Self-organized criticality, long-time correlations, and the standard transport paradigm[a]

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[b]Download from `ftp://ftp.pppl.gov/pub/krommes/Talks/APS99_talk.ps`.

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Current topics of interest

Some interesting (and related) challenges for the theorist:

- submarginal turbulence (linearly stable);
- zonal flows ($k_y = k_z = 0$);
- long-time correlations [$C(\tau) \sim \tau^{-\beta}$].

There has also been considerable discussion of

- self-organized criticality (SOC).

Is SOC an overarching, unifying theme?
Or is it a special case of more general turbulence phenomena?

It's impossible to discuss all of these topics in 25 minutes (see references at end of talk). Today,

- some elementary facets of long-time correlations;
- a (very) few words about the relationship of SOC formalism to general theory.

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Motivation

- Carreras (1998) [APS/DPP98]: "Perhaps the 'standard transport paradigm' must be abandoned."
  - Long-time tails on edge density correlations: $C(\tau) \sim \tau^{-\beta}$.
  - Nonintegrable correlation functions may imply local transport coefficients don't exist.
- Carreras et al. (1998): The measurements [on long-time tails] are "consistent with the SOC paradigm of turbulent transport. However, it does not prove that this model offers the only explanation. . . . At this point, we are not aware of other dynamical mechanisms that may provide an alternative answer, but it may exist."

I will show that there is
an alternative explanation
that does not invoke SOC ideas.
Also, nonintegrability $\not\Rightarrow$ nonlocal transport.

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Goals

The **major goal** of this talk is to argue that

- not all systems with long-time correlations need have nonlocal, nondiffusive transport (nor do they need to be SOC).

To do this, I will

- demonstrate that some stochastic systems can possess long-time correlations while lacking crucial features of SOC models;
- recast (some of) the SOC formalism in the more conventional framework of statistical turbulence theory;
- emphasize the distinction between Eulerian and Lagrangian correlation functions.

Longer-ranged goal: a unified, quantitative theory of

- nonlocal as well as local transport;
- submarginal as well as supermarginal regimes;
- avalanches as well as more conventional fluctuations.

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Summary

- Pure SOC lives in an asymptotic corner of parameter space. It is probably not necessary to look there for explanations of observed long-time correlations.
- Long-time correlations can be present in stochastic systems that do not possess essential features of SOC.
- The spectral balance equation of statistical turbulence theory can unify various approaches to, and formulas for fluctuation spectra.
- Even though Eulerian correlation functions can easily exhibit long-time tails, transport coefficients (determined by Lagrangian correlations) can nevertheless be local.

Long-time tails can coexist with local transport.

This observation should be accounted for in any formal theory of turbulent transport.

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This talk is not about SOC.
It is about the theory of
long-time correlations.

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Selected references on SOC

- Original papers were by Bak, Tang, and Wiesenfeld (Bak et al., 1987; Bak et al., 1988).
- Some pedagogical discussions are by Bak and coworkers (Bak and Tang, 1989; Bak and Paczuski, 1993).
- A recent book for the lay person is by Bak (1996).
- A more technical pedagogical introduction to SOC has been given by Jensen (1998).
- In plasma physics, authors include
  - Diamond and Hahm (1995),
  - Newman et al. (1996),
  - Carreras et al. (1996).
  - See also a variety of talks at this meeting.

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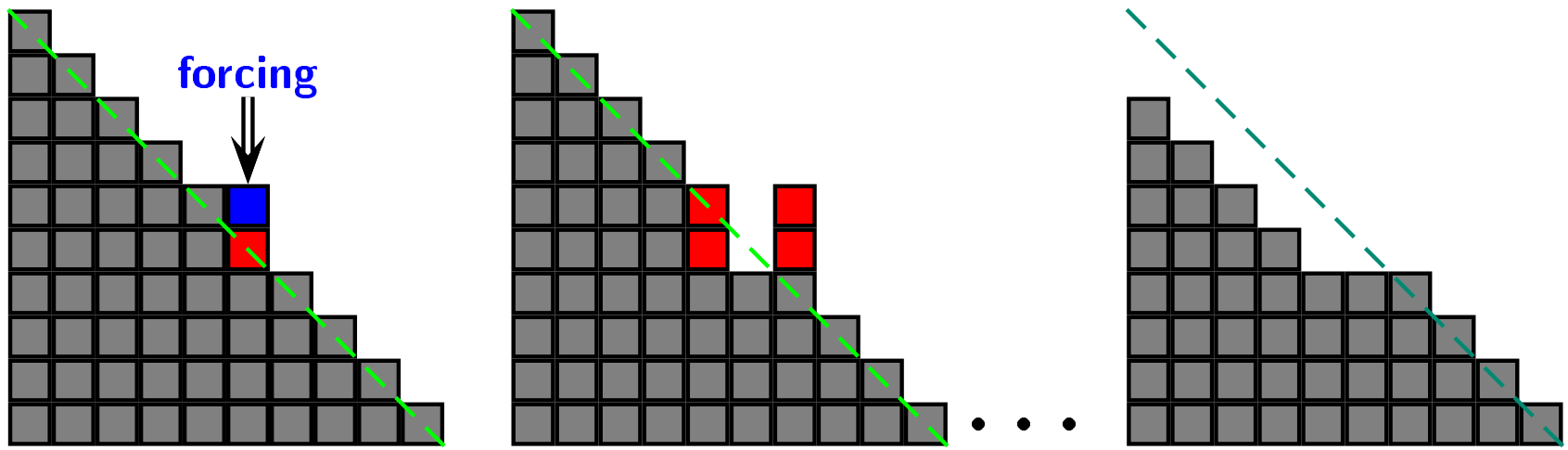
A simple sandpile

On a 1D n-site lattice, implement toppling rule

$$h_i \to h_i - N, \quad h_{i+1} \to h_{i+1} + N \tag{1a,b}$$

when local slope S exceeds $S_{\rm crit}$. The following example is for $n = 10$, $S_{\rm crit} = 1$, $N = 2$.



$t = 0$ $\quad$ $t = 1$ $\quad$ $t = 15 = T$ (lifetime).

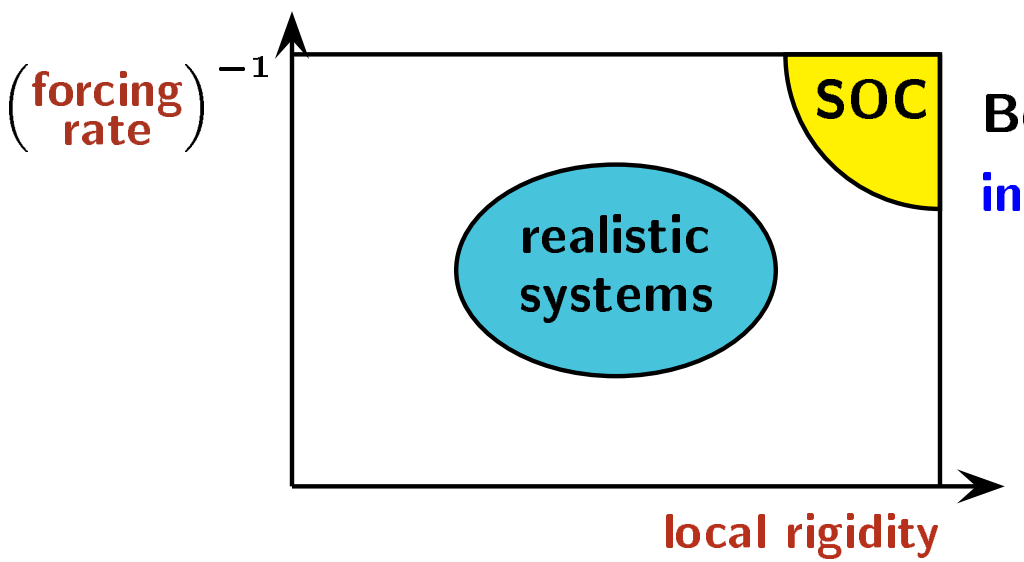
- The initial toppling triggers an avalanche.
- In this example, the final slope is submarginal (lower than critical).

Signatures of SOC

Characteristics of SOC systems include

- excitations ("avalanches") of all scales;
- a "self-selected" nonequilibrium steady state (no obvious tuning parameter);
- frequently submarginal profiles .

In fact, there are parameters: forcing rate ; and local rigidity .



Be careful about interchanges of limits:

- $\tau_f \ll \tau_A$ (conventional);
- $\tau_A \ll \tau_f$ (SOC).

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Long-time tails do NOT require SOC!
And they can coexist with local transport.

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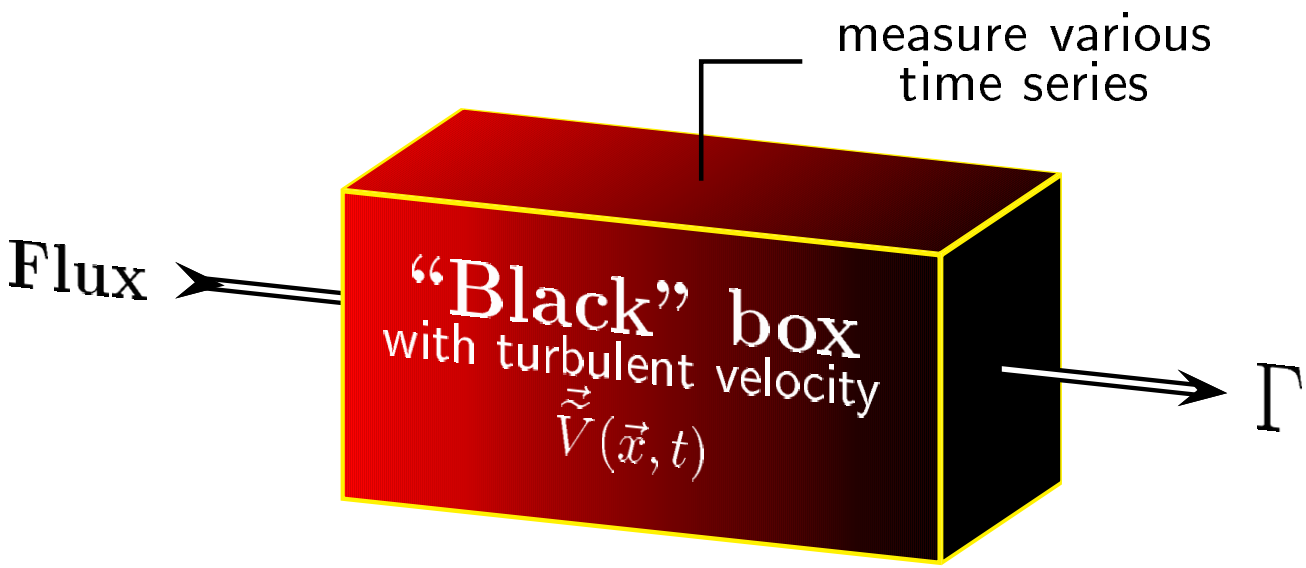
General procedure

I will show that some of the experimental signatures can arise from stochastic systems that lack important features of SOC models.

Consider a "black box" that exhibits (at least some of) the experimental phenomena.



The black box can be either

- a real experiment; or
- a mathematical stochastic model.

1D stochastic models

For the simplest demonstration, Krommes and Ottaviani (1999) studied a 1D stochastic model (with properties that emulate more realistic turbulence).

1D stochastic model
with statistically specified
random velocity $\widetilde{V}(x,t)$

$\Longrightarrow \Gamma$

We specify the statistical properties of $\widetilde{V}(x,t)$. (Random $\widetilde{V}$ implies an effectively multidimensional model.)

- autocorrelation time $\tau_{\rm ac}$; autocorrelation length $L_{\rm ac}$;
- threshold condition (NONE). (Fluctuations are always on.)

NOTE:

NO THRESHOLD $\Rightarrow$ NO AVALANCHES.

A stochastic boundary-value problem

Consider $$\partial_t \widetilde{T}(x,t) + \partial_x \widetilde{\Gamma}(x,t) = 0 \quad (0 < x < 1), \tag{2}$$

where

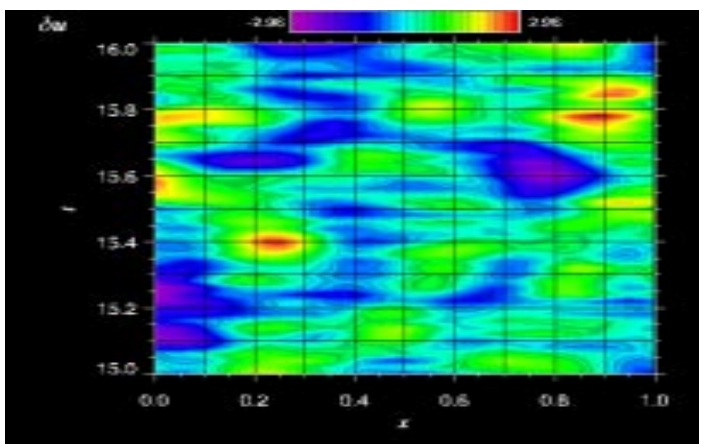
$$\widetilde{\Gamma}(x,t) \doteq \widetilde{\Gamma}_{\mathrm{tb}} + \widetilde{\Gamma}_{\mathrm{cl}} \tag{3}$$

$$\widetilde{\Gamma}_{\mathrm{tb}} \doteq (\delta\widetilde{V}\ \delta T)(x,t) \quad \textbf{(turbulent flux)}, \tag{4a}$$

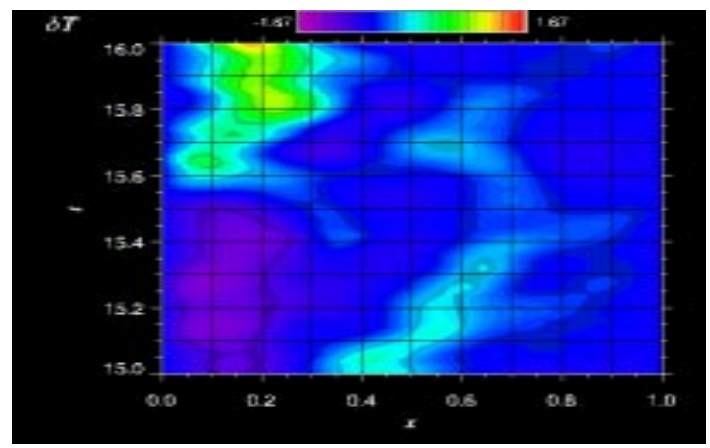
$$\widetilde{\Gamma}_{\mathrm{cl}} \doteq -\mathcal{R}^{-1}\partial_x \widetilde{T} \quad \textbf{(classical flux)}, \tag{4b}$$

and $\delta\widetilde{V}$ **is a centered Gaussian random variable with covariance**

$$\mathcal{V}(\rho,\tau) = \overline{V}^2 \exp(-|\tau|/\tau_{\mathrm{ac}}) \exp(-\tfrac{1}{2}\rho^2/L_{\mathrm{ac}}^2). \tag{5}$$



$\delta\widetilde{V}$



δT

Properties of the model

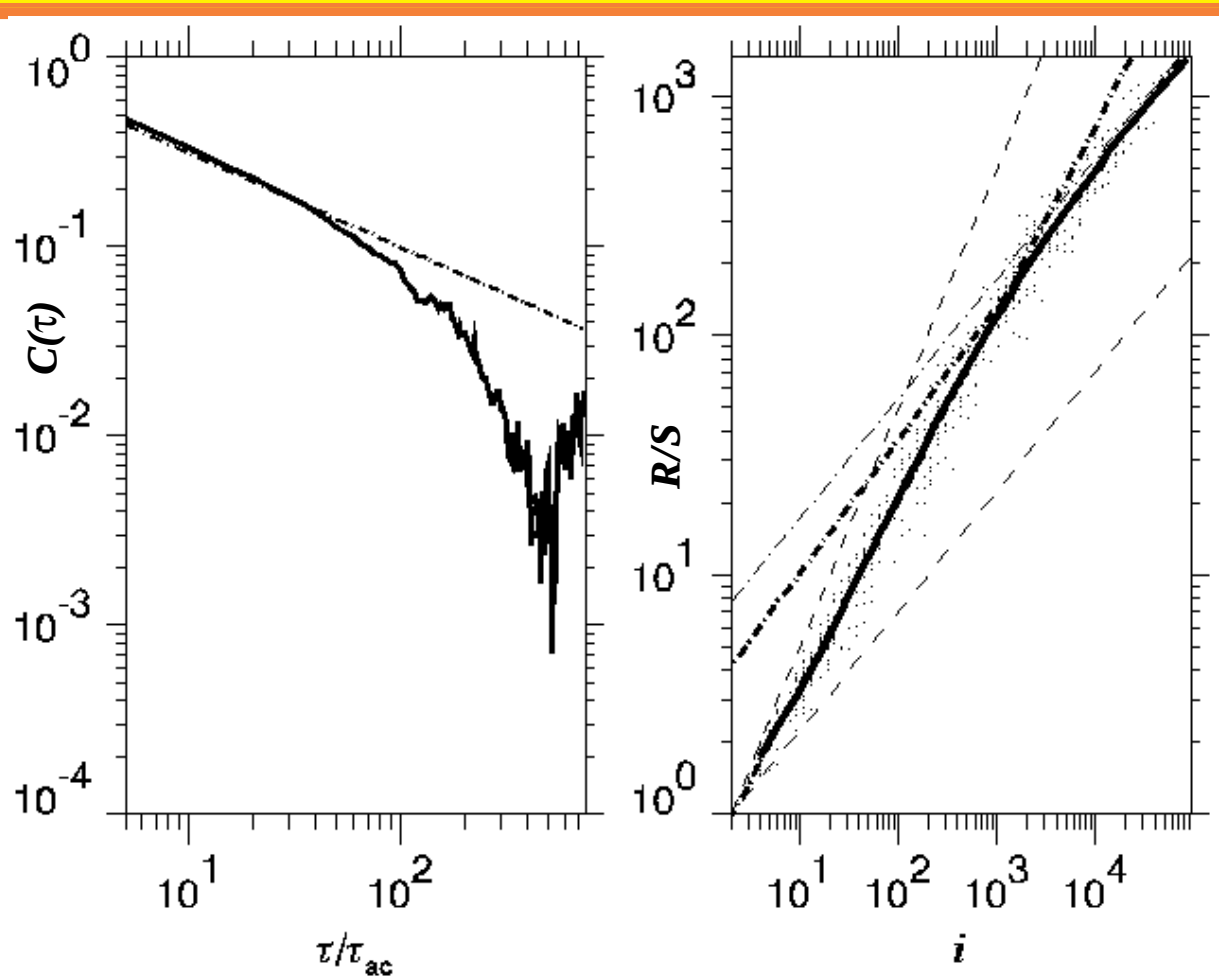
The stochastic model

$$\partial_t \widetilde{T}(x,t) + \partial_x \widetilde{\Gamma}(x,t) = 0 \quad (0 < x < 1) \tag{6}$$

shares with realistic confinement experiments the following properties:

1. The temperature profile is free to evolve.
2. The correlation scales of the velocity field can be taken to be small ("microscopic") with respect to macroscopic values by choosing $\tau_{\rm ac}$ and $L_{\rm ac}$ to be much smaller than 1.
3. The mean temperature obeys $\partial_t T + \partial_x \Gamma = 0$, which in a steady state reduces to the statement that the total (mean) flux is a conserved constant, independent of x.
4. The fluctuations are not forced by arbitrary additive noise on the right-hand side of Eq. (6), but by multiplicative noise of the usual advective variety.

Direct measurement of correlations



(a) Correlation function of the temperature evolved according to Eq. (2) and measured at $x = \frac{1}{4}$. $C(\tau) \sim \tau^{-1/2}$.

(b) Analysis of the Hurst exponent H for the temperature field by the R/S method. $H = 3/4$. [$C(\tau) \sim \tau^{-\beta}$, $H = 1 - \frac{1}{2}\beta$.]

Additional remarks on long-time correlations

- $C(\tau) \sim \tau^{-1/2}$ is nonintegrable.
- Yet the measured transport coefficients (not shown) are essentially quasilinear.

One can already infer that there is no intrinsic link between long-time tails and nonlocal transport.

- Krommes and Ottaviani (1999) also studied long-time correlations in the Hasegawa–Wakatani system.
  - Preliminary interpretation is consistent with long-time tails.
  - However, those measurements were very difficult; much more work should be done here.

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A unification of some formulas for spectral density

- spectral density of independent, overlapping avalanches (Jensen, 1998);
- spectral balance equation (statistical turbulence theory).

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Jensen's power spectrum of overlapping avalanches

Jensen (1998) showed that the power spectrum of the dissipation rate of independent, overlapping avalanches driven at rate ν is

$$C_J(\omega) = 2\nu \int_0^\infty dT \left(\frac{1 - \cos(\omega T)}{\omega^2} \right) \Lambda(T), \tag{7}$$

where the weighted lifetime distribution Λ is

$$\Lambda(T) \doteq \frac{1}{T^2} \int_0^\infty dS \, \underbrace{P(S,T)}_{\text{PDF}} S^2 \tag{8}$$

and

S — time-integrated signal from a single avalanche;

T — avalanche lifetime.

How is this approximate formula related to the general statistical theory of turbulence?

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Spectral balance equation

Consider the primitive amplitude equation

$$\partial_t \psi + iL\psi = \tfrac{1}{2} M\psi\psi = \underbrace{\widetilde{f}}_{\substack{\text{random}\\ \text{internal noise}}} - \underbrace{\eta\,\psi}_{\substack{\text{coherent}\\ \text{nonlinear damping}}} . \qquad (9)$$

$\Rightarrow$ Langevin representation:

$$R^{-1}\psi = \widetilde{f}, \quad \text{with} \quad R^{-1} \doteq \partial_t + \underbrace{iL + \eta}_{\text{“mass operator” } \Sigma} . \qquad (10a,b)$$

Solution: $\psi = R\widetilde{f}$ or $\underbrace{\langle \psi\,\psi \rangle}_{C} = R\,\underbrace{\langle \widetilde{f}\,\widetilde{f} \rangle}_{F}\,R^{\dagger}$. (11a,b)

Spectral balance equation:

$$\boxed{C(\omega) = R(\omega)F(\omega)R(\omega)^{\dagger}.} \qquad (12)$$

There has been considerable analysis of this equation:

- direct-interaction approximation (DIA) (Krommes, 1984);
- clumps (Krommes, 1997);
- Markovian closures and successful comparisons with direct numerical simulations (Bowman and Krommes, 1997; Hu et al., 1997).

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After decades of theoretical and numerical work,

The spectral balance equation of statistical turbulence theory is understood quite well.

Because it is exact, it must include Jensen's results.

What is the relationship between the spectral balance equation and Jensen's formula for the spectra of overlapping avalanches?

Clue: For the classical Langevin equation

$$\dot{v} + \nu v = f(t), \quad F(\tau) = 2D_v \delta(\tau) \tag{13a,b}$$

(with the Einstein relation $D_v = v_t^2 \nu$), one has

$$\underbrace{\langle \delta v \, \delta v \rangle(\omega)}_{C(\omega)} = \underbrace{(\omega^2 + \nu^2)^{-1}}_{|R(\omega)|^2} \underbrace{(2 \times \underbrace{\nu}_{\text{forcing rate}} \times \underbrace{v_t^2}_{\text{intensity}})}_{F(\omega)} . \tag{14}$$

Jensen vs. balance eq'n

By comparing Jensen's approximate formula

$$C_J(\omega) = 2\nu \int_0^\infty dT \left(\frac{1 - \cos(\omega T)}{\omega^2} \right) \Lambda(T), \tag{15}$$

to the rigorous spectral balance equation

$$C(\omega) = |R(\omega)|^2 F(\omega), \tag{16}$$

one can deduce that the weighted lifetime distribution normalized to $\mathcal{I} = \int_0^\infty dT\, \Lambda(T)$ is ($\widehat{\Lambda} \doteq \Lambda/\mathcal{I}$)

$$\boxed{\widehat{\Lambda}(\omega) = 1 - \omega^2 |R(\omega)|^2.} \tag{17}$$

As a check, recall that $R(\omega) = [-i(\omega + i\Sigma_\omega)]^{-1}$. Assume that Σ_ω is real. Then $\widehat{\Lambda}(\omega) = \Sigma_\omega^2/(\omega^2 + \Sigma_\omega^2)$. This formula generalizes the characteristic function of the PDF for Poisson-distributed events. Thus, if $\Sigma_\omega \to \Sigma$ (a constant), then $\widehat{\Lambda}(\tau) = \frac{1}{2} dP(\tau)/d\tau$, where $P(\tau) = 1 - e^{-\Sigma\tau}$ is the probability of at least one event occurring (with rate Σ) in $(0, \tau)$.

Shift focus from Λ (avalanches) to Σ (turbulence).

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Long-time tails can be compatible with local transport.

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Taylor's formula for D

According to Taylor (1921), the particle diffusion coefficient is

$$D = \int_0^\infty d\tau\, C_L(\tau), \tag{18}$$

where C_L is the Lagrangian velocity autocorrelation function:

$$C_L(\tau) \doteq \langle \delta V(\underbrace{x(t+\tau), t+\tau)\delta V(x(t)}_{\text{along trajectory}}, t)\rangle. \tag{19}$$

It is important to distinguish the Lagrangian correlation function from the Eulerian one ($\vec{x}$ and t specified independently; easy to measure):

$$C_E(\tau) \doteq \langle \delta V(\underbrace{x, t+\tau)\delta V(x}_{\text{same point}}, t)\rangle. \tag{20}$$

$C_E(\tau)$ and $C_L(\tau)$ can behave quite differently.

Eulerian vs. Lagrangian correlations

A simple 1D model emphasizes the distinction between Eulerian and Lagrangian correlations. Let

$$\underbrace{V(x,t)}_{\text{arbitrary random field}} = X(x)T(t), \tag{21}$$

where X and T are statistically independent,

- $X(x) = \cos(kx + \theta)$ [θ uniform on $(0, 2\pi)$],
- $\mathcal{T}(\tau) \doteq \langle T(t+\tau)T(t)\rangle$ is arbitrary.

Now

$$C_E(\tau) = \langle V(x,t+\tau)V(x,t)\rangle \tag{22a}$$

$$= \langle X^2(x)\rangle\langle T(t+\tau)T(t)\rangle \tag{22b}$$

$$= \text{const.} \times \underbrace{\mathcal{T}(\tau)}_{\substack{\text{could have}\\ \text{algebraic tails}}} . \tag{22c}$$

C_L can behave differently from C_E

For example, let $x(t) = x + vt$ for Gaussian v. Then

$$C_L(\tau) = \mathcal{X}(\tau)\mathcal{T}(\tau), \tag{23}$$

where

$$\mathcal{X}(\tau) = \langle X(x+v(t+\tau))X(x+vt)\rangle \mathcal{T}(\tau) \tag{24a}$$

$$= \frac{1}{2}\langle[\cos(kv\tau) + \cos(kv\tau+2k(x+vt)+\cancel{2\theta})]\rangle \tag{24b}$$

$$= \frac{1}{2}\langle\cos(kv\tau)\rangle = \frac{1}{2}e^{-k^2\overline{v}^2\tau^2/2}. \tag{24c}$$

Thus, for $V(x,t) = X(x)T(t)$, one finds

$$C_E(\tau) \propto \mathcal{T}(\tau), \qquad C_L(\tau) \propto \underbrace{e^{-k^2\overline{v}^2\tau^2/2}}_{\substack{\text{dominates} \\ \tau \text{ decay}}} \mathcal{T}(\tau). \tag{25a,b}$$

A more realistic random trajectory need not change this conclusion.

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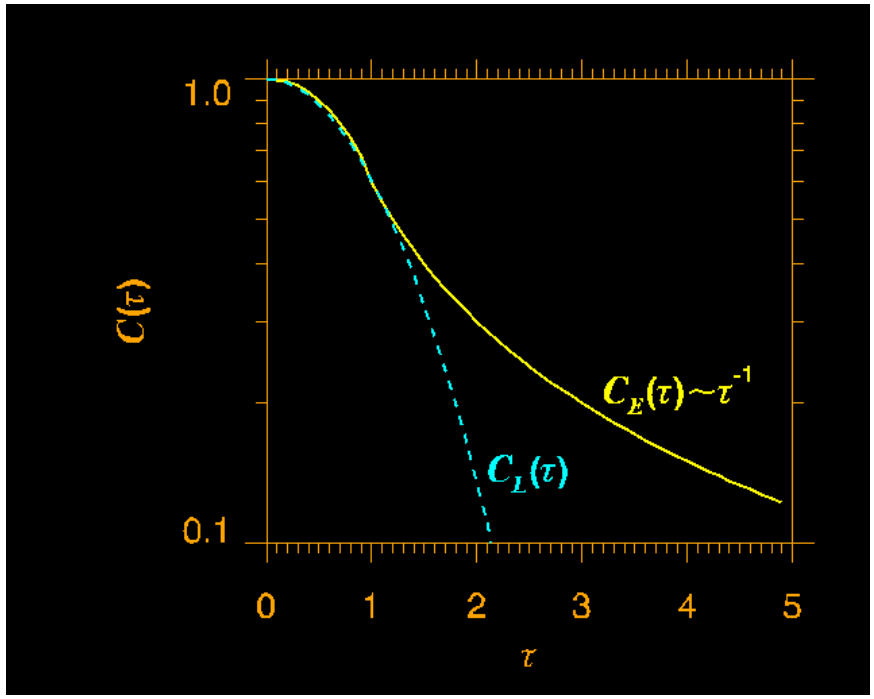

In some models, the Lagrangian correlation function decays more rapidly than the Eulerian one, due to linear and/or turbulent phase mixing.

$C_L(\tau)$ can be integrable
even when $C_E(\tau)$ is nonintegrable.

What about more realistic velocity fields?

Eulerian correlations in the SCF

The **Fluctuation–Dissipation Ansatz** is

$$C_{\vec{k}}(\tau) = R_{\vec{k}}(\tau) C_{\vec{k}}(\tau = 0) \quad (\tau > 0). \tag{26}$$

(This is exact in thermal equilibrium; otherwise, it is known as the **Self-Consistent Field Approximation**.[a]) Then

$$C_E(\tau) = \int \frac{d\vec{k}}{(2\pi)^d} R_{\vec{k}}(\tau) C_{\vec{k}} \tag{27a}$$

$$= \frac{\Omega_d}{(2\pi)^d} \int_0^\infty dk\, k^{d-1}\, R_k(\tau) C_k, \tag{27b}$$

If one assumes that $R_k(\tau) \sim e^{-k^2 D\tau}$ as $k \to 0$, then by stationary phase

$$\boxed{C_E(\tau) \sim \tau^{-d/2}.} \quad \text{More generally,} \quad \boxed{C_E(\tau) \sim \tau^{-\beta}.} \tag{28}$$

[a] Rediscovered by Krommes and Oberman (1976) in the context of convective cells.

Lagrangian correlations in turbulence

$C_E(\tau) \sim \tau^{-d/2}$ is reminiscent of the famous long-time tails in many-body kinetic theory (Alder and Wainwright, 1970; Krommes and Oberman, 1976). But those were for Lagrangian correlations!

Note: If $C_L(\tau) \sim \tau^{-d/2}$, then $\mu = \int_0^\infty d\tau\, C_L(\tau)$ is nonintegrable for $d \leq 2$! In that case, local transport coefficients do not exist.

How does one assess the integrability of Lagrangian correlations in conventional turbulence theory?

Either

- study (the time dependence of) the mass operator $\Sigma_{\vec{k}}(\tau)$, say in the DIA (this is difficult);
- study (the convergence of) the turbulent damping rate $\eta_{\vec{k}}$ in a Markovian closure. E.g., $\mu = \lim_{k\to 0} \eta_{\vec{k}}/k^2$. [If limit exists: local transport; C_L is integrable.]

Turbulent damping in the EDQNM

In the eddy-damped quasi-normal Markovian (EDQNM) approximation,

$$\eta_{\vec{k}} = -\sum_{\Delta} M_{\vec{k}\vec{p}\vec{q}} M^*_{\vec{p}\vec{q}\vec{k}} \theta^*_{\vec{k}\vec{p}\vec{q}} C_{\vec{q}}. \tag{29}$$

For example, consider the turbulent damping rate of vorticity in the Hasegawa–Mima equation:

$$\eta_{\vec{k}} = -\sum_{\Delta} \frac{|\widehat{\mathbf{z}} \cdot \vec{p} \times \vec{q}|^2 (p^{-2} - q^{-2})(q^{-2} - k^{-2})}{([k^{-2}] + 1)([p^{-2}] + 1)} \times \frac{C_{\vec{q}}}{(\eta_{\vec{k}} + \eta_{\vec{p}} + \eta_{\vec{q}})^*}. \tag{30}$$

- The bracketed terms in the denominator stem from adiabatic electron response; when they are omitted, the model reduces to the 2D guiding-center model (Krommes and Similon, 1980).
- Assume isotropy for simplicity.

Markovian $\eta_{\vec{k}}$, cont'd

$$\eta_{\vec{k}} = -\frac{2}{(2\pi)^2}\int_{\Delta} dp\, dq\, p^2 q^2 |\sin\alpha| \times \frac{(p^{-2}-q^{-2})(q^{-2}-k^{-2})}{([k^{-2}]+1)([p^{-2}]+1)} \frac{C_q}{(\eta_k+\eta_p+\eta_q)^*}, \qquad (31)$$

- μ_k converges for HM.
- But it diverges (logarithmically) for the guiding-center model (adiabatic electron response omitted). That is a manifestation of long-time tails in the Lagrangian Green–Kubo integrand.

$C_E(\tau)$	—	$\tau^{-\beta}$ (quite general)
$C_L(\tau)$	—	??? (either integrable or nonintegrable)

Don't infer predictions about transport from Eulerian correlation functions.

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Related research[a]

Burgers' equation: For some recent thoughts, see

J. A. Krommes, Renormalized dissipation in the nonconservatively forced Burgers equation, Phys. Plasmas, submitted.

Stochastic models and submarginal turbulence: See

J. A. Krommes, Submarginal profiles and turbulent transport: An exactly solvable model, Phys. Plasmas 4, 1342 (1997).

Dynamical mechanisms for submarginal turbulence: For a summary of the current status, see

J. A. Krommes, Recent results on analytical plasma turbulence theory: Realizability, intermittency, submarginal turbulence, and self-organized criticality, Plasma Phys. Control. Fusion 41, A641 (1999).

Zonal flows: For some new perspectives, see

J. A. Krommes, The influence of random zonal flows on saturation levels in simple stochastic models, Phys. Plasmas (in press).

[a] See `http://www.pppl.gov/docs/publications/pages/publications.html`.

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Summary

- Pure SOC lives in an asymptotic corner of parameter space. It is probably not necessary to look there for explanations of observed long-time correlations.
- Long-time correlations can be present in stochastic systems that do not possess essential features of SOC.
- The spectral balance equation of statistical turbulence theory can unify various approaches to, and formulas for fluctuation spectra.
- Even though Eulerian correlation functions can easily exhibit long-time tails, transport coefficients (determined by Lagrangian correlations) can nevertheless be local.

These points ought to be taken into account in future discussions of paradigms for transport and interpretations of experimental results.

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