

PPPL-4133

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December 2005



Princeton Plasma Physics Laboratory

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Controlled Fusion with Hot-Ion Mode in a Degenerate Plasma

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Abstract

In a Fermi-degenerate plasma, the rate of electron physical processes is much reduced from the classical prediction, possibly enabling new regimes for controlled nuclear fusion, including the hot-ion mode, a regime in which the ion temperature exceeds the electron temperature. Previous calculations of these processes in dense plasmas are now corrected for partial degeneracy and relativistic effects, leading to an expanded regime of self-sustained fusion.

PACS numbers:

Keywords:

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I. INTRODUCTION

The fuel which can sustain thermonuclear reactions most easily is a mixture of deuterium and tritium (D-T). The D-T reaction, however, has serious drawbacks. First, tritium does not exist in substantial quantities. Second, the D-T reaction produces neutrons; neutrons activate other material and damage the first wall. Therefore, it would be desirable to utilize controlled thermonuclear reactions that produce the fewest neutrons or no neutrons. The most promising fuel with no neutron (sometimes called advanced fuel) is proton-boron-11 [$P + B^{11} \rightarrow 3\alpha(2.7 \text{ MeV})$] and deuterium-helium-3 [$D + He^3 \rightarrow p(14.7 \text{ MeV}) + \alpha(3.6 \text{ MeV})$]. But all advanced fuels require a much higher temperature than does the D-T mixture. There are many costs to maintaining the high temperature. In particular, the bremsstrahlung losses in this regime might be greater than the fusion power, which makes self-burning of advanced fuels unlikely [1].

In classical plasmas, both the fusion power and the radiation losses are proportional to the square of the density. Thus, the power balance is essentially a function only of the temperature and the ratio of the fuel concentration since the power balance is insensitive to the density, and self-sustained aneutronic fusion burning remains unlikely [2]. However, in Fermi degenerate plasmas, the prospect of the aneutronic fuel burning can be very different due to the reduction of electron collisions, which both allows the ion temperature to exceed the electron temperature and reduces the bremsstrahlung loss.

In previous work [3, 4], we showed that certain properties of degenerate plasmas such as reduced i-e collisions enable an attractive fusion regime. We also showed that the fusion byproducts are primarily stopped not by electrons but by ions, even in the limit when the electron temperature goes to zero, thus allowing a regime of operation in which ions are hotter than electrons, the so-called “hot-ion mode” of operation. We estimated that the density should be more than 10^5 g/cm^3 , at which the ion temperature is more than 100 keV and the electron temperature is 30 keV. This regime has more favorable energy balance than the equal temperature mode, and so can facilitate the self-sustained burning of aneutronic fuel. This reduction of i-e coupling also can affect the current drive efficiency [5]. However, we did not consider the effects of partial degeneracy and the relativistic effects on the i-e collisions, the reduction of the bremsstrahlung, and the energy that goes from fusion byproducts into the electrons. In this paper, we calculate these effects more accurately. As

a consequence, we can show that the possible self-sustained burning regime is larger than what we have predicted previously, and how even larger ion-electron temperature differences can be sustained.

This paper is organized as follows. In Sec. (II), we explain the self-burning requirements of aneutronic fuels, and discuss the power balance in classical plasmas. We also summarize prior work on fusing advanced fuel using inertial confinement. In Sec. (III), we explain essential differences between degenerate electron plasmas and classical plasmas. In Sec. (III A), we show how the i-e collision rate in a degenerate plasma is greatly reduced from the classical prediction. In Sec. (III B), we apply the result by Maynard and Deutsch [6] to obtain the partial degeneracy effects on the i-e collision rate. In Sec. (III C), we calculate how the stopping power formula changes in a relativistic plasma. In Sec. (III D), we calculate, based on the reduced stopping power formula, the fraction of energy that goes from the fusion byproducts to electrons or ions. In Sec. (III E), we apply the result by Eliezer [7] to calculate the effects of degeneracy on bremsstrahlung. These new effects tend to enlarge the regime for hot-ion fusion that we identified previously. The calculation of the precise enlargement is left to a separate paper. In Sec. (IV), our conclusions are summarized.

II. SELF-BURNING REQUIREMENT OF ANeutronic FUEL

A. Self-Burning requirements

Ignition requires that the fusing fuel be maintained at high ion-temperature long enough to produce enough fusion reactions, and that the fusion power (energy produced from fusion) should be more than the total power losses (e.g. losses due to plasma instability, continuous bremsstrahlung losses, cyclotron radiation, line radiation and particle losses from transport) [8]. Due to the large fusion cross-section at relatively low kinetic energy, Deuterium-Deuterium (D-D) and Deuterium-Tritium (D-T) in magnetic fusion satisfy the above requirements. However, aneutronic fuel (e.g. P-B-11 and D-He-3) is only self-burning with difficulty. This is because the P-B-11 fusion reaction is appreciable only when $T_i \geq 200$ keV, but at this temperature, the fusion power, P_F , of the P-B-11 is less than the bremsstrahlung losses, P_B , if the electron temperature is the same as the ion temperature, i.e. $P_B(T_e = T_i) > P_F(T_i)$ [1]. Apart from the apparent insurmountable task of recycling or reflecting these power

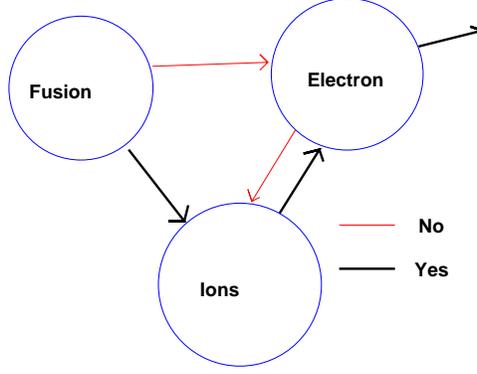


FIG. 1: To achieve a hot-ion mode, the fusion by-products must dissipate their energy mostly not into electrons but ions. The ions then will dissipate their energy into the electrons via Coulomb collisions, and the electrons will lose energy by bremsstrahlung losses.

losses, the only way to overcome this difficulty would be if the fuel were burned at high ion temperature and low electron temperature ($T_i > T_e$). To achieve this hot-ion mode, the fusion by-products (α -particle for the P-B-11, $\alpha + p$ for the D-He-3) should primarily dissipate their energy not on electrons but on ions as shown in Fig. (1).

To find an attractive parameter regime for the hot-ion mode in an ideal plasma, consider that the rate of energy transfer from α -particle to electrons via Coulomb collisions is given by the classical formula:

$$\nu_{i,e}^C = 3.2 \times 10^{-9} \frac{1}{\mu_i T_e^{3/2}} n_e Z_i^2 \log \Lambda, \quad (1)$$

where $\log \Lambda$ is the Coulomb logarithm, T_e in eV, μ_i in esu and Z_i is the charge number of the alpha particle and n_e in cm^{-3} . Note that $\nu_{i,e}^C$ is inversely proportional to the electron temperature. The rate of energy transfer from α particles to ions via Coulomb collisions can be given as

$$\nu_{i,j}^C = 1.8 \times 10^{-7} \left(\frac{\mu_i^{1/2}}{\mu_j} \right) \frac{1}{E^{3/2}} n_j Z_i^2 Z_j^2 \log \Lambda, \quad (2)$$

where Z_j is the charge number of background ions and E is the energy of the alpha particle in eV. Note that $\nu_{i,j}^C$ is independent of the electron temperature and the ion temperature. We note from Eqs. (1) and (2) that the energy of the α particle will be dissipated on electrons if the electron temperature is too low ($\nu_{i,e}^C$ is inversely proportional to the electron temperature). This critical electron temperature is 30 keV.

Assuming that the energy of the fusion byproducts goes into ions ($T_e > 30$ keV), we can obtain the electron temperature for given ion temperature by the balance between the bremsstrahlung losses P_B and the energy transfer rate P_{ie} from ions into electrons via Coulomb collision: $P_B(T_e) = P_{ie}(T_i, T_e)$. Now, with electron temperature known, we can obtain the ratio $F = P_F(T_i)/P_B(T_e, T_i)$. There then appears to exist a band around $T_i \cong 300$ keV at which F is larger than unity [1]. Unfortunately, this computation [1] was based on an old data of the fusion rate: the recent data suggests much reduced rate [9], so there is no self-burning regime.

In the case of the D-He-3, the fusion rate is already appreciable at 50 keV and there exists a self-burning regime. However, deuterium ions do produce neutrons via $D + D \rightarrow He^3 + n$, $D + D \rightarrow T + p$ and $D + T \rightarrow n + \alpha$. Therefore, the minimum amount of deuterium density should be small; something like $n_D/n_{He} \leq 0.1$ might be desirable. However, under the condition $n_D/n_{He} \leq 0.1$, one can show that there exists no self-burning regime.

In summary, it is possible to self-burn advanced fuel only by maintaining a hot-ion mode. For the P-B-11, according to the recent reduced activity data, it seems impossible to sustain fusion reaction even in a hot-ion mode. In the D-He-3 case, due to the production of neutrons from deuteriums, it is desirable to have small deuterium density such as $n_D/n_{He} \leq 0.1$. However, at this fuel concentration, there exists no self-burning regime.

B. Hot-Ion Regime in advanced fuel burning

The hot ion mode is always desirable in fusion devices. For example, in the D-T reactor, it can enhance the performance and the confinement vastly [10]. In magnetic fusion devices, the hot-ion mode can be obtained, in principle, by catalyzing alpha particle power to ions using injected rf waves, i.e. alpha channeling [11]. Note, however, that the hot-ion mode is a necessary condition for advanced fuel while it is just advantageous for in the D-T fuel.

To achieve a hot-ion mode in inertial confinement fusion using P-B-11, there have been proposals to generate a detonation wave [12–15]. Eliezer [13] showed that compressed fuel can be burned by an expanding ion fusion-burning wave preceded by an electron-conduction heat detonation wave. A large gap between the electron temperature $T_e \cong 80$ keV and the ion temperature $T_i \cong 200$ keV might then be achievable. However, they withdrew this claim later because the activity data was revised lower [14]. More recently, however,

the bremsstrahlung was predicted to be much reduced, giving brighter prospect for P-B-11 fusion [7]. Leon et al. [16] showed that plasma degeneracy lower the ignition temperature for D-T, and that for P-B-11, the ignition temperature can be lower than 20 keV when $\rho = 3.3 \times 10^7 \text{ g/cm}^3$. They implied that the density is too large for economical fusion reactor.

In the D-He-3 ICF, Honda [17] pointed out that, due to the nuclear elastic scattering, there will be more energy transfer to ions from the 14 MeV proton. While this is still smaller than energy transfer to electrons, it nevertheless improves the fusion reactivity. However, the electron temperature is still the same as the ion temperature according to their scenario.

III. DEGENERATE PLASMA

In dense plasmas, the power balance becomes very different, in ways that favor the hot-ion mode. In quantum electron plasmas, the electron distribution satisfies Fermi-Dirac statistics, since electrons obey the exclusion principle. In the Fermi distribution, the occupation number g is defined as $g(E) = 1/(\exp[(E - \mu)/T_e] + 1)$, where μ is the chemical potential, $E = \hbar^2 k^2 / 2m_e$ is the kinetic energy of an electron, and g is normalized as $n_e = \int 2gd^3k / (8\pi^3)$. If the final state has the occupation number g_s , a transition from the initial state to the final state is forbidden with the probability $1 - g_s$. If the electron De Broglie wave length is comparable to the inter-particle spacing, this exclusion principle becomes important to consider. This is usually when $\theta = T_e / E_F < 10$, where $E_F = \hbar^2 (3\pi^3 n_e)^{2/3} / 2m_e$ is the Fermi energy.

An example, consider metals at $\rho = 10 \text{ g/cm}^3$, and $n_e \cong 10^{22} \text{ cm}^{-3}$ corresponding to the Fermi energy E_F of a few eV. At room temperature, we then obtain $\theta \cong 0.01$. Consider, as another example, a D-T ICF target with $\rho = 10^3 \text{ g/cm}^3$ (E_F being a few keV) and the temperature $T_e \cong 10 \text{ keV}$. We obtain $\theta \cong 2 - 5$. As shown later, the relevant parameter regime for aneutronic burning will be $\rho \cong 10^5 \text{ g/cm}^3$, at which the Fermi energy is a few tens of keV. For the electron temperature of a few tens of keV, the degeneracy parameter θ is order of unity.

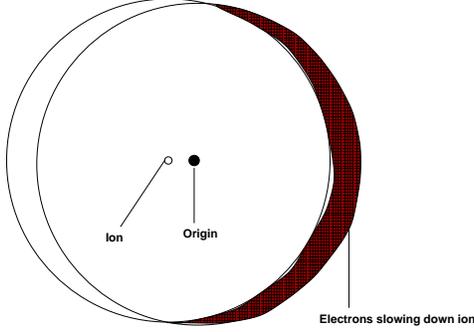


FIG. 2: Electron velocity space. Because the Fermi-sea (inside the circle) is already occupied, only electrons on the Fermi-surface (the area filled with black) can participate in slowing down the ion.

A. Reduction of Ion-Electron Collisions in a Degenerate Plasma with $T_e = 0$

In a degenerate plasma, certain collisions are forbidden because of the exclusion principle, which reduces the total collision rate. If the velocity of an ion is slower than the electron Fermi-velocity, then as the ion moves in the plasma, it slows down giving its kinetic energy away to electrons. However, because the Fermi-sea is already occupied by electrons, only electrons on the Fermi-surface can take part in these collisions as shown in Fig. (2). The original calculation of the collision rate has been obtained by Fermi [18] and Lindhard [19]. Later on, the electronic stopping power in an electron degenerate metal has been intensively studied theoretically [20–27] and experimentally [27–33], in the case when the velocity of an ion is smaller than the electron Fermi-velocity.

Lindhard derived the stopping power formula:

$$\frac{dK}{dt} = \frac{q^2}{2\pi^2} \int \left[\frac{\mathbf{k} \cdot \mathbf{v} \operatorname{Im} D(\mathbf{k}, \mathbf{k} \cdot \mathbf{v})}{k^2 v |D(\mathbf{k}, \mathbf{k} \cdot \mathbf{v})|^2} \right] d^3 \mathbf{k} . \quad (3)$$

where K is the energy of the ions, q is the charge of the ion, and D is the electron dielectric function. Note that the right hand side is the kinetic energy loss of the ion per length. By using the dielectric function from the quantum random phase approximation, he obtained the stopping frequency,

$$\frac{dE}{dt} = C(\chi) \frac{8}{3\pi} \frac{m^2 Z^2 e^4}{\mu \hbar^3} E, \quad (4)$$

where μ is the ion mass, E is the ion energy, m is the electron mass, $\chi^2 = e^2 / \pi \hbar v_F$, v_F is the Fermi velocity, and $C(\chi) \cong (1/2)[\log(1 + 1/\chi^2) - 1/(1 + \chi^2)]$ [20]. The above formula is valid

if $v \ll v_F$ and $r_s \ll 1$, where v is the ion velocity, and $r_s = (me^2/\hbar)(3/4\pi n_e)^{1/3}$ [18, 19, 21]. The collisions occur between the ion and the fastest electrons rather than, as in a weakly-coupled hot plasma, between the ion and the thermal electrons. The collisional cross-section decreases as $1/v_F^4$. This strong dependence of the cross-section on v_F just suffices to cancel the effect of the greater electron density, the greater energy loss per collision, and the great relative velocity of the colliding particles. A surprising result is that the stopping frequency is almost independent of the electron density. From Eq. (4), the i-e collision frequency is given as

$$\nu_{ie} = 3.47 \times 10^{13} (Z^2/\mu) \frac{1}{\text{sec}} \quad (5)$$

where μ is the nucleus mass in the unit of the proton mass, and $C(\chi) \cong 2$ when $n \cong 10^{28} (1/\text{cm}^3)$.

B. Reduction of Ion-Electron Collision: $T_e \neq 0$

The stopping power in a partially degenerate plasma has been obtained by Maynard and Deutsch [6]. We apply their formula to our regime of interest. The starting equations is Eq. (3) with the dielectric function,

$$D^l(\mathbf{k}, \omega) = 1 + \frac{2m^2\omega_{pe}^2}{\hbar^2k^2} \sum_n \frac{g(E_n)}{N} \left(\frac{1}{k^2 + 2\mathbf{k} \cdot \mathbf{k}_n - \frac{2m}{\hbar}(\omega + i\gamma)} \right) \\ + \frac{2m^2\omega_{pe}^2}{\hbar^2k^2} \sum_n \frac{g(E_n)}{N} \left(\frac{1}{k^2 + 2\mathbf{k} \cdot \mathbf{k}_n - \frac{2m}{\hbar}(\omega - i\gamma)} \right), \quad (6)$$

where g is the occupation number and $N = \sum_n g(E_n)$ is the total number of electrons. We numerically integrate Eq. (3) for the density $n_e = 10^{29} \text{ cm}^{-3}$, and obtain the stopping power as a function of the temperature in Fig. (3). As shown in the figure, for non-zero electron temperature, the i-e collision frequency decreases further with the electron temperature. See for details [6, 34, 35]

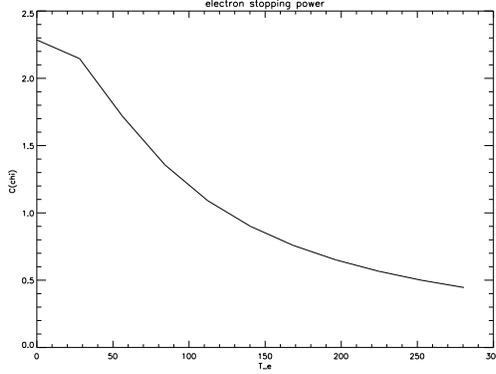


FIG. 3: $C(T_e)$ in Eq. (4) for $0 < T_e < 250$ keV with $n_e = 1.3 \times 10^{29}$ cm $^{-3}$ (the Fermi energy $E_F = 93$ keV, Y-axis: $\chi(T_e)$, X-axis: electron temperature T_e in keV).

C. Relativistic Effect and Reduction of Ion-Electron Collision

For a plasma with the density $n_e \cong 10^{29}$ cm $^{-3}$, the Fermi energy is not negligible with respect to the rest-mass energy (500 keV), and the relativistic effect should be taken into account. While the exact dielectric function with full consideration of the relativistic effect can be found in the literature [36], a simpler approach with approximations is adopted here.

The stopping power formula for a degenerate plasma [19] can be written as

$$\frac{dK}{dl} = \frac{4\pi Z^2 e^4}{m_e v^2} n_e L, \quad (7)$$

where K is the kinetic energy of the ion, and L is

$$L = \frac{6}{\pi} \int_0^{\frac{v}{v_F}} u du \int_0^\infty z^3 dz \frac{f_i(u, z)}{(z^2 + \chi^2 f_r(u, z))^2 + \chi^4 f_i(u, z)^2}, \quad (8)$$

where v_F is the Fermi velocity, $z = k/2k_F$, $u = |\omega|/kv_F$, $\chi^2 = e^2/\pi\hbar v_F$, and f_i (f_r) is related to the longitudinal dielectric function as $D^l(k, \omega) = 1 + (3\omega_{pe}^2/k^2 v_F^2)(f_r + i f_i)$, where $\omega_{pe} = \sqrt{4\pi n e^2/m_e}$ is the plasma frequency.

For the relativistic dielectric function, we use Lindhard's except that the dispersion relation between the momentum and the energy is different from the classical one, i.e. $E(k) = \hbar\omega_k = \sqrt{m^2 c^4 + \hbar^2 k^2 c^2}$ rather than $E_k = \hbar^2 k^2 / 2m_e$. Then, the dielectric function is given as

$$\begin{aligned}
D^l(\mathbf{k}, \omega) = & 1 + \frac{m\omega_{pe}^2}{k^2} \sum_n \frac{g(E_n)}{N} \frac{1}{E(k+k_n) - E(k_n) + \hbar(\omega + i\gamma)} \\
& + \frac{m\omega_{pe}^2}{k^2} \sum_n \frac{g(E_n)}{N} \frac{1}{E(-k+k_n) - E(k_n) - \hbar(\omega + i\gamma)},
\end{aligned} \tag{9}$$

In Eq. (8), we need to integrate f_r and f_i with respect to u and z . Lindhard has shown firstly that when the velocity of the ion is much smaller than the electron Fermi velocity, the major contribution to L comes from the region in the integration over the region $z \ll 1$ and $u \cong 0$, and secondly that f_i is proportional to u when $u \ll 1$. Based on these observations, we can use two approximations. First, in the denominator of Eq. (8), we ignore f_i and consider $f_r(u, z)$ only when $u = 0$ and $z \ll 1$. Second, $f_i(z, u)$ in the numerator only needs to be obtained to the first order in u as a function of z . Therefore, we only need to evaluate $f_r(z, 0)$ when $\omega = 0$ and $z \ll 1$, and $f_i(z, u)$ to the first order in u .

Firstly, let us evaluate f_r . We can write f_r from Eq. (9) as

$$f_r = \frac{3 \hbar^2 k_F^2}{2 2m_e} \sum_n \frac{f(E_n)}{N} \left[\frac{2E(k_n)}{E(k+k_n)^2 - E(k_n)^2} + \frac{2E(k_n)}{E(-k+k_n)^2 - E(k_n)^2} \right]. \tag{10}$$

After integration of the angle between k and k_n , Eq. (10) becomes

$$f_r = \frac{3}{2} k_f^2 \frac{1}{2\pi^2 n_e} \int g(k_n) \frac{E(k_n)}{mc^2} \frac{k_n}{k} \log\left(\frac{k_n + k/2}{k_n - k/2}\right) dk_n, \tag{11}$$

where $g(k)$ is the occupation number. Assuming $k/2k_n \ll 1$, we can expand the logarithmic terms in terms of $k/2k_n$ and simplify Eq. (11) as

$$f_r \cong \frac{1}{k_F} \int g(k_n) \frac{E(k_n)}{mc^2} dk_n, \tag{12}$$

For a plasma with zero-temperature, we obtain $f_r \cong 1$ since $g(k_n) = 1$ if $k_n < k_F$, and $g(k_n) = 0$ if $k_n \geq k_F$. If we calculate Eq. (12) to the first order, we obtain $f_r(0, 0) = 1 + (1/6)(\hbar^2 k_F^2 / m_e^2 c^2)$. For a plasma with non-zero temperature, Eq. (12) should be used with appropriate $g(k)$.

Secondly, we consider f_i in the denominator of the right-hand side of Eq. (8). As mentioned, we only need to evaluate f_i to the first order in u . From Eq. (9), we can write f_i as

$$\begin{aligned}
f_i = & + \frac{3 \hbar^2 k_F^2}{2} \frac{1}{2m_e} \sum_n \frac{f(E_n)}{N} \pi \delta(E(k+k_n) - E(k_n) + \hbar\omega) \\
& - \frac{3 \hbar^2 k_F^2}{2} \frac{1}{2m_e} \sum_n \frac{f(E_n)}{N} \pi \delta(E(-k+k_n) - E(k_n) - \hbar\omega) .
\end{aligned}$$

After integrating out the angle between k and k_n , it becomes

$$\begin{aligned}
f_i(z, u) = & + \frac{3}{2} \frac{k_F^2}{2m_e c^2} \frac{1}{2\pi^2 n_e} \int_{R_1} \left[g(k_n) \frac{k_n}{2k} (E(k_n) + \hbar\omega) \right] dk_n \\
& + \frac{3}{2} \frac{k_F^2}{2m_e c^2} \frac{1}{2\pi^2 n_e} \int_{R_2} \left[g(k_n) \frac{k_n}{2k} (-E(k_n) + \hbar\omega) \right] dk_n ,
\end{aligned}$$

where R_1 is a set in real axis with $R_1 = \{k > 0 : (E(|k| + |k_n|) > E(k_n) + \hbar\omega)\}$, and $R_2 = \{k > 0 : (E(-|k| + |k_n|) > E(k_n) + \hbar\omega)\}$. Assuming $\hbar\omega/m_e c^2 \ll 1$, to the first order in u , it can be shown that

$$f_i(k, \omega) \cong \frac{\pi}{2} u \left[g(k) \frac{E(k)}{m_e c^2} - \frac{\hbar^2}{m_e^2 c^2} \int g(k_n) k_n dk_n \right]. \quad (13)$$

where the first term of the right-hand side in Eq. (13) is from $\int_{R_1} g(k_n) (k_n/2k) E(k_n) dk_n - \int_{R_2} g(k_n) (k_n/2k) E(k_n) dk_n$ and the second term is from $\int_{R_1} g(k_n) (k_n/2k) \hbar\omega dk_n + \int_{R_2} g(k_n) (k_n/2k) \hbar\omega dk_n$. For a plasma with zero-temperature, in the limit $z \ll 1$, we obtain f_i from Eq. (13) as

$$f_i(u, z) = \frac{\pi}{2} u \left(1 - \frac{1}{2} \frac{\hbar^2 k_F^2}{m_e^2 c^2} \right), \text{ if } z < 1$$

$$f_i(u, z) = 0, \text{ if } z > 1, \quad (14)$$

which is the same as the classical formula in the limit $(\hbar k_F^2/m_e^2 c^2) \ll 1$.

We now return to the original problem of the relativistic correction to the stopping power. If the temperature is zero, from Eqs. (8) and (14), we can conclude that the stopping power is smaller due the relativistic effect than the classical calculation by a factor:

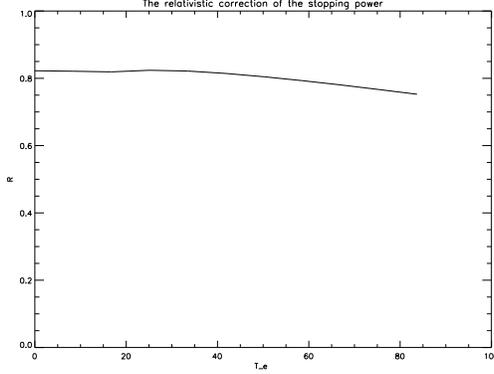


FIG. 4: The ratio of the relativistic stopping power to the classical stopping power for $0 < T_e < 80$ keV with $n_e = 10^{29}$ cm $^{-3}$ (the Fermi energy $E_F = 78$ keV, Y-axis: $R = L_{\text{rel}}/L_{\text{classical}}$, X-axis: electron temperature T_e in keV).

$$\frac{L_{\text{rel}}}{L_{\text{cla}}} = \left[1 - \left(\frac{1}{2} - \frac{1}{12 \log(\chi)} \right) \frac{\hbar^2 k_F^2}{m_e c^2} \right]. \quad (15)$$

For a non-zero temperature, we should use Eqs. (8), (12) and (13). An approximate correction can be made by calculating the stopping power from classical mechanics [34], and then correcting for the relativistic effect, so that

$$\frac{L_{\text{rel}}}{L_{\text{cla}}} \cong \left[1 - \frac{\hbar^2}{m_e^2 c^2 g(0)} \int g(k_n) k_n dk_n + \frac{1 \log(A)}{2 \log(\chi)} \right]. \quad (16)$$

where

$$A = \frac{\int g(k_n) \frac{E(k_n)}{m_e c^2} dk_n}{\int g(k_n) dk_n}. \quad (17)$$

Eqs. (15) and (16) are the major results of this section. In Fig. (4), $R = L_{\text{rel}}/L_{\text{classical}}$ is plotted as a function of the electron temperature for $n_e = 10^{29}$ cm $^{-3}$. In short, the reduction of the stopping power due to relativistic effects are 10-20 % of the classical result. Nagy [37] has obtained the exact stopping power formula for zero electron temperature:

$$C(\chi) = \frac{1}{2} \frac{1+a^2}{a^2} \left[\log\left(1 + \frac{\pi}{\alpha(1+a^2)^{1/2}}\right) - \frac{\pi}{\pi + \alpha(1+a^2)^{1/2}} \right], \quad (18)$$

where $\alpha = e^2/\hbar c \simeq 1/137$ and $a = mc/p_F$. By comparing this formula with a classical formula in Eq. (4):

$$C(\chi) = \frac{1}{2} \left[\log\left(1 + \frac{1}{\chi^2}\right) - \frac{1}{1 + \chi^2} \right], \quad (19)$$

where $\chi^2 = \alpha a/\pi$, we can also predict 10-20 % of reduction in the stopping due to the relativistic correction. Our result agrees well with Nagy's for zero temperature.

D. Fraction η of energy that goes from fusion by-products to electrons

In this section, the fraction η of energy that goes from fusion byproducts into the electrons is calculated, more exactly, using the result in Secs. (III A) (III B) and (III C), as a function of the electron temperature, density and fuel concentrations.

First, remember that the ion-ion collision frequency is given as in Eq. (2), where $\log(\Lambda)$ is the Coulomb logarithm. The Coulomb logarithm, $\log \Lambda$ can be obtained from the integration of the impact parameter [38]: $\log \Lambda = \int_0^{\rho_{\max}} d\rho\rho/(\rho_C^2 + \rho^2)$, where $\rho_C = Z_i Z_j e^2/2E_0$ is the distance at the closest approach, ρ_{\max} is the maximum impact parameter, and E_0 is the kinetic energy of the fusion byproduct. In our partially degenerate relativistic plasma, the maximum impact parameter can be estimated as the screening length D_s , which has been calculated, through the quantum random phase approximation [39]:

$$a/D_s = 0.1718, \quad (20)$$

where a is the inter-particle spacing. Then, we can estimate

$$\log \Lambda \cong \log [(n_e)^{-1/3}/0.1718\rho_C]. \quad (21)$$

For an example, an alpha particle with $\epsilon = 3.7$ MeV and $n_e = 3 \times 10^{28}$ cm⁻³, we obtain $\log(\Lambda) \cong 10.8$, which is larger than the Coulomb logarithm used in [3] by a factor 2. With the change of the Coulomb logarithm and the reduction of the electron stopping power in mind, we now obtain η -equation:

$$\eta = r_e(T_e) = \int_0^{E_0} \frac{dE}{E_0} \frac{\nu_{ie}(T_e)}{\nu_{ie}(T_e) + \sum_j \nu_{\alpha,j}(E)}. \quad (22)$$

where E_0 is the initial energy of the α particle, and ν_{ie} is given in Eq. (4). This equation can be simplified to

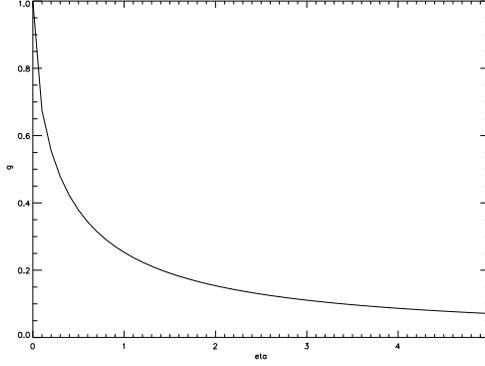


FIG. 5: The function g [Y-axis: $g(\zeta)$, X-axis: ζ].

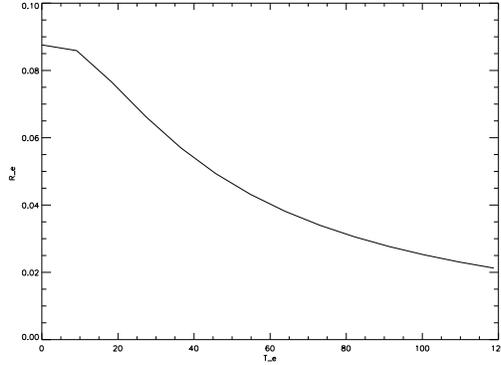


FIG. 6: The fraction of energy transfer from an alpha particle (2.7 MeV) to electrons as a function of electron temperature. The P-B-11 with density $n_e = 4 \times 10^{28}$ 1/cm³, $n_B/n_P = 0.3$, and the Fermi energy $E_F = 43$ keV, (Y-axis: r_e , X-axis : the electron temperature T_e in keV)

$$\eta(T_e) = \int_0^1 \frac{1}{1 + \zeta(T_e)/s^{3/2}} ds, \quad (23)$$

where

$$\zeta(T_e) = \sum_j 1.8 \times 10^{-7} \frac{n_j Z_i^2 Z_j^2 \lambda}{\epsilon^{3/2} \nu_{ie}(T_e)} \left(\frac{m_i^{1/2}}{m_j} \right). \quad (24)$$

We plot $g(\zeta) = \eta$ as a function of ζ in Fig. (5). For an example, in the P-B-11 fuel with $n_B/n_P = 0.3$ and $n_e = 4 \times 10^{28}$ cm⁻³, we plot $\eta(T_e)$ as a function of T_e in Fig. (6).

In the D-He-3, proton ($E_0 = 14.7$ MeV) and alpha particle ($E_0 = 3.6$ MeV) are fusion by-products. The fraction of the energy from an alpha particle to electrons, $r_{e,\alpha}$, can be obtained in the same way in the case of the P-B-11. For proton, the nuclear elastic collision

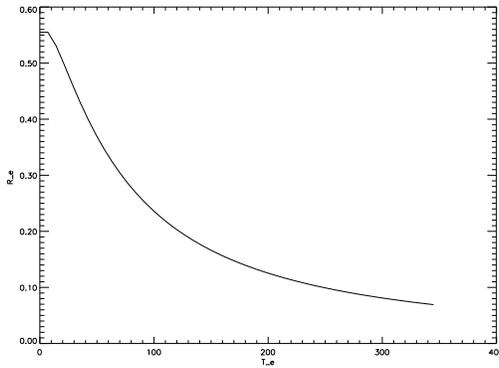


FIG. 7: The fraction of energy transfer from fusion by-products (a 3.6 MeV alpha particle and a 14.7 MeV proton) to electrons as a function of electron temperature. The D-He-3 with density $n_e = 3 \times 10^{28} \text{ 1/cm}^3$, $n_D/n_{he} = 0.3$, and the Fermi energy $E_F = 35 \text{ keV}$, (Y-axis: r_e , X-axis : the electron temperature T_e in keV.)

(NEC) must be also taken into account. The NEC is an elastic collision between nuclei in which nuclei only exchange their kinetic energy. Especially, the NEC between proton and He-3 is quite large. Including the NEC, we can express $r_{e,p}$ as

$$r_{e,p}(T_e) = \int_0^{E_0} \frac{dE}{E_0} \left[\frac{\nu_{ie}(T_e)}{\nu_{ie}(T_e) + \sum_j \nu_{\alpha,j}(E) + \sigma_N(E)v(E)f(E)} \right]. \quad (25)$$

where E_0 is the initial energy of the proton, $v(E)$ is the proton velocity, σ_N is the NEC cross-section, and $f(E)$ is the fraction of the proton energy per a NEC. By assuming $\sigma_N(E)v(E)f(E)$ as constant with respect to energy (this is a good approximation for the NEC between proton and He-3), we can use $r_{e,p}(T_e) = \gamma(T_e)g(\gamma(T_e)\zeta)$, where $\gamma(T_e) = \nu_{ie}(T_e)/(\nu_{ie}(T_e) + \sigma_N(E)v(E)f(E))$. Then, the total fraction can be obtained as $\eta = (14.7r_{e,p} + 3.6r_{e,\alpha})(14.7 + 3.6)$ with $r_{e,\alpha}$ is the fraction of energy from the alpha particle to electrons given in Eq.(22). For an example, we plot η as a function of T_e in Fig. (7) when $n_d/n_{he} = 0.2$ and $n_e = 3 \times 10^{28} \text{ cm}^{-3}$.

E. Reduction of Bremsstrahlung due to Partial Degeneracy

In previous work [3], the classical formula for bremsstrahlung was used. But bremsstrahlung power is greatly reduced in a degenerate plasma. While any reduction of bremsstrahlung leads to make electrons hotter, the reduction of the i-e collision frequency

due to higher electron temperature further reduces the energy transfer from ions to electrons. Therefore, the degeneracy effects on bremsstrahlung is important in limiting the energy flow from the ions. The exact formula for bremsstrahlung, which Eliezer [7] has recently derived, is applied in our particular regime. An electron emits photons as it collides with background ions, and the rate of the emission of photons by this electron is given by Greene [40]:

$$B(\omega, E) = n_i Z^2 v_e(E) \frac{d\sigma}{d\hbar\omega}, \quad (26)$$

where B is the rate of photons emission per second in the units of $\text{erg}^{-1} \text{sec}^{-1}$, and $v_e(E) = \sqrt{2E/m_e}$ is the velocity of the electron, and $(d\sigma/d\hbar\omega) = (A/\hbar\omega)(\eta_0^2/\pi\sqrt{3})$, where $A = (8\pi/3)(e^2\hbar/m^2c^3) = 5.728 \times 10^{-22} \text{ cm}^2$, and $\eta_0 = (e^2/\hbar v_e)$. The Cramer's approximation [40], which is valid when $\eta_0 \gg 1$, is used in Eq. (26). Considering Fermi-Dirac statistics, the radiation power P_B then can be written as [7]:

$$P_B = K \left(\frac{E_F}{\hbar} \right) \theta^2 f(T_e), \quad (27)$$

where μ is the chemical potential, E_F is the Fermi energy, $\theta = T_e/E_F$ and $K = (32\pi/3\sqrt{3})(Z^2 n_i e^6 / c^3 \hbar^3)$. and $f(T_e)$ is

$$f(T_e) = \int_0^\infty \left[\frac{\log\left(\frac{1+\exp(\tilde{\mu}-a)}{1+\exp(\tilde{\mu})}\right)}{\exp(a)-1} \right] da, \quad (28)$$

where $\tilde{\mu} = \mu(T_e)/T_e$. P_B is in the units of $\text{eV cm}^{-3} \text{sec}^{-1}$, or energy per volume per second. The radiation power density $W(\omega)$ is given as

$$W(\omega) = K \frac{\log\left(\frac{1+\exp(\mu/T_e - \hbar\omega/T_e)}{1+\exp(\mu/T_e)}\right)}{\exp(\hbar\omega/T_e) - 1} T_e, \quad (29)$$

where $P_B = \int d\omega W_B(\omega)$.

In Fig. (8), for the density $n_e = 10^{29} \text{ cm}^3$ ($E_F = 79 \text{ keV}$), the ratio of P_B to the classical result is plotted as function of the temperature. For the calculation of the radiation re-absorption, the radiation power density should be used, which is quite different from the classical black body radiation. As an example, for the density $n_e = 10^{29} \text{ cm}^3$ and the temperature $T_e = 15 \text{ keV}$, the ratio of $W(\omega)$ to the classical calculation is plotted in Fig. (9).

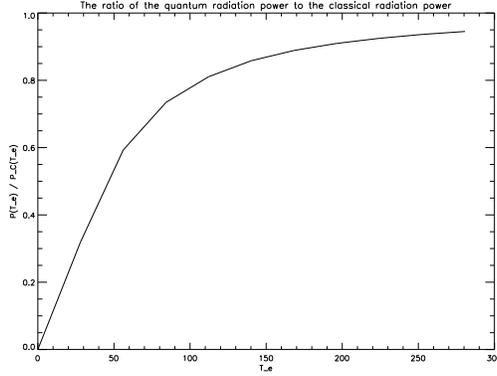


FIG. 8: The ratio of the exact radiation power P to the classical radiation power for $0 < T_e < 300$ keV with $n_e = 10^{29}$ cm $^{-3}$ and the Fermi energy $E_f = 78$ keV (Y-axis: P/P_C , X-axis : the electron temperature T_e in keV)

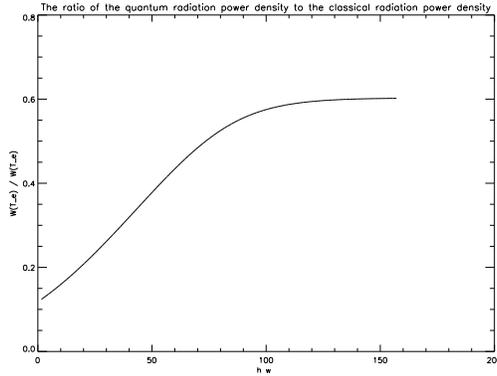


FIG. 9: The ratio of the exact radiation power density W to the classical radiation power density at $T_e = 15.7$ keV with $n_e = 10^{29}$ cm $^{-3}$ and the Fermi energy $E_F = 78$ keV (Y-axis: W/W_C , X-axis : the photon energy $\hbar\omega$ in keV).

IV. CONCLUSION

In degenerate plasmas, the electronic processes are much slower than the classical prediction due to Fermi-Dirac statistics. This aspect of degenerate plasmas enhances the prospect of controlled fusion of advanced fuels, since the reduction of the ion-electron coupling and the bremsstrahlung losses eventually impede energy dissipation from hot fusing ions. In particular, the ion energy losses are no longer proportional to the square of the density. The power balance is then quite sensitive to density, which makes the advance fuel burning feasible.

Building on our previous work [3], we show here that the partial degeneracy and the relativistic effects reduces the ion-electron collision frequency considerably compared to the zero temperature result previously assumed. This reduction means that the fraction of energy that goes from fusion byproducts into ions is larger than previously obtained. Since the bremsstrahlung is simultaneously much reduced compared to the classical result that was used in previous work, the parameter regime that we identified previously [3] will expand considerably. The precise calculation of the self-sustained burning regime is treated in a companion paper.

The authors thank R. Kulsrud, G. Hammett and S. Cohen for useful discussion. This work was supported by DOE Contract No. AC02-76CH0-3073 and by the NNSA under the SSAA Program through DOE Research Grant No. DE-FG52-04NA00139.

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