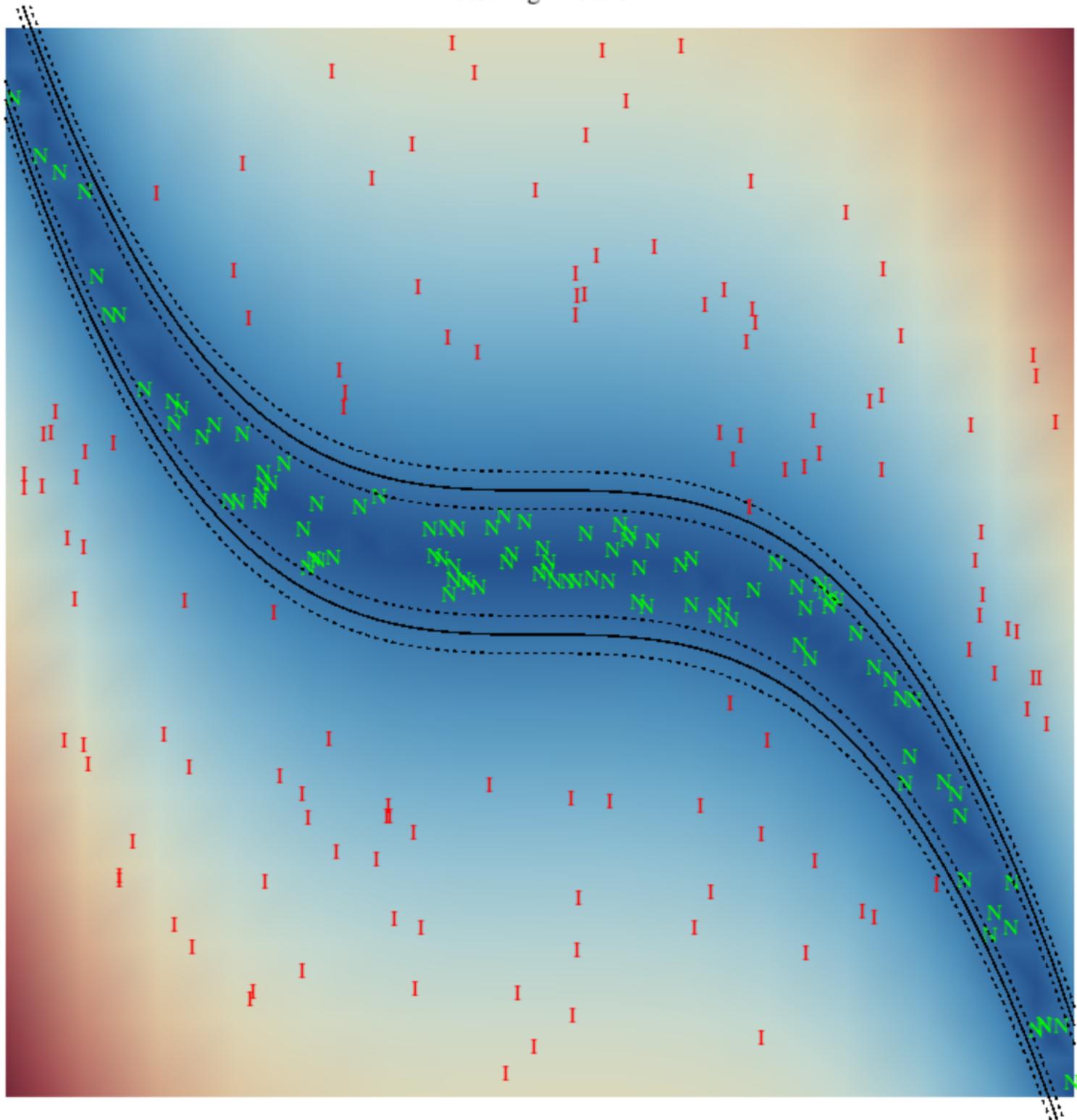
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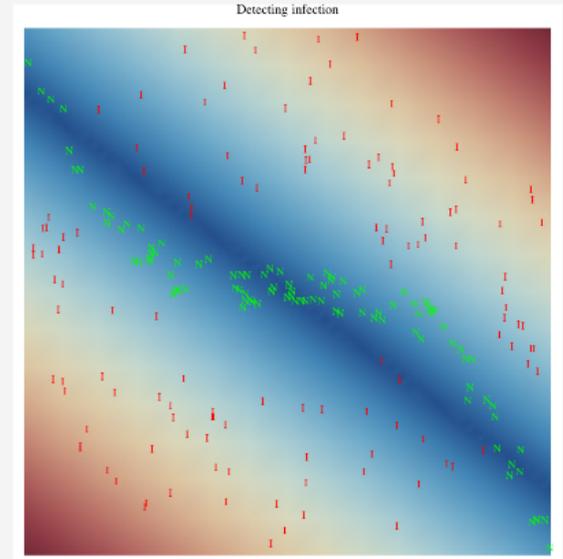
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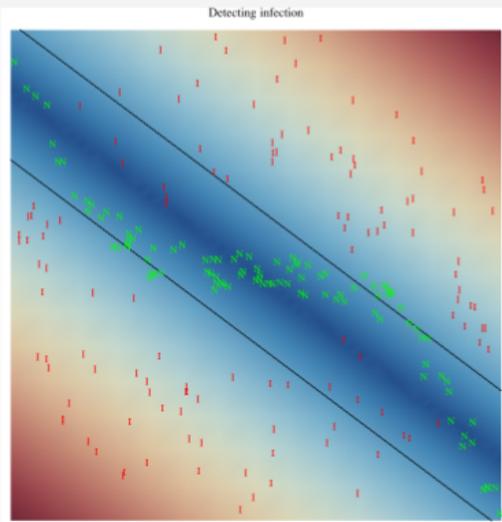
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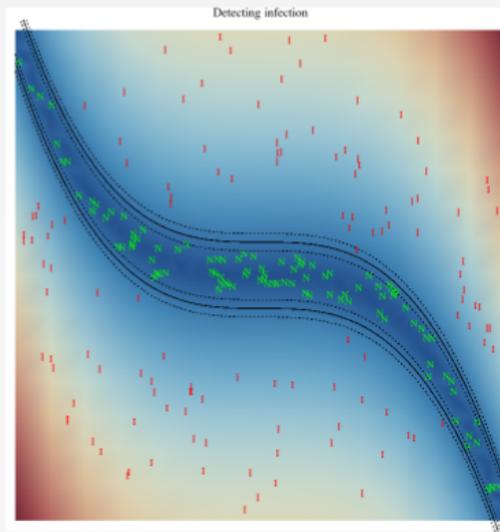
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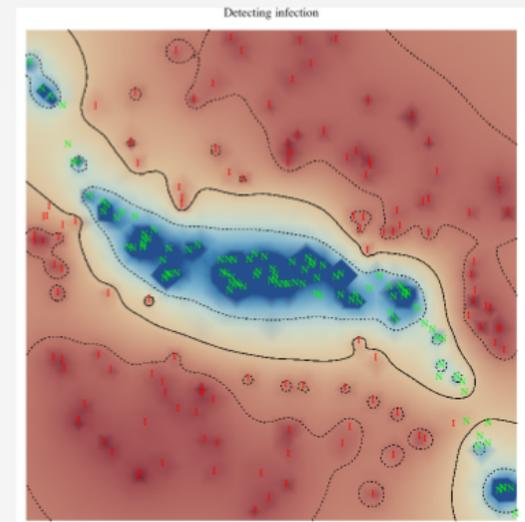
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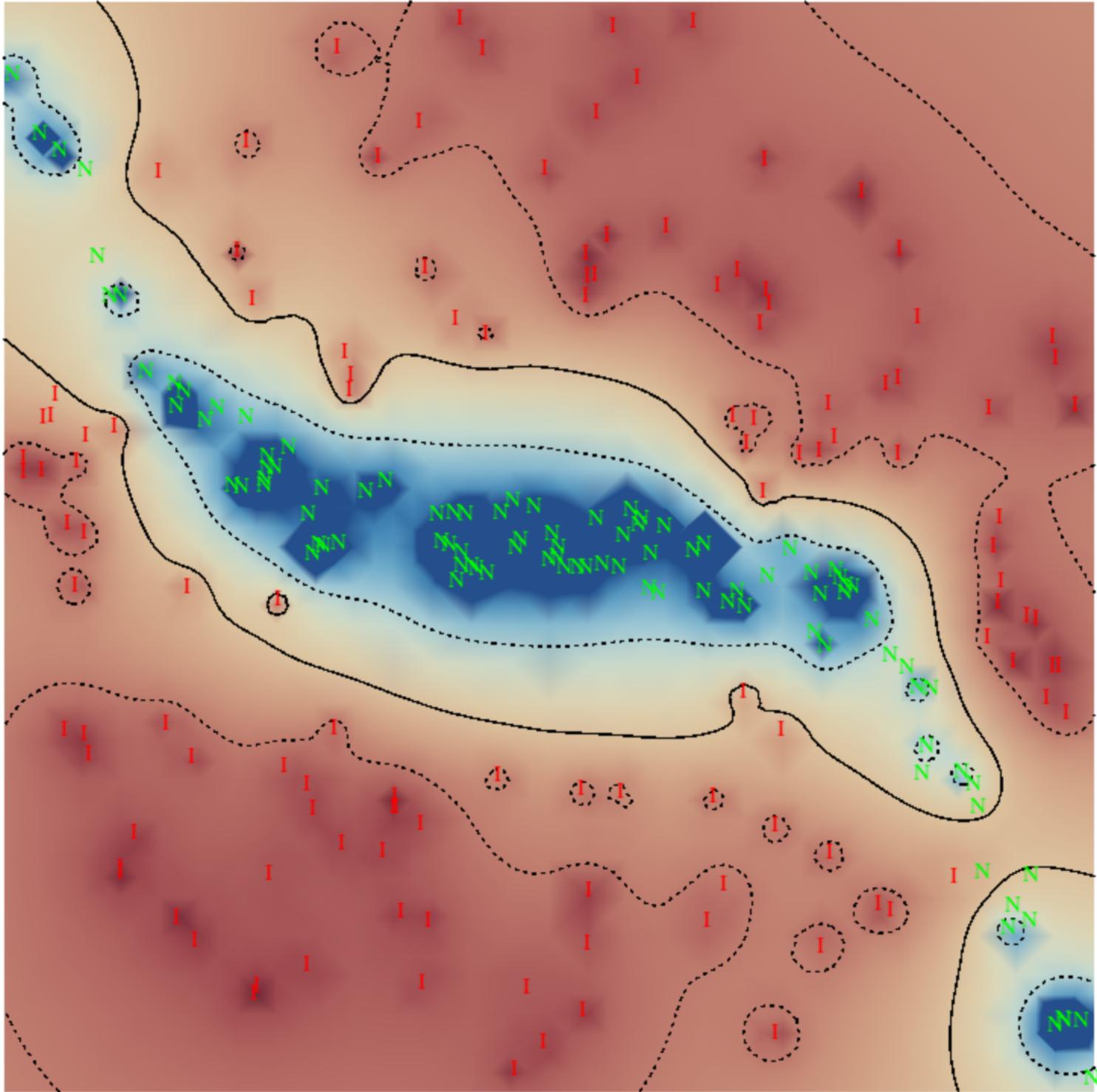
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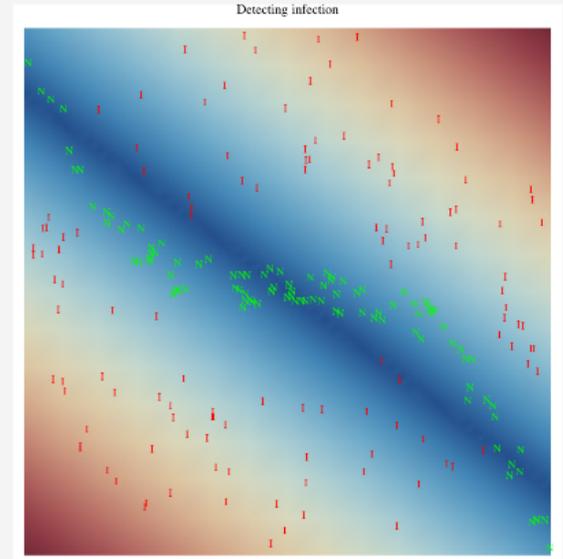
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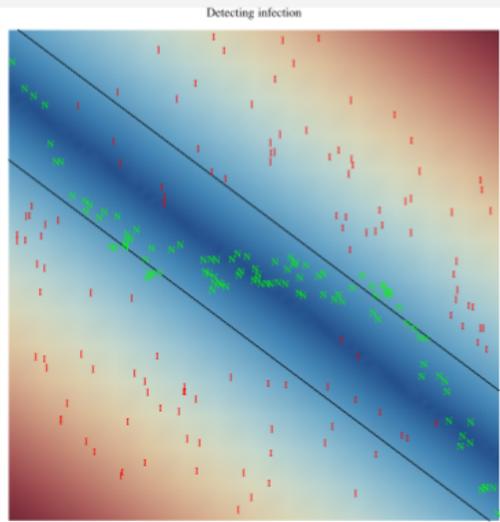
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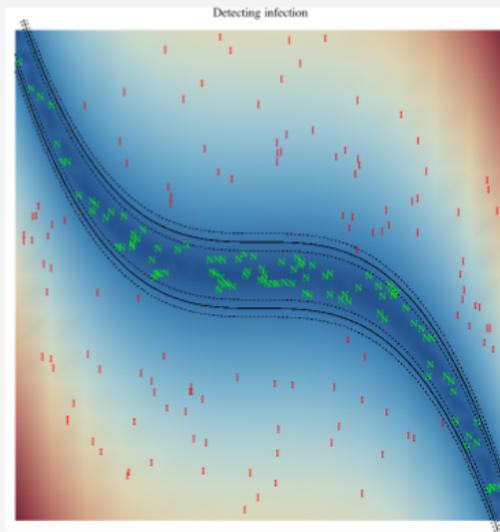
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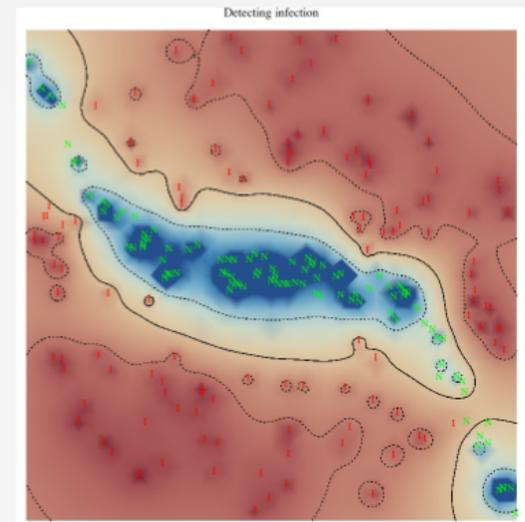
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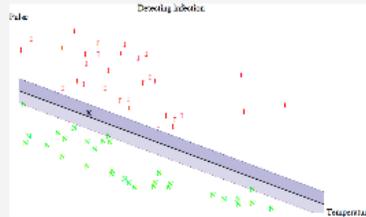
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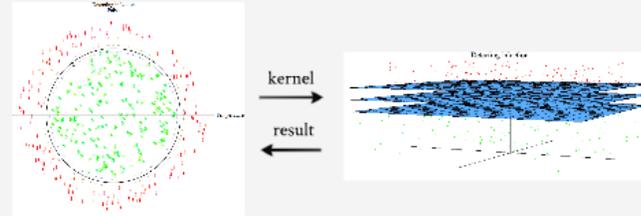
Machine learning and detecting infection

IBM Artemis project

Prediction/Classification

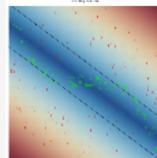


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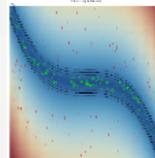


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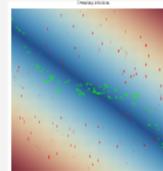
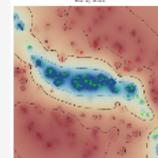
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Too many?



Summary

Theoretical math is cool!

Three apparant paradoxes:

Hilbert's Hotel, The pea and the sun, The dough and the pan.

Predict length of stay. Help detect infections.

Thank you!



Thanks to:

Retsef Levi, Peter Dunn, Bethany Daily, Cecilia Zenteno
MIT and MGH

Alex Scheinker - mountain & tree
www.alexscheinker.com

Sam Rodriguez - medical illustrations
www.sampaintings.com

David Scheinker
dscheink@mit.edu

is cool!

paradoxical results.

Predict length of stay.

The pea and the sun

Area
Young man in mathematics you don't understand things, you just get used to them.
-John von Neumann

The dough and the pan

Infinite dimensional space

A mathematician is a blind man in a dark room looking for a black hat which isn't there.
might be

-Charles Darwin

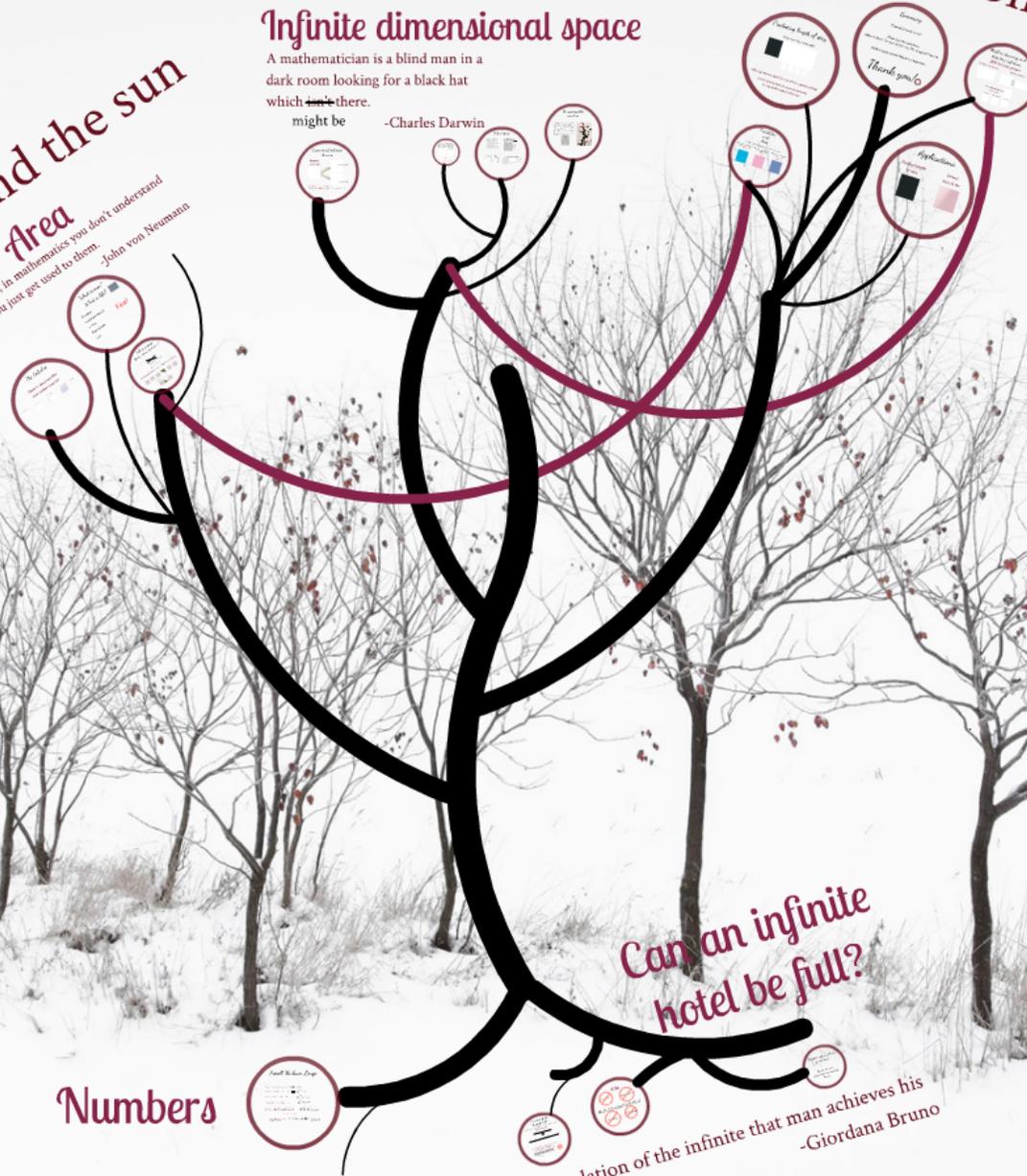
Applications

Computers have limitations

Numbers

Can an infinite hotel be full?

It is in contemplation of the infinite that man achieves his greatest good.
-Giordano Bruno



Can studying infinite dimensional space help improve health care?

Theoretical math
is cool!

Three seemingly
paradoxical results.

Detect infections.
Predict length of stay.

