Conditions for the Breakdown of the Phase Screen Approximation in Reflectometry*

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Statement of the Problem

- Reflectometry (microwave scattering) measurements in large fusion scale plasmas has yielded exciting new results on the properties of plasma turbulence.
- Strong scattering (multiple diffraction orders and loss of coherent reflection) typically mean that simple analytic models are inadequate.
- In this paper we address the issue of the breakdown of one such analytic model, the WKB or Phase screen approximation, in fusion scale experiments.

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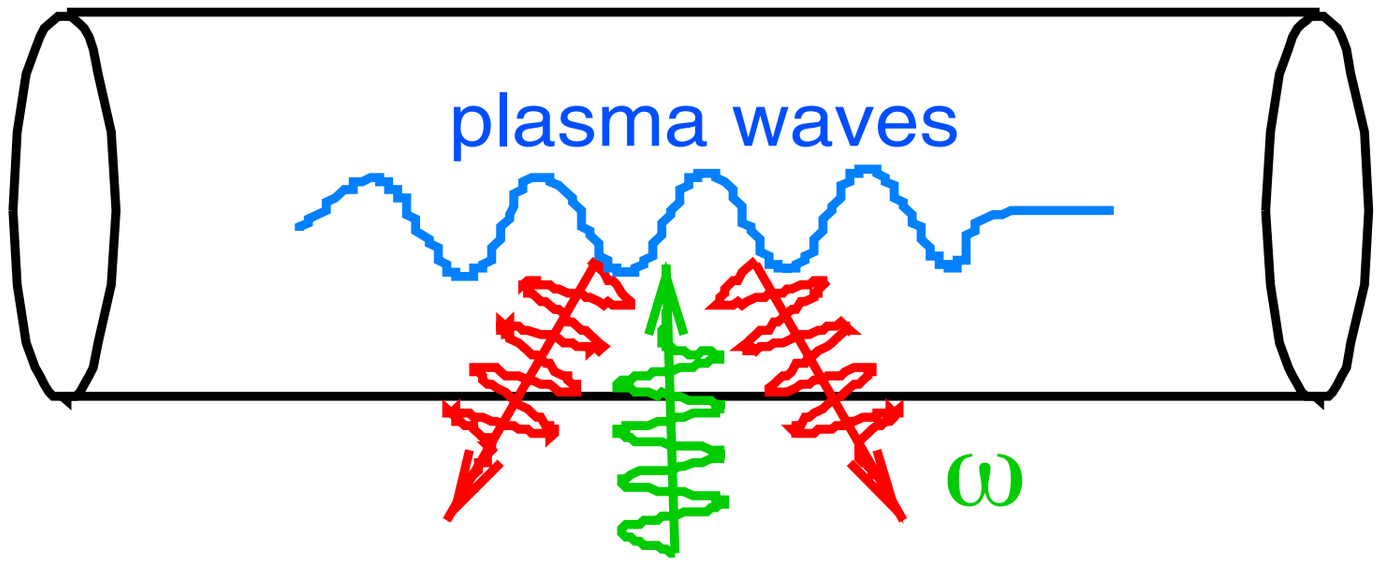
The Two Dimensional Scalar Wave Equation: Wave Propagation in an Inhomogeneous Unmagnetized Plasma (Relevant for E||B propagation)



$$\nabla^2 E + k_0^2 \varepsilon \cdot E = 0 \quad ; \quad \text{assume} \quad \nabla(\nabla \cdot E) = 0$$

$$\varepsilon = \varepsilon_0 (1 + \hat{\varepsilon}); \quad \varepsilon_0 = 1 - x / L_\varepsilon \quad ; \quad \hat{\varepsilon} = \frac{\delta n_e}{n_e} \sin(K_y y)$$

$$k_0 = \frac{2\pi}{\lambda_0} \quad ; \text{ Boundary condition } E_{in} = e^{ik_0 x}$$

Numerical Solution is Sought for System of ODEs

Fourier transform in y :

$$E = \sum_{m=-N}^{N} E_m(x) e^{imK_y y}$$

Coupled set of ODEs :

$$\frac{d^2}{dx^2} E_m + k_0^2(\varepsilon_0 - m^2\alpha^2)E + \frac{k_0^2 \varepsilon_0 \hat{\varepsilon}}{2i}(E_{m-1} - E_{m+1}) = 0; \quad \alpha = K_y / k_0$$

Boundary condition :

$E_{inc} = e^{ik_0 x}$ in vacuum, and $E \to 0$ as $x \to \infty$

Note that the cutoff position for diffraction order *m* is given by: $\varepsilon_0 - m^2\alpha^2 = 0$

Linearized Phase Screen Approximation for Fluctuation Measurements

Geometric optics: For $Kx,\ Ky \ll k_0$

$$E = e^{i\phi};\quad \phi = 2k_0\int_0^{x_c}\sqrt{\varepsilon}\ dx,\ \text{where } \varepsilon = \varepsilon_0(1+\hat{\varepsilon})$$

Linearizing for small $\hat{\varepsilon}$ gives;

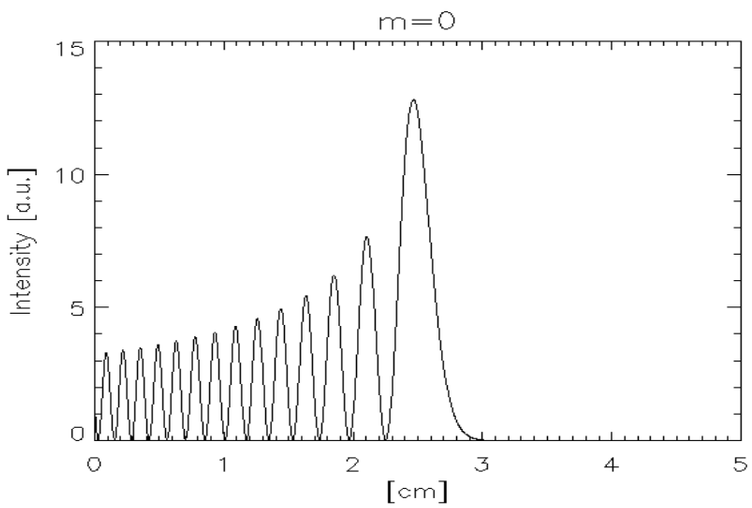
$$\delta\phi = \int_0^{x_c}\frac{\delta\varepsilon}{\sqrt{\varepsilon_0}}\ dx,\ \delta\varepsilon = \varepsilon_0\cdot\hat{\varepsilon}$$

Apart from a phase term, the outgoing spectrum in K_y is given by the Fourier transform:

$$E_m = \int_{-\infty}^{\infty} dy\ e^{imK_y y}e^{i\delta\phi}$$

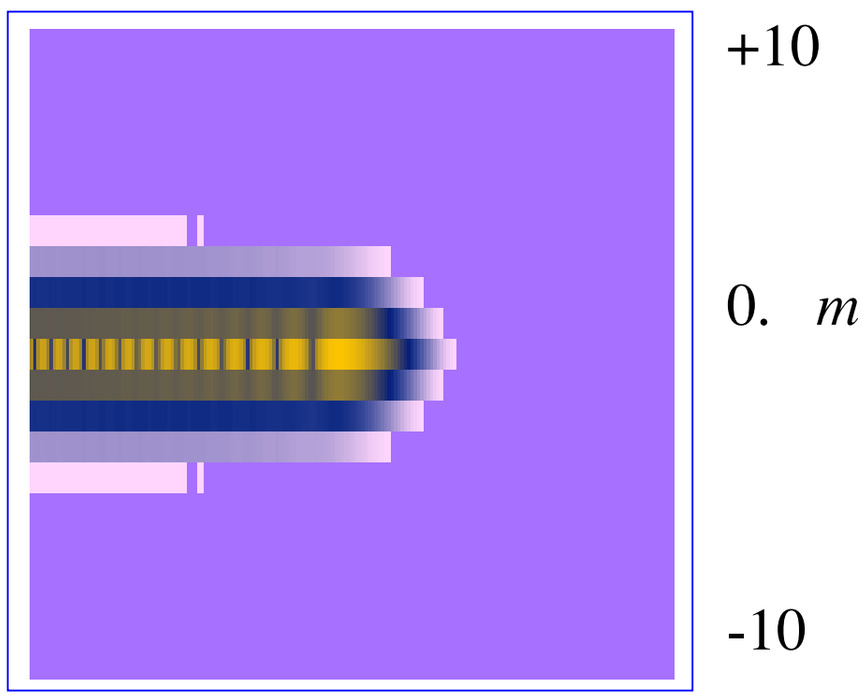
Qn: Under what condition (ignoring Bragg reflection) is this expression valid for the amplitude of the scattered field?

Case I: Weak Scattering.
Cuttoff layer displaced with increasing *m*



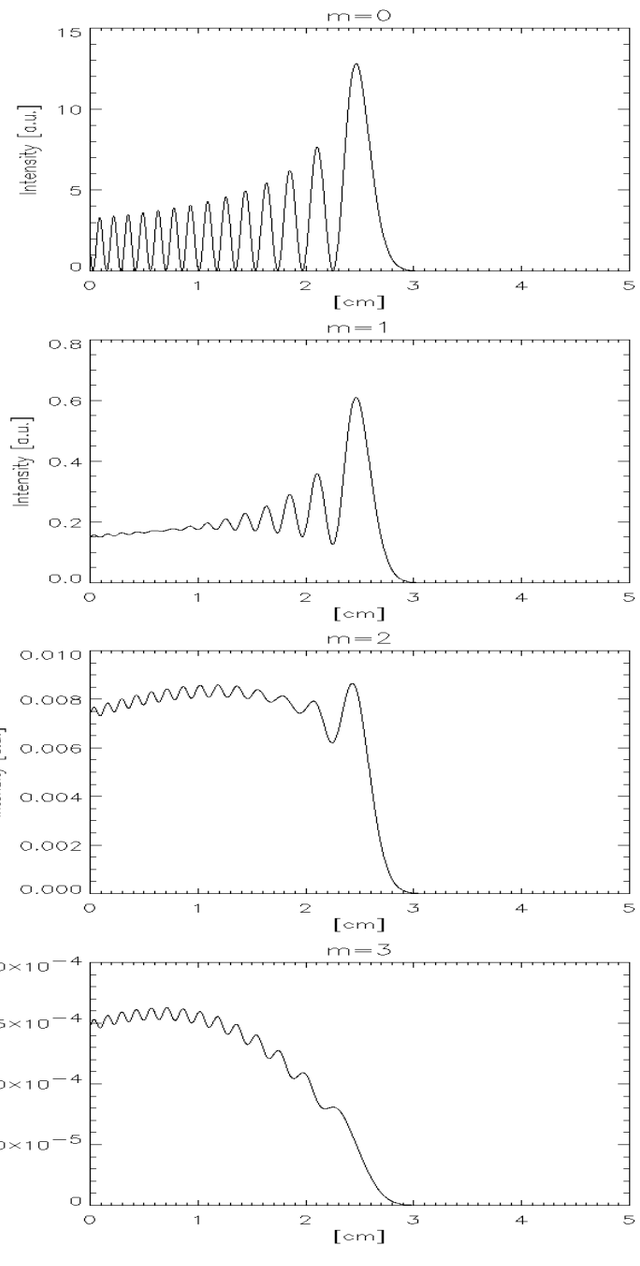
Incoming wave propagates from left to right. Incident wave reflects from ε=0.

Run parameters: k_0=25 cm^{-1}, l_x=5.26 cm, K_y=0.5 cm^{-1}, $\delta n/n$=0.005 across the medium.



Log plot of intensity for each diffraction order. Note outward displacement of diffraction orders for increasing m. This leads to weaker coupling than expected from phase screen model.

Case I: Weak Scattering Continued: Note that the higher *m* components can in principle "walk" out of the box



Airy pattern clearly observed in m=0 standing wave component.

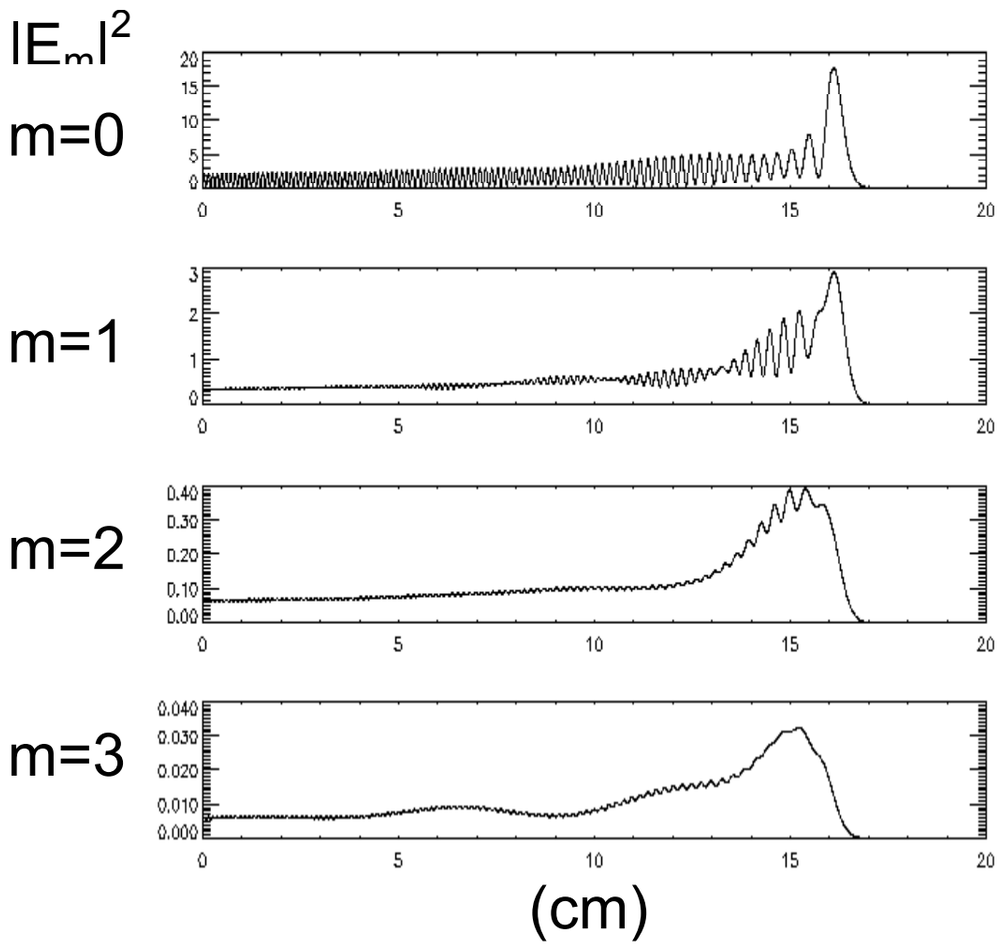
Standing wave structure disappears in m=1 and higher m components. Outward propagating solutions.

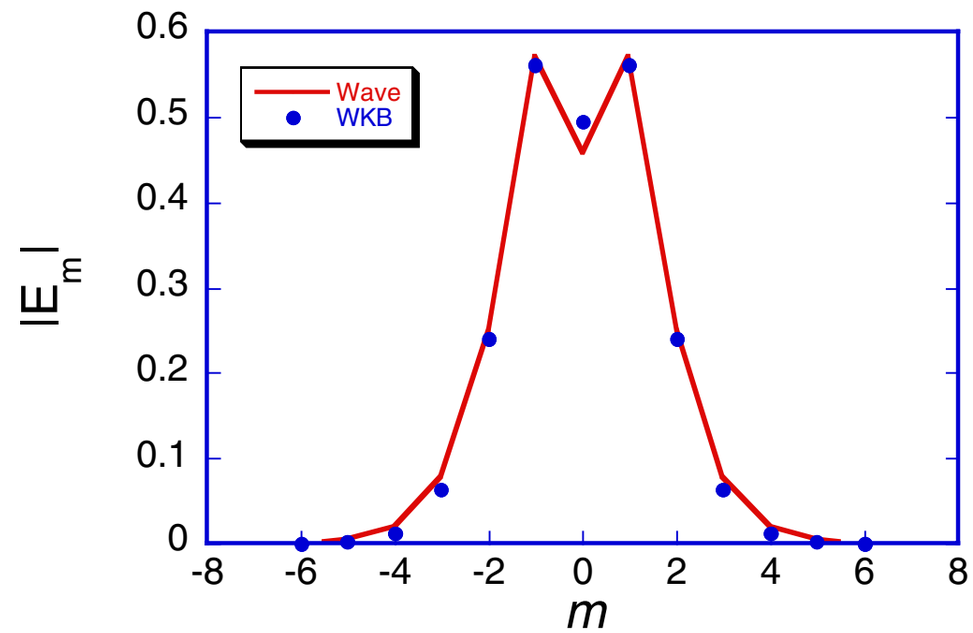
Enhancement at the cutoff disappears for m=2 component.

M=3 no longer samples the cutoff region.

Case II: Phase Screen Approximation Is Valid for Long Wavelength Fluctuations

(k_0=25 cm^{-1}, K_y=1cm^{-1}, K_x=1cm^{-1}, $\delta n/n$=0.005, L=20 cm, Ny=6, Nx=5000)



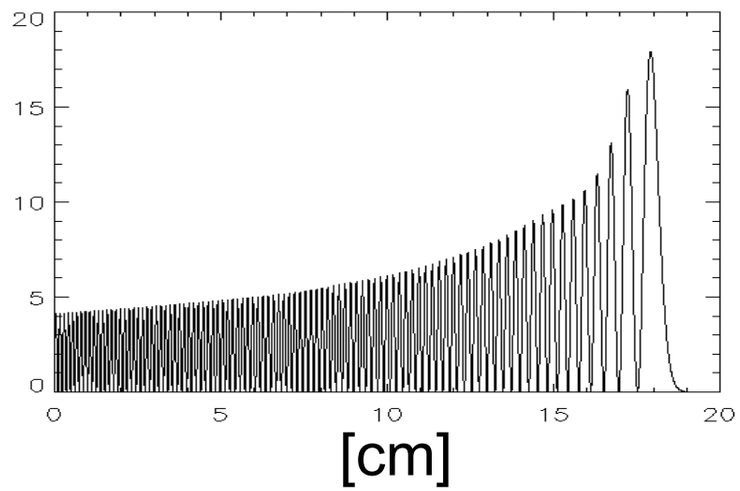


Wave amplitudes agree with WKB within 10% for |m|≤3

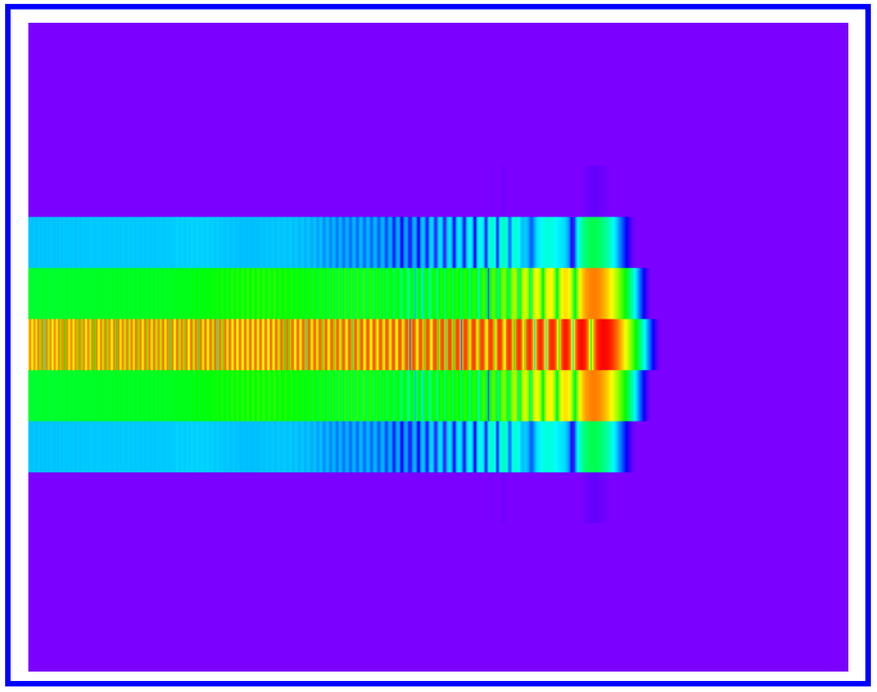
Total outgoing power : 1.001
→ Numerically stable

Case III: Phase Screen Approximation Breaks Down at Large K_y

(k_0=25 cm^{-1}, **K_y=4cm^{-1}**, K_x=1cm^{-1}, $\delta n/n$=0.005, L=20 cm, N_y=6, N_x=5000)



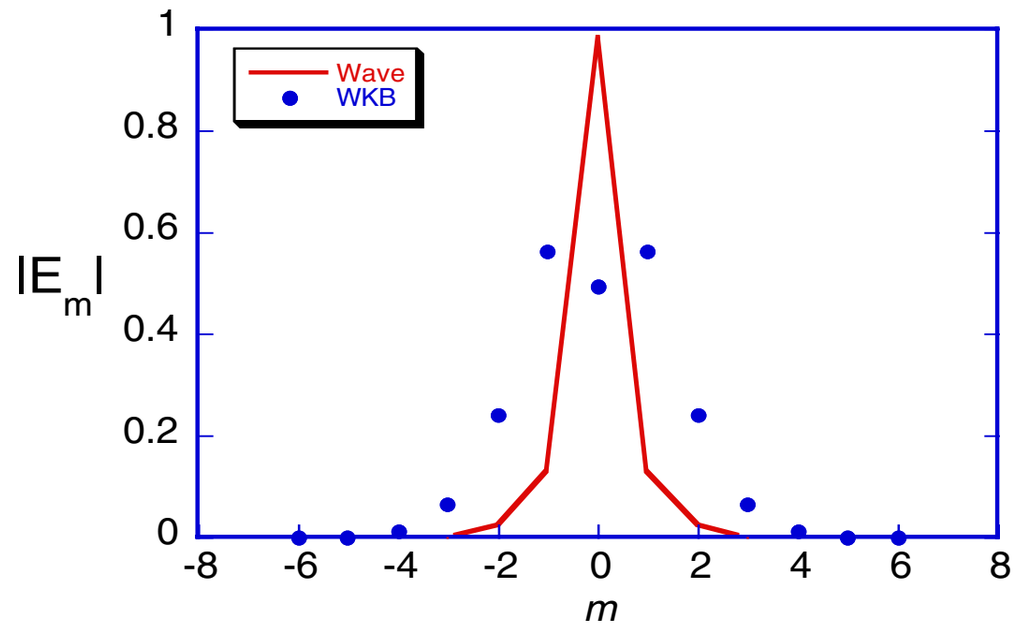
$|E_0|^2$



6

0 *m*

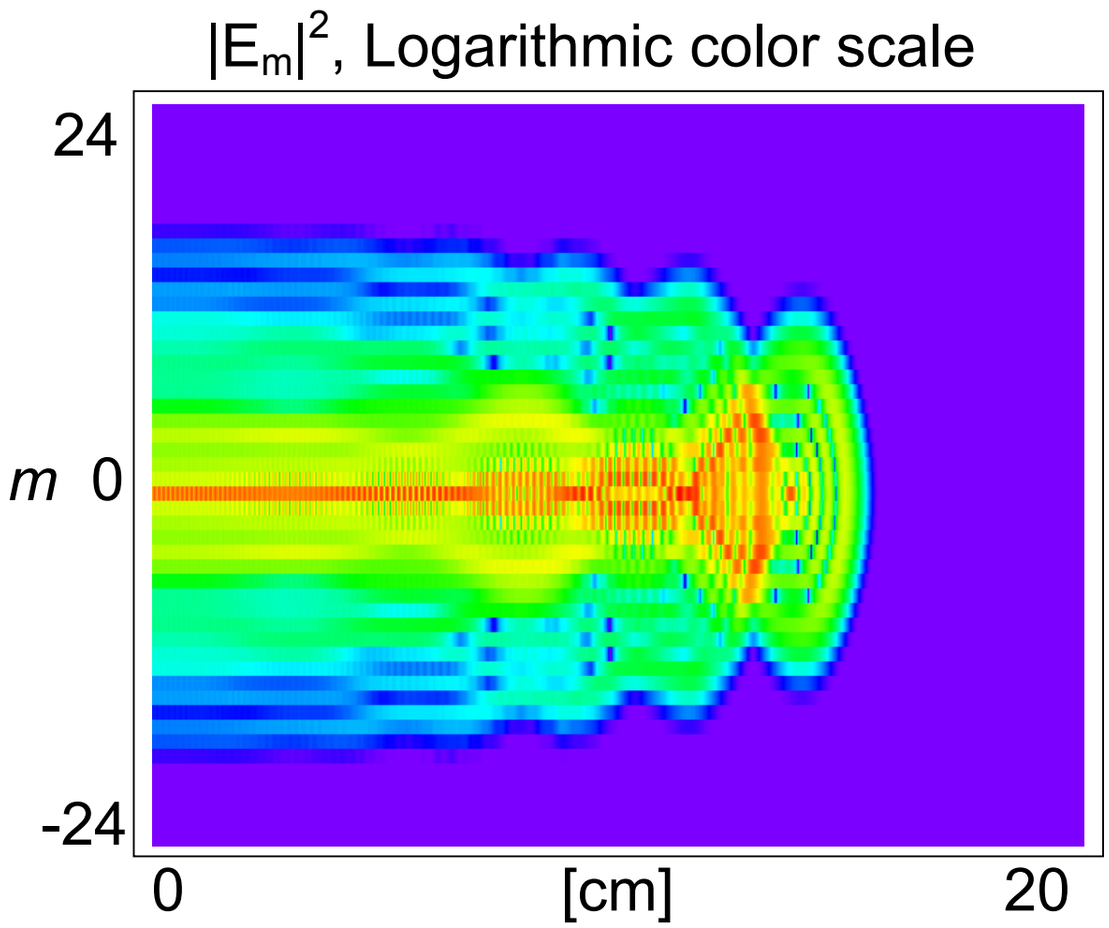
-6



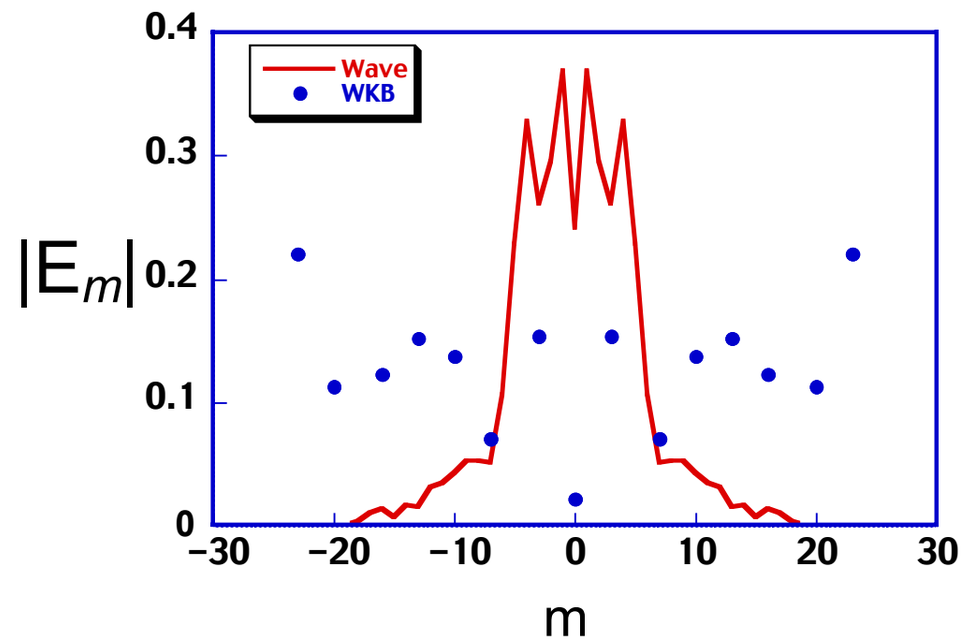
The wave amplitudes shows a strong coherent reflection (for m=0) indicating <u>weak</u> scattering. This would represents an underestimate of the fluctuation level based on the WKB model shown in blue.

Case IV: Phase Screen Approximation Breaks Down for Large Fluctuation Levels and Multiple Cutoff Layers

(k_0=25 cm^{-1}, **K_y=1cm^{-1}**, K_x=1cm^{-1}, **$\delta n/n$=0.08,** L=20 cm, N_y=6, N_x=5000)



Radiation trapping and localization of scattered field observed in numerical solution.



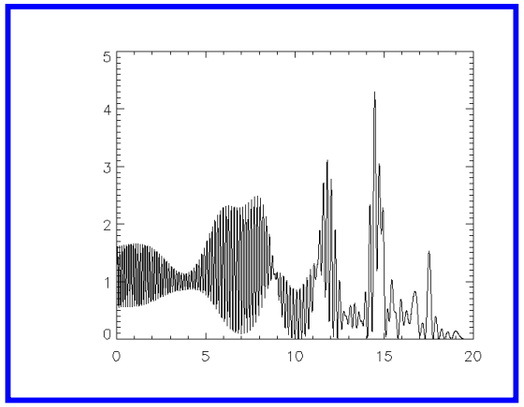
Note the High-m (m≈6) cutoff, which gives the impression of weaker fluctuation levels.

Total outgoing power : 1.01

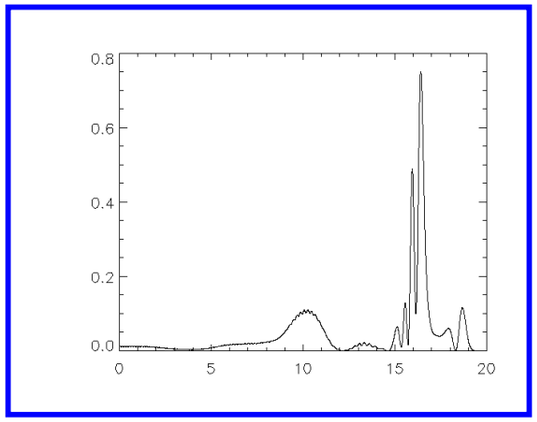
→ Numerically stable !!

Case IV: $|E_m|^2$ (K_v=1. cm^{-1}, k_0=25. cm^{-1})

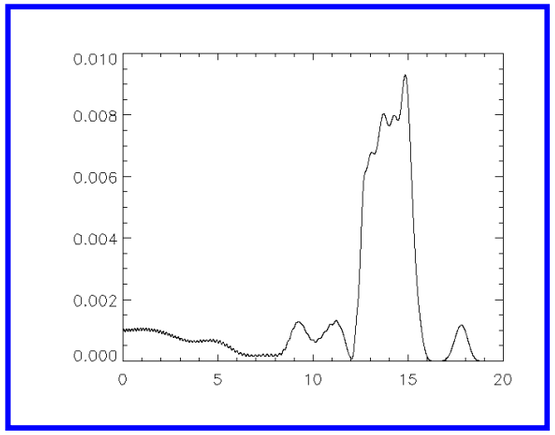
M=0



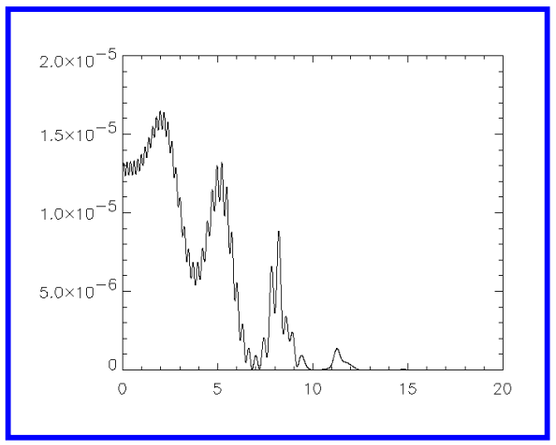
M=6



M=12



M=18



M=0: Depletion of signal near cutoff due to coupling to higher-m components.

M=6, 12: At moderate m, radiation appears strongly trapped near cutoff.

M=16: At high-m, there is no contribution from m=0 cutoff region.

Summary: At Long Wavelength and Moderate Fluctuation Levels the Phase Screen Model Is Valid (Relevant to Core Localized Measurements)

- Conditions for the break down of the WKB (or phase screen approximation) in reflectometry were investigated.

- For long wavelength fluctuations, and moderate fluctuation levels, the deviation from WKB estimates is small.

- However, for increasing K_y and/or large fluctuation levels, the phase screen model can significantly underestimate the ambient level of fluctuations.

- For core fluctuation measurements $\delta n/n \leq 1\%$, the phase screen model is in reasonable agreement with full wave analysis. However, this situation can break down near the plasma edge where large fluctuation levels ($\delta n/n > 10\%$) and/or short scale structures can occur.

- Future work will focus on identifying more general criteria for the validity of the phase screen model, together with detailed benchmarking to other full wave codes (see next poster), WKB and perturbative full wave analytic models.