AST-565 Homework Set 11

Problem 1: Numerical solutions of the surface wave dispersion relation

The dispersion relation of the $k_z = 0$ surface waves for a KV beam is given by [Davidson & Qin, Eq. (8.57)]

$$0 = D_\ell(\omega) = 1 + \frac{\hat{\omega}_{pb}^2}{2\ell\gamma_b^2\nu^2}[1 - (\frac{r_b}{r_w})^{2\ell}]\Gamma_b^\ell(\omega)$$

$$= 1 - \frac{\hat{\omega}_{pb}^2}{2^{\ell+1}\ell\gamma_b^2\nu^2}[1 - (\frac{r_b}{r_w})^{2\ell}]\sum_{m=0}^{\ell}\frac{\ell!}{m!(\ell-m)!}\frac{(\ell-2m)\nu}{[\omega-(\ell-2m)\nu]},$$

where $\nu / \nu_0 = (1 - \hat{\omega}_{pb}^2 / 2\gamma_b^2\omega_{\beta\perp}^2)^{1/2}$, and $\nu_0 = \omega_{\beta\perp}$. Let $\hat{\omega}_{pb}^2 / 2\gamma_b^2\omega_{\beta\perp}^2 = 0.36$ and $r_b / r_w = 0.5$. Numerically calculate the 5 eigen-frequencies ω / ν_0 for the quadrupole mode ($\ell = 4$).

Problem 2: δf method

In the δf method, the distribution function f and the fields are split into two parts

$$
\begin{aligned}
&f = f_0 + \delta f = f_0 + wF, \\
&\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}, \\
&\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B},
\end{aligned}
$$

where $(f_0, \mathbf{E}_0, \mathbf{B}_0)$ is a known solution of the Vlasov-Maxwell equations, and $(\delta f, \delta\mathbf{E}, \delta\mathbf{B})$ are the perturbed distribution function and fields relative to $(f_0, \mathbf{E}_0, \mathbf{B}_0)$. The known solution $(f_0, \mathbf{E}_0, \mathbf{B}_0)$ is allowed to depend on time. The perturbed distribution function $\delta f \equiv wF$ is constructed from the distribution function of simulation particles F and the weight function w in phase space. Because the simulation particles follow the same trajectories as the physical particles, F satisfies the Vlasov equation as well. But F needs not to be the same as f, and does not necessarily satisfy the Maxwell equations. Shows that the dynamics of w is determined from

$$
\begin{aligned}
\frac{dw}{dt} &= \frac{1}{F}\frac{df_0}{dt} = \frac{q}{m}\left(\delta\mathbf{E} + \frac{\mathbf{v}\times\delta\mathbf{B}}{c}\right)\cdot\frac{1}{F}\frac{\partial f_0}{\partial\mathbf{v}} \\
&= \frac{w-g}{f_0}\frac{df_0}{dt} = \frac{q}{m}\left(\delta\mathbf{E} + \frac{\mathbf{v}\times\delta\mathbf{B}}{c}\right)\cdot\frac{(w-g)}{f_0}\frac{\partial f_0}{\partial\mathbf{v}},
\end{aligned}
$$

where $g \equiv f / F$ is a constant of the motion for each simulation particle, *i.e.*, $dg / dt = 0$, because $df / dt = 0$ and $dF / dt = 0$.