Nonneutral Flow in High-Power Diodes

- Electron emissions from cathode
- Child-Langmuir flow
- Magnetically-insulated electron flows
- Magnetically-insulated ion diodes
- Magnetron instability – theory
- Multi-resonator magnetrons
- Instabilities in magnetically-insulated ion deodes
- Applied-B diodes with virtual cathode

Electron emission from cathode

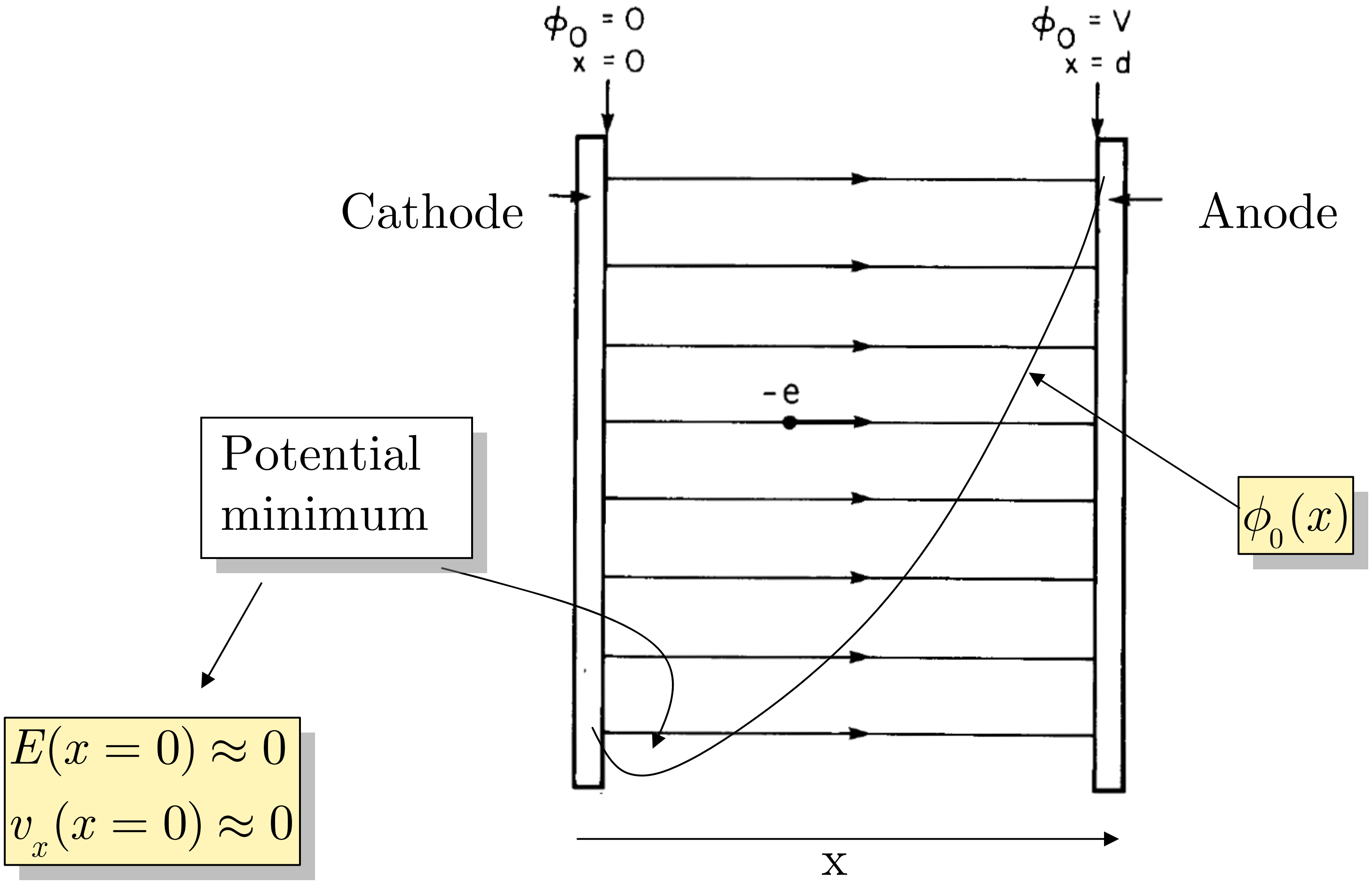
- Thermionic emission
- Photoemission
- Secondary emission

Electron obtain energy to escape

- Field emission
  - Deform potential barrier

- Explosive emission
  - Singular field melts and vaporizes wiskers by resistive heating

Space charge limited flow



Child-Langmuir flow (planar, non-relativistic)

$$\frac{eV}{m_e c^2} \ll 1,\ \ v_x(x=0) \approx 0$$

$$\left.\begin{aligned} -en_e^0(x)V_{xe}^0(x) &\equiv J_e = \text{ const.} \\ \frac{m_e}{2}V_{xe}^{02}(x) - e\phi_0(x) &= \text{ const.} \\ \frac{\partial^2}{\partial x^2}\phi_0(x) &= 4\pi e n_e^0(x) \end{aligned}\right\}$$

$$\boxed{\frac{\partial^2}{\partial x^2}\phi_0(x) = -\frac{4\pi(m_e/2e)^{1/2}J_e}{[\phi_0(x)]^{1/2}},}$$

$$\boxed{\frac{1}{2}\left(\frac{\partial\phi_0}{\partial x}\right)^2 = -8\pi\left(\frac{m_e}{2e}\right)^{1/2} J_e\phi_0^{1/2}}$$

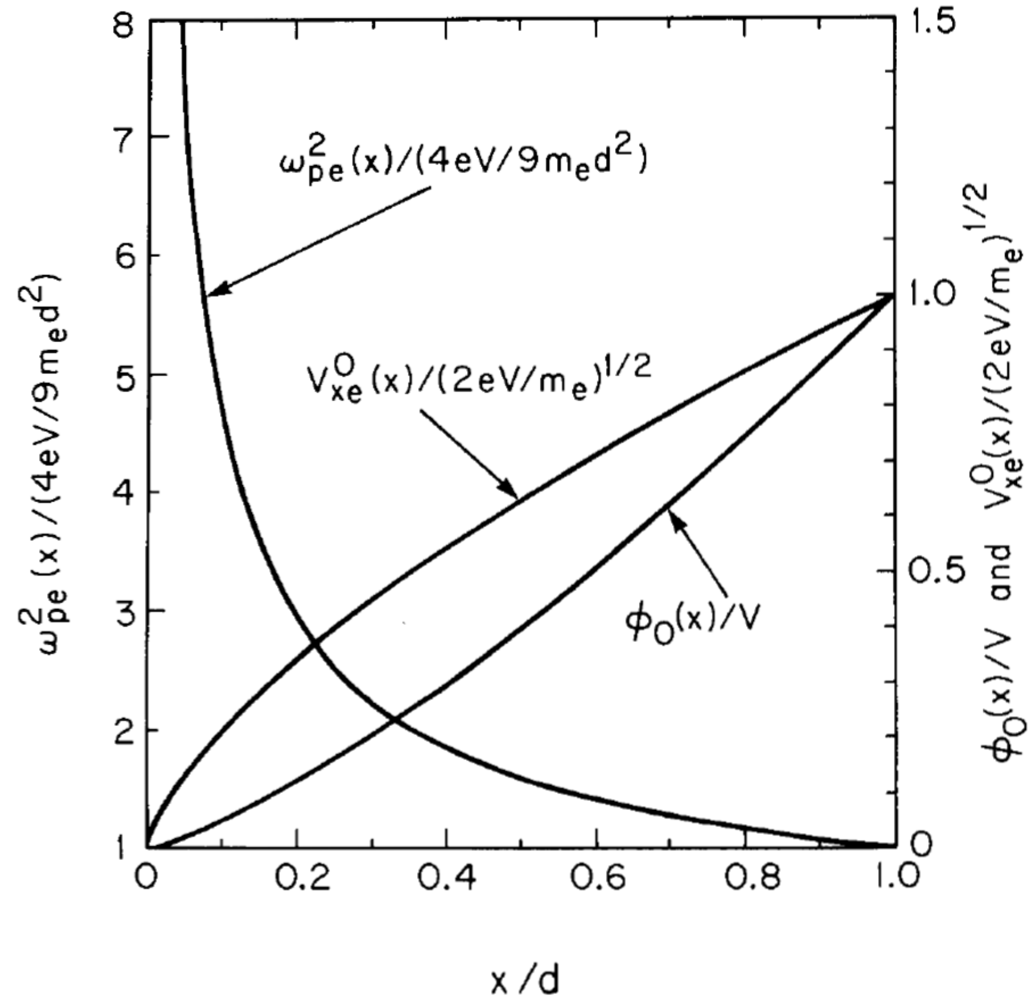
$$\boxed{\phi_0(0) = 0,\ \ \left.\frac{\partial\phi_0}{\partial x}\right|_{x=0} = 0,\ \ \phi_0(d) = V}$$

$$\boxed{\phi_0(x) = \left[-9\pi\left(\frac{m_e}{2e}\right)^{1/2} J_e d^2\right]^{2/3}\left(\frac{x}{d}\right)^{4/3}}$$

Why 3 B.C. for 2nd ODE?

$$\phi_0(x) = V\left(\frac{x}{d}\right)^{4/3}, \quad 0 \le x \le d,$$

$$J_e = J_{\mathrm{CL}} \equiv -\frac{2^{1/2}}{9\pi d^2}\frac{m_e c^3}{e}\left(\frac{eV}{m_e c^2}\right)^{3/2}$$



$$\boxed{\text{Assume } \left.\frac{\partial \phi_0}{\partial x}\right|_{x=0} = E_0, \text{ maxiumize } J \text{ wrt } E_0.}$$

Child-Langmuir flow (planar, relativistic)

$$-en_e^0(x)V_{xe}^0(x) \equiv J_e = \text{ const.}$$

$$\boxed{\left[\gamma_e^0(x)-1\right]m_e c^2 - e\phi_0(x) = \text{ const.}}$$

$$\frac{\partial^2}{\partial x^2}\phi_0(x) = 4\pi e n_e^0(x)$$

$$\gamma_e^0(x) = [1 - V_{xe}^{02}(x)/c^2]^{-1/2}$$

$$\boxed{\frac{\partial^2}{\partial x^2}\gamma_e^0(x) = -\frac{4\pi J_e e}{m_e c^3}\frac{\gamma_e^0(x)}{[\gamma_e^{02}(x)-1]^{1/2}}}$$

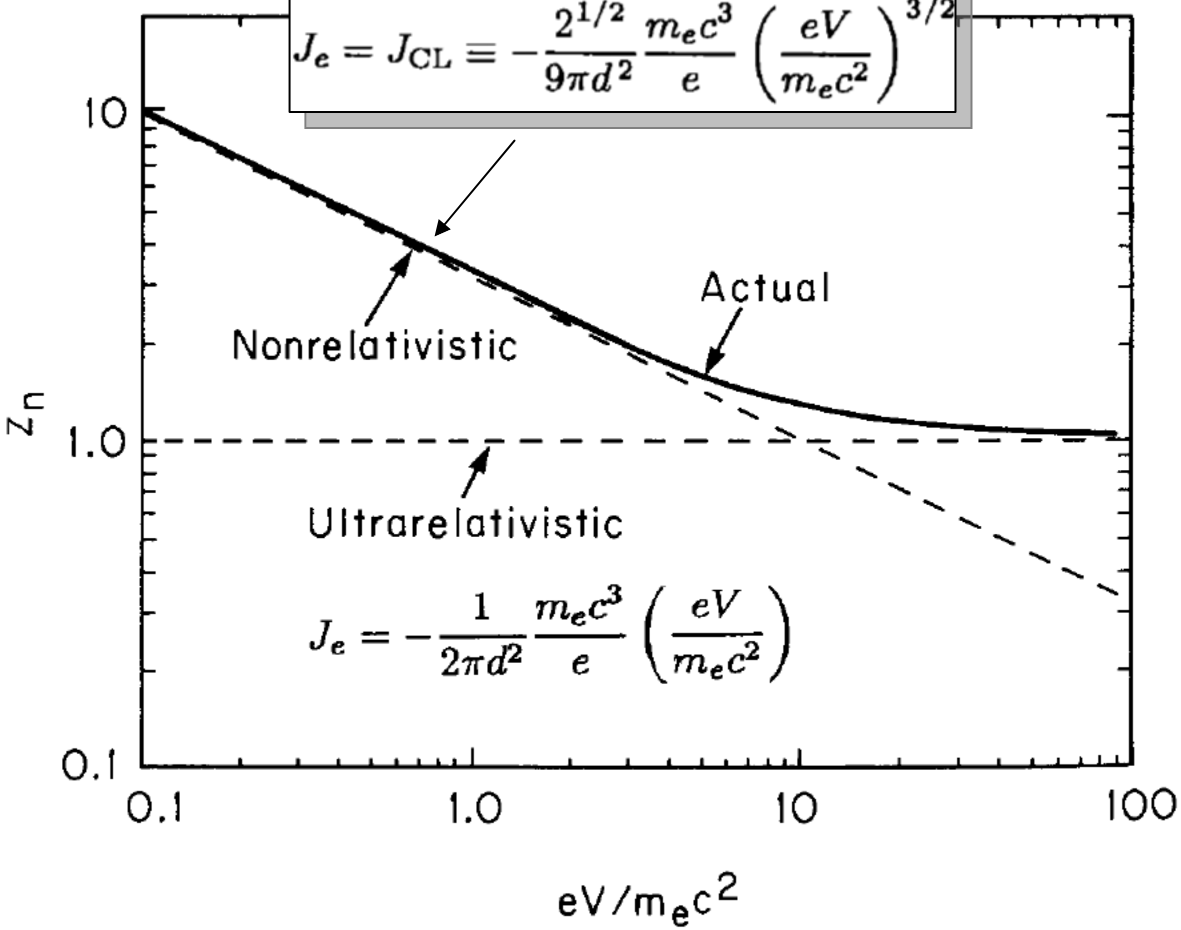
$$\boxed{\frac{1}{2}\left(\frac{\partial\gamma_e^0}{\partial x}\right)^2 = -\frac{4\pi J_e e}{m_e c^3}\left[\gamma_e^{02}-1\right]^{1/2}}$$

$$\boxed{\gamma^0(0)=1,\quad \left.\frac{\partial\gamma^0(x)}{\partial x}\right|_{x=0}=0,\quad \gamma^0(d)=1+\frac{eV}{m_e c^2}}$$

$$\boxed{\int_1^{\gamma_e^0(x)}\frac{d\gamma_e}{(\gamma_e^2-1)^{1/4}} = \left(\frac{-8\pi J_e e}{m_e c^3}\right)^{1/2} x}$$

Impedance

$$Z_n = -\frac{cV}{2\pi d^2 J_e}$$



$$J_e = J_{\mathrm{CL}} \equiv -\frac{2^{1/2}}{9\pi d^2}\frac{m_e c^3}{e}\left(\frac{eV}{m_e c^2}\right)^{3/2}$$

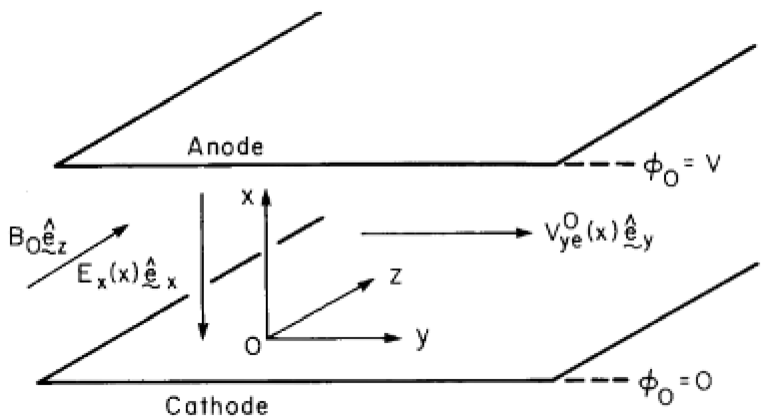
$$J_e = -\frac{1}{2\pi d^2}\frac{m_e c^3}{e}\left(\frac{eV}{m_e c^2}\right)$$

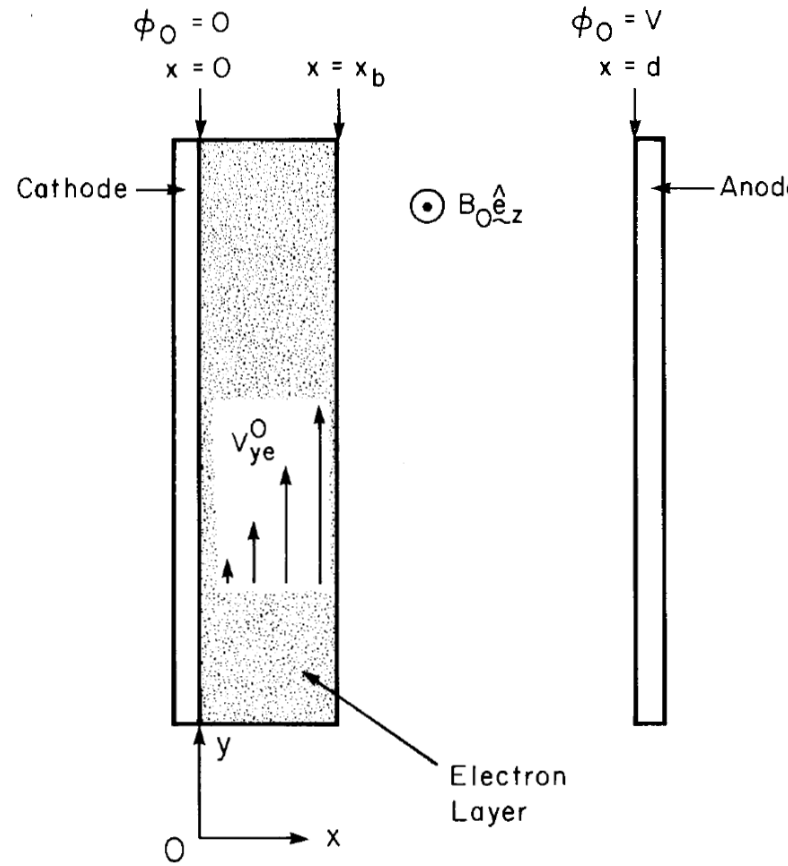
$$\boxed{\left(\frac{-8\pi J_e e}{m_e c^3}\right)^{1/2} x = \int_0^{\gamma(x)} \frac{d\gamma}{\left(\gamma^2 - 1\right)^{1/4}}}$$

$$\boxed{\int_0^{\gamma(x)} \frac{d\gamma}{\gamma^{1/2}} = \int_0^{\phi(x)} \phi^{-1/2} d\phi, \quad \gamma \gg 1}$$

$$\boxed{\int_0^{\gamma(x)} \left(\frac{2e\phi}{m_e c^2}\right)^{-1/4} d\,\frac{e\phi}{m_e c^2}, \quad \gamma \approx 1}$$

<u>Magnetically insulated Brillouin flow (planar, nonrelativistic)</u>





Brillouin flow condition

$$\frac{1}{2} m_e V_{ye}^{02}(x) - e\phi_0(x) = \text{ const.}$$

$$0 = -n_e^0(x)\, e \left[E_x(x) + \frac{1}{c} V_{ye}^0(x)\, B_0 \right]$$

$$\frac{\partial}{\partial x} E_x(x) = -4\pi e n_e^0(x)$$

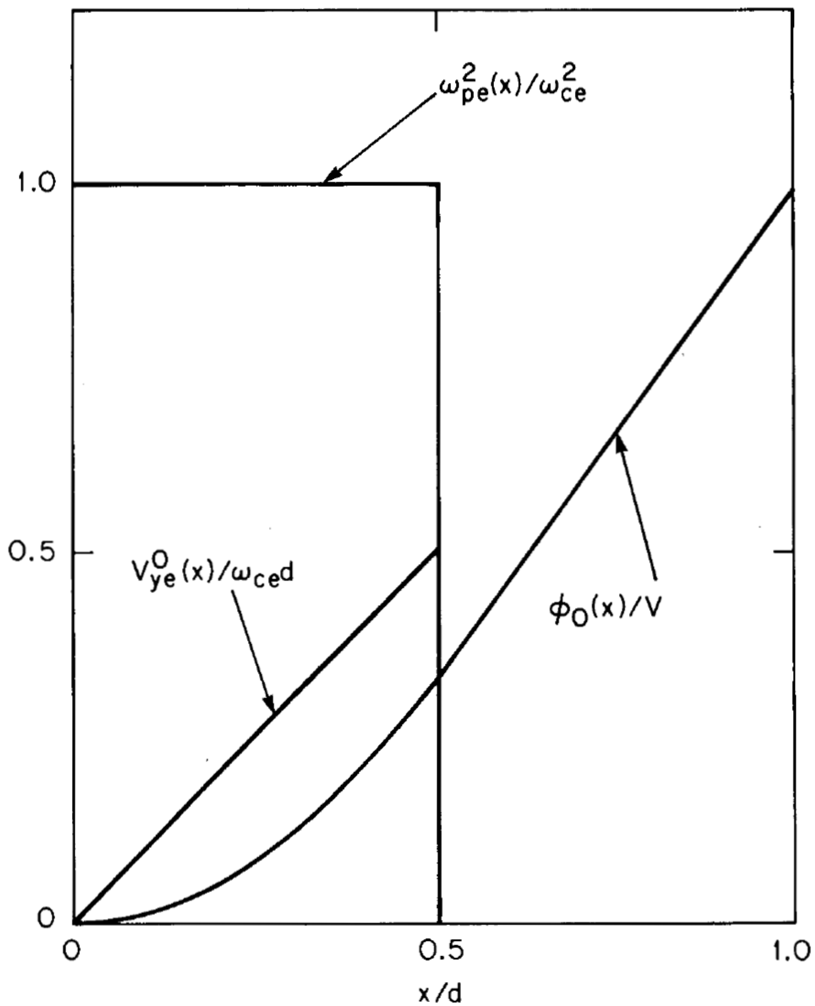
$$m_e V_{ye}^0(x) \frac{\partial}{\partial x} V_{ye}^0(x) + e E_x(x) = 0$$

$$\frac{\partial}{\partial x} V_{ye}^0(x) = \omega_{ce}$$

$$V_{ye}^0(x) = \omega_{ce} x$$

$$\omega_{pe}^2(x) = \omega_{ce}^2$$

$$\frac{e\phi_0(x)}{m_e c^2} = \begin{cases} \dfrac{\omega_{ce}^2}{c^2} \dfrac{x^2}{2} \,, & 0 \leq x < x_b \,, \\[2ex] \dfrac{\omega_{ce}^2}{c^2} \dfrac{x_b^2}{2} + \dfrac{\omega_{ce}^2}{c^2} x_b (x - x_b) \,, & x_b < x \leq d \,, \end{cases}$$



$$\frac{eV}{m_e c^2} = \frac{\omega_{ce}^2 d^2}{c^2}\left[\frac{x_b}{d} - \frac{1}{2}\frac{x_b^2}{d^2}\right]$$

$$\frac{x_b}{d} = 1 - \left(1 - \frac{2eV/m_e c^2}{\omega_{ce}^2 d^2/c^2}\right)^{1/2}$$

$$\boxed{x_b < d}$$

$$\updownarrow$$

$$\boxed{\frac{eV}{m_e c^2} < \frac{eV_H}{m_e c^2} \equiv \frac{1}{2}\frac{\omega_{ce}^2 d^2}{c^2}}$$

Hall cut-off potential

$$eV_H = \frac{1}{2}\omega_{ce}^2 d^2 m_e$$

$\boxed{\text{For a given } V \text{ and } B\text{, how to maximize } J_e\text{?}}$

Magnetically insulated Brillouin flow (planar, relativistic)

$$\frac{\partial}{\partial x} B_z(x) = \frac{1}{c} 4\pi e n_e^0(x) V_{ye}^0(x) = -\frac{4\pi e n_e^0(x) E_x(x)}{B_z(x)}$$

$$\frac{\partial}{\partial x} E_x(x) = -4\pi e n_e^0(x)$$

$$\left[\gamma_e^0(x) - 1\right] m_e c^2 - e\phi_0(x) = \text{const.}$$

$$V_{ye}^0(x) = -cE_x(x)/B_z(x)$$

$$\boxed{B_z^2(x) - E_x^2(x) = \text{const.}}$$

$$\gamma_e^0(x) = [1 - V_{ye}^{02}(x)/c^2]^{-1/2}$$

$$\boxed{\frac{B_z(x)}{\gamma_e^0(x)} = \text{const.}}$$

$$+$$

$$\gamma_e^{03}(x) \frac{E_x(x)}{B_z(x)} \frac{\partial}{\partial x} \left[\frac{E_x(x)}{B_z(x)}\right] = -\frac{eE_x(x)}{m_e c^2}$$

$$\left[\frac{eB_z(x)}{\gamma_e^0(x) m_e c}\right]^2 = \frac{4\pi n_e^0(x) e^2}{\gamma_e^0(x) m_e}$$

$$\boxed{n_e^0(x)/\gamma_e^0(x) = \text{const}}$$

$$\boxed{\frac{\partial^2}{\partial x^2} B_z(x) - \kappa^2 B_z(x) = 0}$$

$$\frac{\partial^2}{\partial x^2} B_z(x) - \kappa^2 B_z(x) = 0$$

$$\kappa \equiv \frac{4\pi e n_e^0(x)}{B_x(x)} = \text{const.}$$

$$B_z(x) = \begin{cases} B_0 \dfrac{\cosh(\kappa x)}{\cosh(\kappa x_b)}, & 0 \le x < x_b , \\ B_0 , & x_b < x \le d , \end{cases}$$

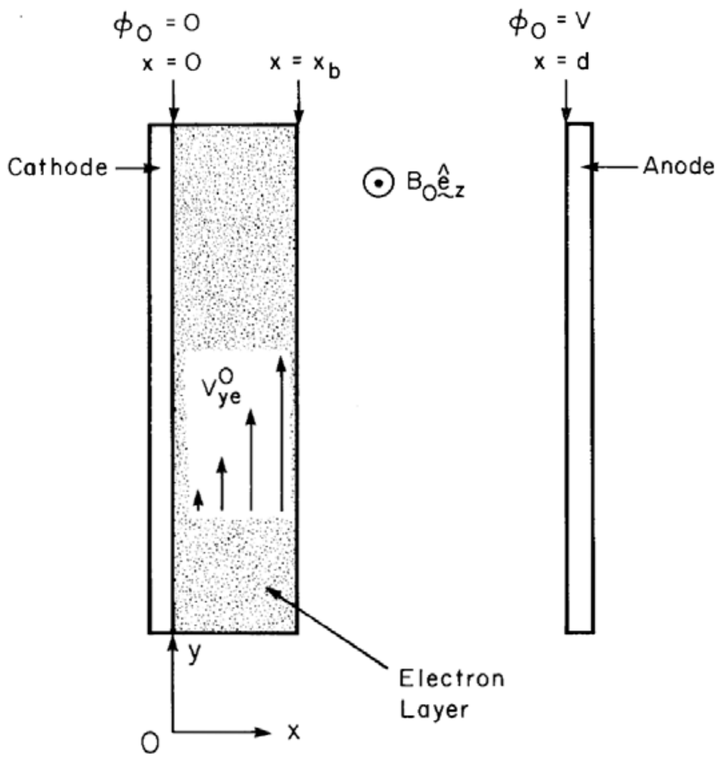
Before and after electron filling:

$$\int_0^d B_z(x) dx = const.$$

$$B_f d = \frac{B_0}{\kappa} \tanh(\kappa x_b) + B_0(d - x_b)$$

$$\frac{eV_H}{m_e c^2} = \left[1 + \frac{e^2 B_f^2 d^2}{m_e^2 c^4}\right]^{1/2} - 1$$

Relativistic Hall cut-off potential



Magnetron instability

Relativistic magnetron diodes in MV generates GW EM radiations.

$$\boxed{\begin{aligned}\delta\mathbf{E}(\mathbf{x},t) &= \delta E_x(x,y,t)\hat{\mathbf{e}}_x + \delta E_y(x,y,t)\hat{\mathbf{e}}_y \ , \\ \delta\mathbf{B}(\mathbf{x},t) &= \delta B_z(x,y,t)\hat{\mathbf{e}}_z \ .\end{aligned}}$$

$$\boxed{\delta\psi(x,y,t) = \sum_{k=-\infty}^{\infty} \delta\psi(x,k)\exp(iky - i\omega t)}$$

Magnetron instability (linearized fluid and Maxwell's equations)

$$\left(\frac{\partial}{\partial t}+V_{ye}^{0}(x)\frac{\partial}{\partial y}\right)\delta n_e(\mathbf{x},t)+\frac{\partial}{\partial \mathbf{x}}\cdot[n_e^0(x)\delta\mathbf{V}_e(\mathbf{x},t)]=0\,,$$

$$\left(\frac{\partial}{\partial t}+V_{ye}^{0}(x)\frac{\partial}{\partial y}\right)[\gamma_e^0(x)m_e\delta\mathbf{V}_e(\mathbf{x},t)+\delta\gamma_e(\mathbf{x},t)m_eV_{ye}^0(x)\hat{\mathbf{e}}_y]$$
$$+\,\delta V_{xe}(\mathbf{x},t)\frac{\partial}{\partial x}[\gamma_e^0(x)m_eV_{ye}^0(x)\hat{\mathbf{e}}_y]$$
$$=-e\left(\delta\mathbf{E}(\mathbf{x},t)+\frac{V_{ye}^0(x)\hat{\mathbf{e}}_y\times\delta\mathbf{B}(\mathbf{x},t)}{c}+\frac{\delta\mathbf{V}_e(\mathbf{x},t)\times B_z(x)\hat{\mathbf{e}}_z}{c}\right)$$

$$\delta E_x(x,k)=-\frac{1}{(1-\omega^2/c^2k^2)}\left[\frac{\partial}{\partial x}\Phi_k(x)+\frac{i\omega}{c^2k^2}4\pi e n_e^0(x)\delta V_{xe}(x,k)\right]\,, \tag{B-9}$$

$$\delta B_z(x,k)=\frac{1}{(1-\omega^2/c^2k^2)}\left[\frac{\omega}{ck}\frac{\partial}{\partial x}\Phi_k(x)+\frac{i}{ck}4\pi e n_e^0(x)\delta V_{xe}(x,k)\right]. \tag{B-10}$$

$$\frac{\partial}{\partial x}\delta E_x(x,k)-k^2\Phi_k(x)=-4\pi e\delta n_e(x,k)\,.$$

Magnetron instability (eigenmode eq.)

$$\Phi_k(x) = \frac{i}{k}\delta E_y(x,k)$$

$$\Phi_k(x=0) = 0 = \Phi_k(x=d)$$

$$\frac{\partial}{\partial x}\left\{[1+\chi_\perp(x,k,\omega)]\frac{\partial}{\partial x}\Phi_k(x)\right\} - k^2\left[1+\chi_\parallel(x,k,\omega)\right]\Phi_k(x)$$
$$= \frac{k\,\Phi_k(x)}{\left[\omega - kV_{ye}^0(x)\right]}\left[1-\frac{V_{ye}^0(x)}{c}\frac{\omega}{ck}\right]\frac{\partial}{\partial x}\left[\frac{\omega_{pe}^2(x)\omega_{ce}(x)}{\gamma_e^{02}(x)\nu_e^2(x,k,\omega)}\right]$$

$$\chi_\perp(x,k,\omega) = -\left[1-\frac{V_{ye}^0(x)}{c}\frac{\omega}{ck}\right]^2\frac{\omega_{pe}^2(x)\gamma_e^0(x)}{(1-\omega^2/c^2k^2)\,\nu_e^2(x,k,\omega)}\ ,$$

$$\chi_\parallel(x,k,\omega) = -\frac{\omega^2}{c^2k^2} - \frac{\omega_{pe}^2(x)}{\gamma_e^0(x)\nu_e^2(x,k,\omega)}\left[1-\frac{\omega^2}{c^2k^2}+\frac{\omega_{pe}^2(x)}{\gamma_e^0(x)c^2k^2}\right]$$

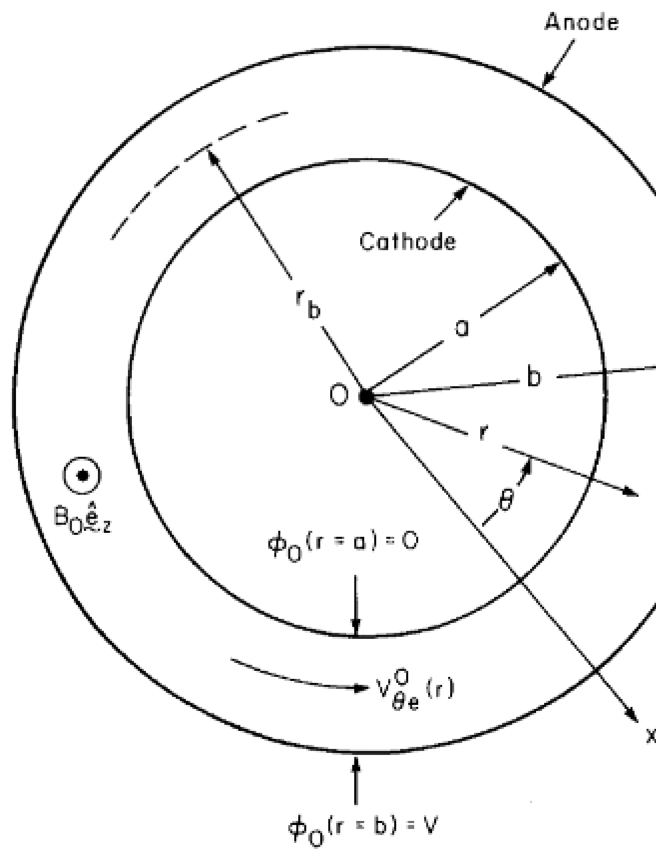
$$\nu_e^2(x,k,\omega) = \gamma_e^{02}(x)\left[\omega - kV_{ye}^0(x)\right]^2\left[1+\frac{\omega_{pe}^2(x)/\gamma_e^0(x)c^2k^2}{1-\omega^2/c^2k^2}\right]$$
$$-\left[\omega_{ce}^2(x)/\gamma_e^{02}(x) - \omega_{pe}^2(x)/\gamma_e^0(x)\right].$$

Magnetron instability happens when $1 + \mathrm{Re}\,\chi_\perp(x_i, k, \omega) \simeq 0$

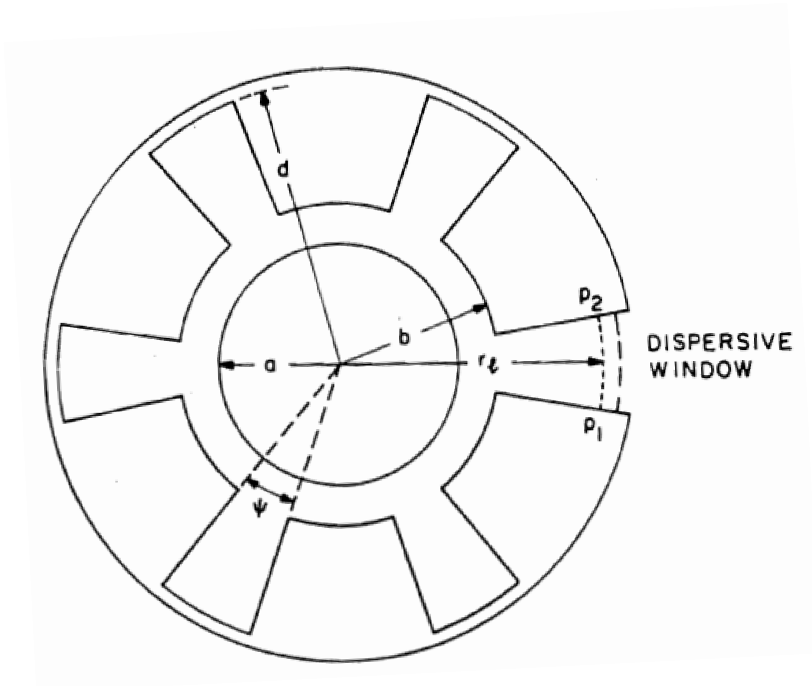
$$\left[\mathrm{Re}\,\omega - kV^0_{ye}(x_i)\right]^2 = \omega^2_{ce}(x_i)/\gamma^{04}_e(x_i) \quad \text{for some } x_i < x_d$$

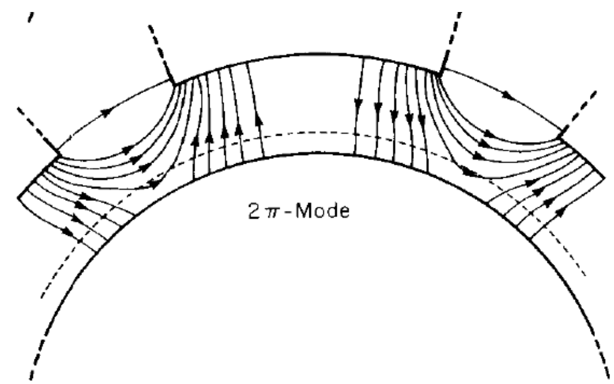
Interesting thing happens at singularity!

Magnetically insulated Brillouin flow (cylindrical)



A6 relativistic magnetron [Palevsky & Bekefi, Phys. Fluids 22, 986 (1979)]

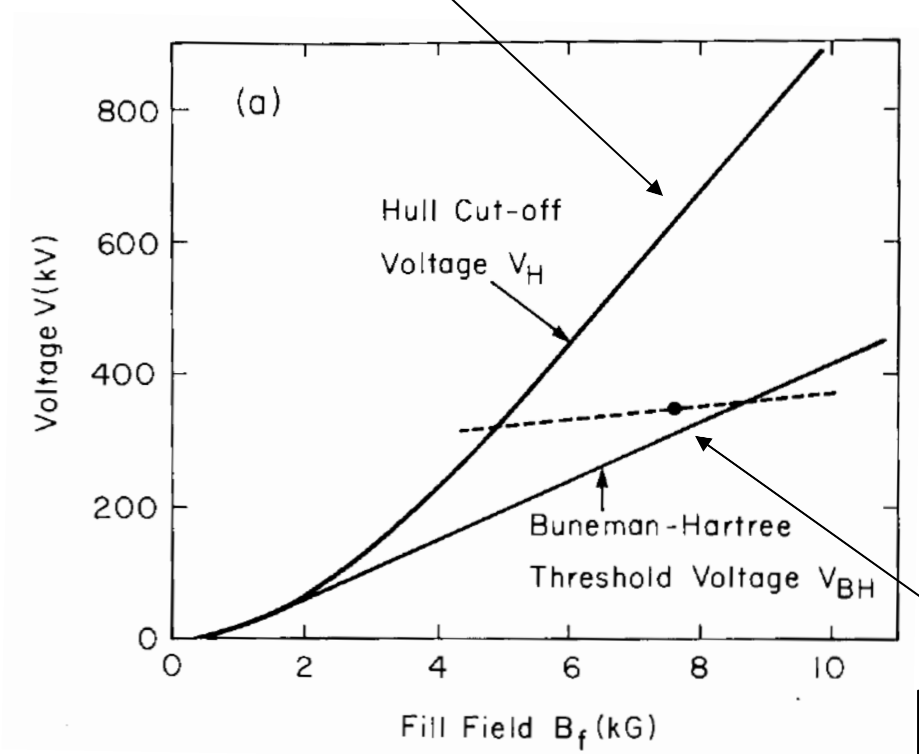




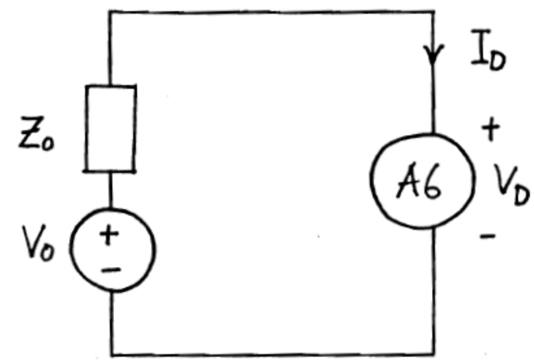
$a = 1.58$cm, $b = 2.11$cm, $d = 4.11$cm, $B_f = 7.6$KG
P=0.45GW per open port

<u>A6 relativistic magnetron</u> [Palevsky & Bekefi, Phys. Fluids 22, 986 (1979)]

$$x_b < d \Leftrightarrow V < V_H$$



Circuit



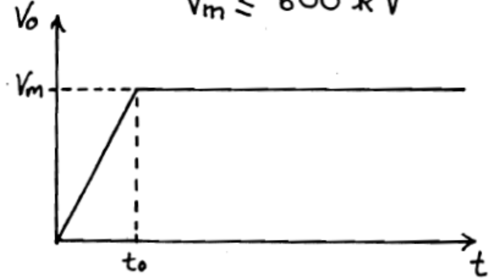
$$v(x_b) > \frac{\omega}{k} \Leftrightarrow V > V_{BH}$$

Particle-in-cell simulation [Chan, Chen, and Davidson Appl. Phy. Lett. 57, 1271(1990)]

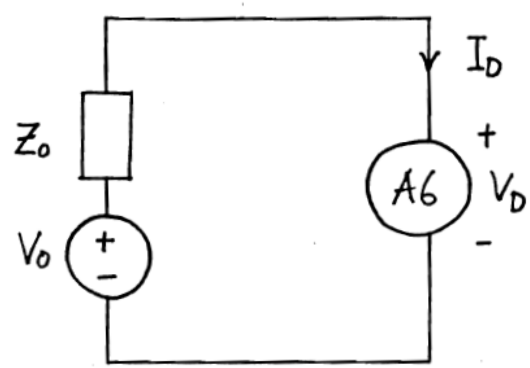
Voltage Pulse

$t_0 = 4\,\mathrm{ns}$

$V_m \leq 600\,\mathrm{kV}$

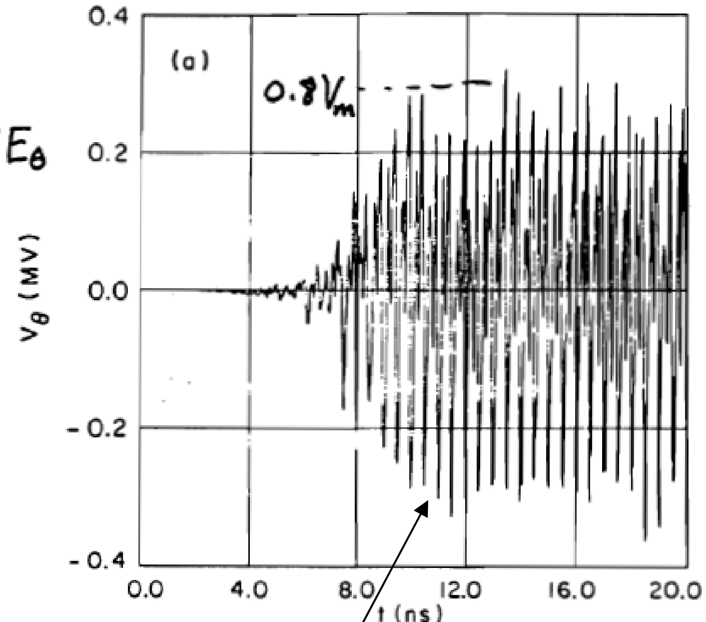


Circuit



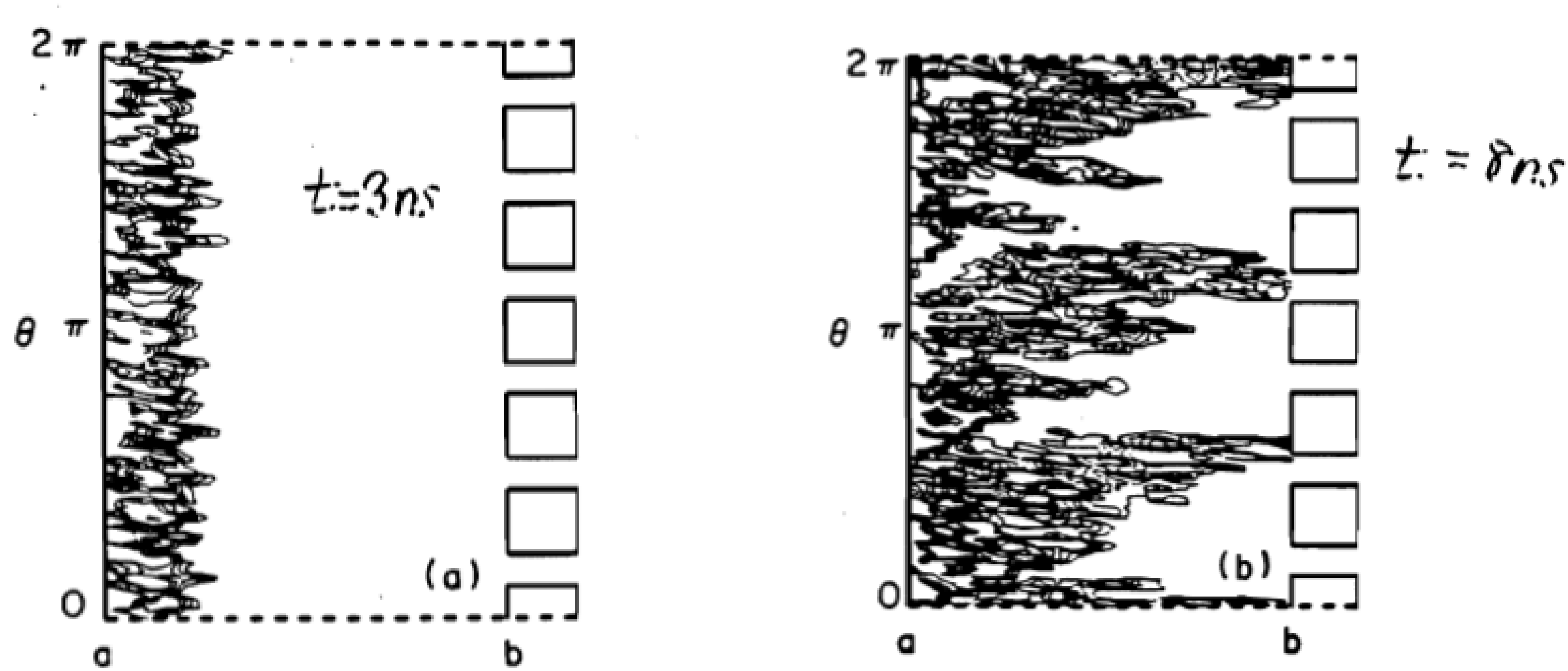
$$V_\theta(t) = \int_{\varphi_1}^{\varphi_2} d\theta\, r_\ell\, \delta E_\theta$$

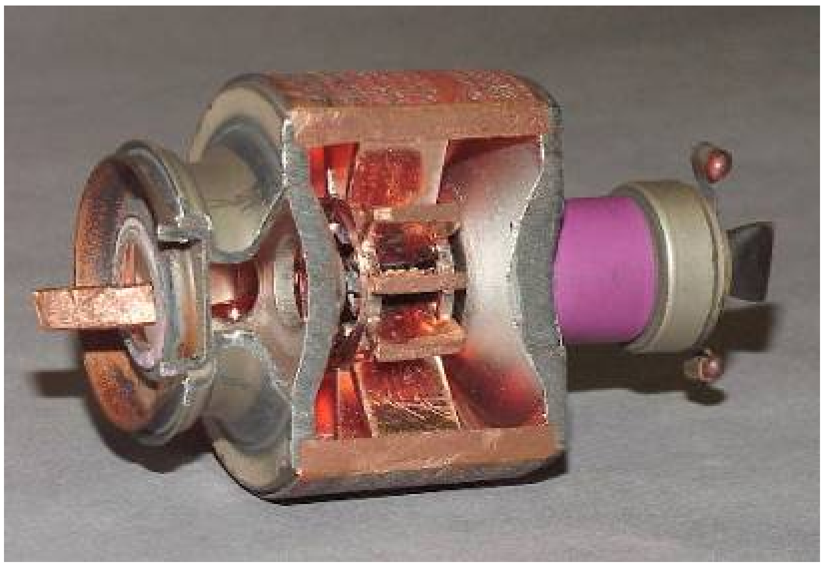
$r_\ell = 3.7\,\mathrm{cm}$

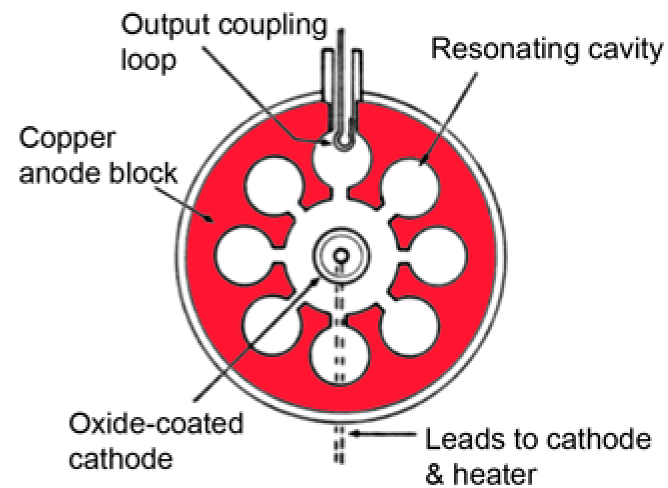


Magnetron instability

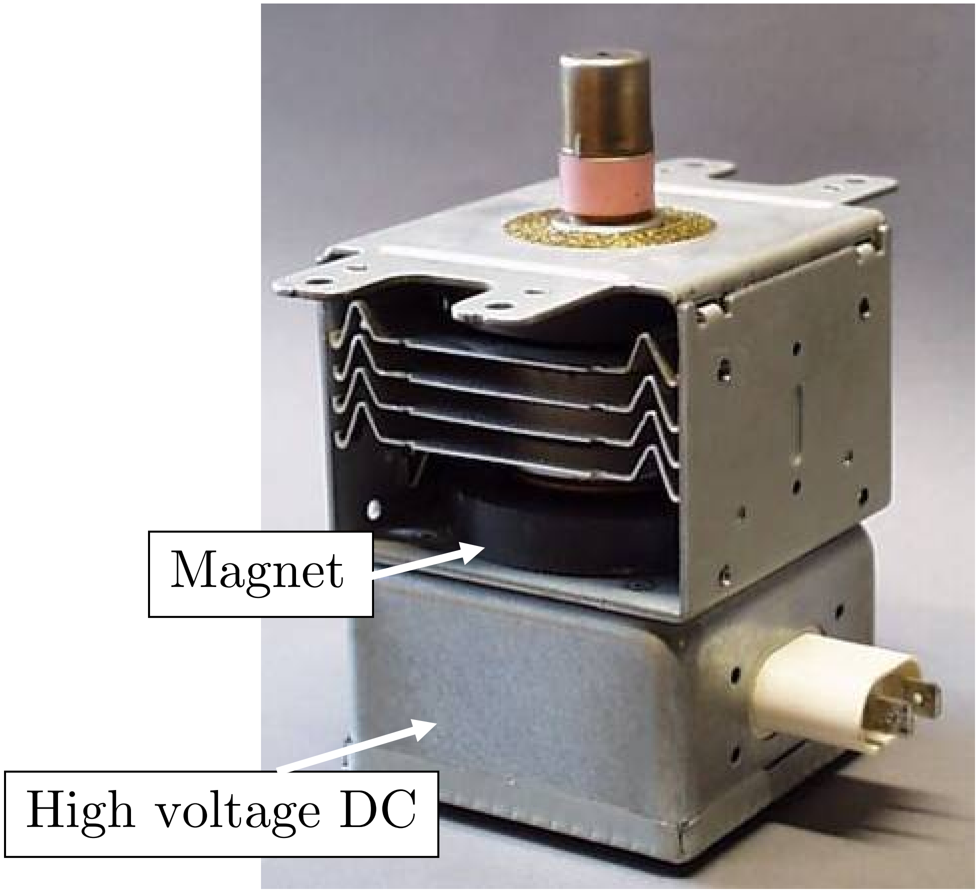
<u>Particle-in-cell simulation</u> [Chan, Chen, and Davidson Appl. Phy. Lett. 57, 1271(1990)]





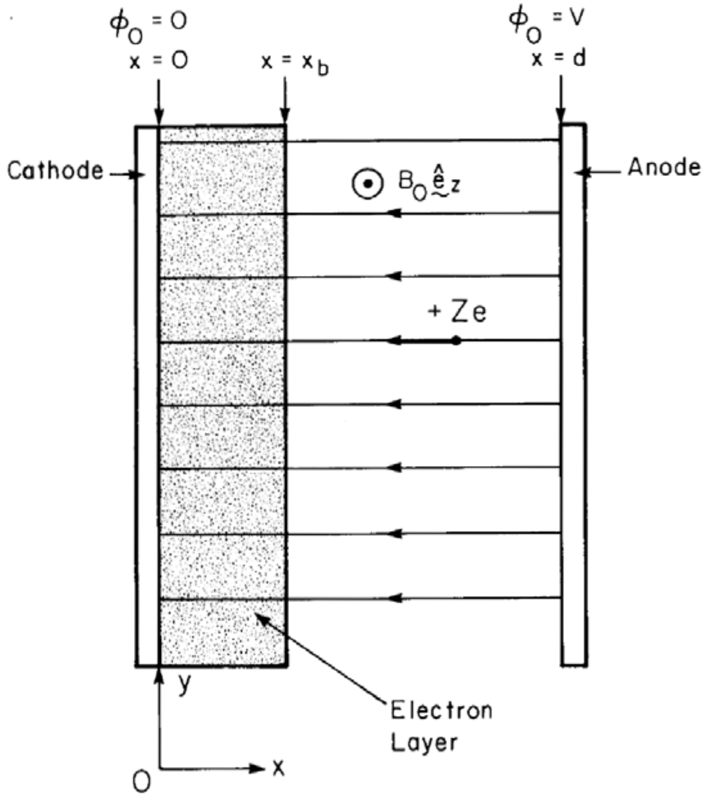


Resonant cavity magnetron high-power high-frequency oscillator

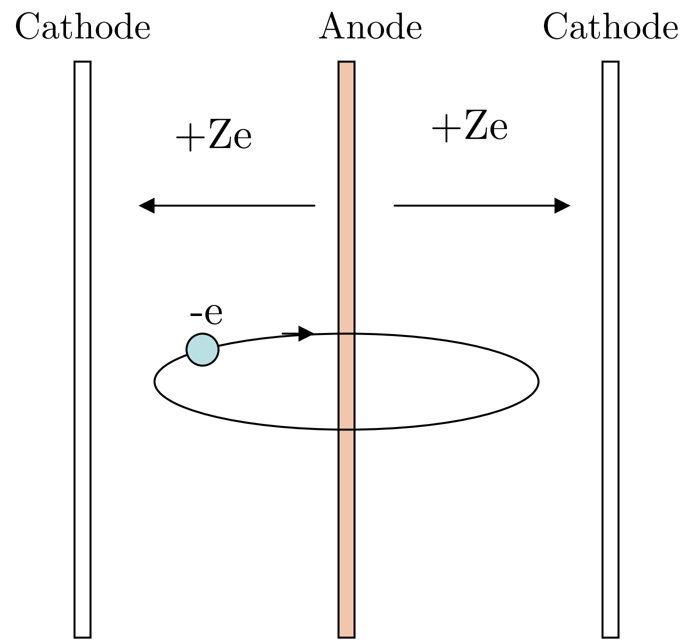


1KW, 8$

Ion diodes



Magnetic insulation



Triode

Magnetically insulated ion diode (planar, relativistic)

$$\frac{1}{2} m_e V_{ye}^{02}(x) - e\phi_0(x) = \text{const.}$$

$$0 = -n_e^0(x)\, e \left[E_x(x) + \frac{1}{c} V_{ye}^0(x)\, B_0 \right]$$

$$m_e V_{ye}^0(x) \frac{\partial}{\partial x} V_{ye}^0(x) + e E_x(x) = 0$$

$$\frac{\partial}{\partial x} V_{ye}^0(x) = \omega_{ce} \longrightarrow V_{ye}^0(x) = \omega_{ce} x$$

$$Z_i\, e n_i^0(x) V_{xi}^0(x) = J_i = \text{const.}$$

$$\frac{m_i}{2} V_{xi}^{02}(x) + Z_i e\phi_0(x) = Z_i eV$$

$$Z_i e n_i^0(x) = -\frac{(m_i/2Z_i eV)^{1/2} J_i}{[1 - \phi_0(x)/V]^{1/2}}$$

$$\frac{\partial^2}{\partial x^2}\phi_0(x) = \begin{cases} 4\pi e \left[n_e^0(x) - Z_i n_i^0(x) \right], & 0 \le x < x_b\,, \\ -4\pi e Z_i n_i^0(x)\,, & x_b < x \le d\,, \end{cases}$$

$0 < x < x_b$

$$\omega_{ce}^2 = \omega_{pe}^2(x) - \frac{4\pi n_i^0(x) Z_i e^2}{m_e}$$

$$\phi_0(x) = \frac{1}{2}\frac{B_0}{c}\omega_{ce} x^2$$

$x_b < x < d$

$$\frac{\partial^2}{\partial x^2}\phi_0(x) = -\frac{K^2 V}{[1 - \phi_0(x)/V]^{1/2}}$$

$$K^2 = -\frac{4\pi}{V}\left(\frac{m_i}{2Z_i eV}\right)^{1/2} J_i$$

$$\phi_0(x) = V\left\{1 - \left[\frac{3}{2}K(d-x)\right]^{4/3}\right\}$$

$$1 - \frac{V_H}{V}\frac{x_b^2}{d^2} = \left[\frac{3}{2}Kd\left(1 - \frac{x_b}{d}\right)\right]^{4/3}$$

$$V_H \equiv \frac{m_e}{2e}\omega_{ce}^2 d^2$$

$$\boxed{\frac{x_b}{d} = \frac{3}{2}\left\{1 - \left(1 - \frac{8}{9}\frac{V}{V_H}\right)^{1/2}\right\}}$$

$$J_i = J_{\rm CL}\left(\frac{3}{2}\frac{V_H}{V}\frac{x_b}{d}\right)^{3/2}\left(1-\frac{x_b}{d}\right)^{-1/2}$$

$$J_{\rm CL} = -\frac{2^{1/2}V}{9\pi d^2}\left(\frac{Z_i e V}{m_i}\right)^{1/2}$$