Weight growth due to resonant particles and a modified δf method with smooth switching between δf and total-f methods

Hong Qin

Plasma Physics Laboratory, Princeton University

US Heavy Ion Fusion Science Virtual National Laboratory

www.princeton.edu/~hongqin/

Present at

Courant Institute, New York University

Feb. 5, 2008

Vlasov-Maxwell system and particle simulation

$$\frac{df}{dt} = \left\{ \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} + \left[\boldsymbol{F}_{ext} + q \left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} \right) \cdot \frac{\partial}{\partial \boldsymbol{p}} \right] \right\} f(\boldsymbol{x}, \boldsymbol{p}, t) = 0$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \int d^3p \;\; q\boldsymbol{v} f(\boldsymbol{x}, \boldsymbol{p}, t) + \frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial B}{\partial t},$$

$$\nabla \cdot \boldsymbol{E} = 4\pi \int d^3p \;\; q f(\boldsymbol{x}, \boldsymbol{p}, t)$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Nonlinear, differential-integral

- Particle-in-cell simulation -- effective solution method.
- Difficulty -- simulation noise.

$$\boxed{\frac{\tilde{f}}{f} \sim \frac{1}{\sqrt{N}}}$$

δf particle simulation method reduces noise

$$\begin{pmatrix} f \\ \boldsymbol{E} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} f_0 \\ \boldsymbol{E}_0 \\ \boldsymbol{B}_0 \end{pmatrix} + \begin{pmatrix} \delta f \\ \delta \boldsymbol{E} \\ \delta \boldsymbol{B} \end{pmatrix}$$

A know solution;
Control variates.

$$\frac{dw}{dt} = \frac{1}{F}\frac{df_0}{dt} = \frac{q}{m}\left(\delta\boldsymbol{E} + \frac{\boldsymbol{v}\times\delta\boldsymbol{B}}{c}\right)\cdot\frac{1}{F}\frac{\partial f_0}{\partial \boldsymbol{v}}$$
$$= \frac{w-g}{f_0}\frac{df_0}{dt} = \frac{q}{m}\left(\delta\boldsymbol{E} + \frac{\boldsymbol{v}\times\delta\boldsymbol{B}}{c}\right)\cdot\frac{(w-g)}{f_0}\frac{\partial f_0}{\partial \boldsymbol{v}}$$

$$\delta f = wF$$

simulation particle distributuion

$F = gf, \;\; g = const;$

importance sampling

$$\nabla\times\delta\boldsymbol{B} = \frac{4\pi}{c}\int d^3p \;\; q\boldsymbol{v}\delta f(\boldsymbol{x},\boldsymbol{p},t) + \frac{1}{c}\frac{\partial\delta\boldsymbol{E}}{\partial t}$$
$$\nabla\times\delta\boldsymbol{E} = -\frac{1}{c}\frac{\partial\delta\boldsymbol{B}}{\partial t}$$
$$\nabla\cdot\delta\boldsymbol{E} = 4\pi\int d^3p \;\; q\delta f(\boldsymbol{x},\boldsymbol{p},t)$$
$$\nabla\cdot\delta\boldsymbol{B} = 0$$

$$\frac{\tilde{f}}{f} \sim \frac{\widetilde{\delta f}}{f} \sim \frac{w}{\sqrt{N}} \ll \frac{1}{\sqrt{N}}, \text{ if } w \ll 1.$$

Weight growth due to resonant simulation particles

$$\frac{\tilde{f}}{f} \sim \frac{\widetilde{\delta f}}{f} \sim \frac{w}{\sqrt{N}} \gg \frac{1}{\sqrt{N}}, \text{ if } w \gg 1.$$

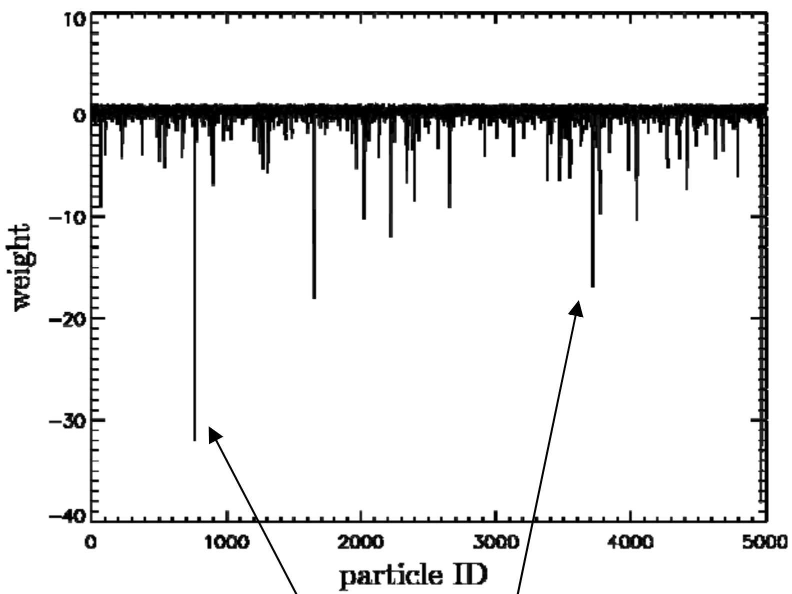
Can happen for many reasons.

$$f_0 = \frac{n_0(\boldsymbol{x})}{(2\pi T / m)^{3/2}} \exp\left(-\frac{v^2}{2T / m}\right)$$

$$\frac{d}{dt}\ln(1-w) = \frac{q}{T}\delta\boldsymbol{E}\cdot\boldsymbol{v}$$

$$w(t) = 1 - [1 - w(t=0)]\exp[h(t)]$$

$$h(t) = -\int_0^t \frac{q\delta\boldsymbol{E}\cdot\boldsymbol{v}}{T} dt$$



$q\delta\boldsymbol{E}\cdot\boldsymbol{v} < 0$ for all $t \Rightarrow w \rightarrow -\infty$

Modified δf method with smooth switching between δf and total-f methods

$$f = \alpha f_0 + wF$$

$$\delta f = (\alpha - 1) f_0 + wF$$

$0 \le \alpha \le 1$

$\alpha = 0$: total-f

$\alpha = 1$: δf

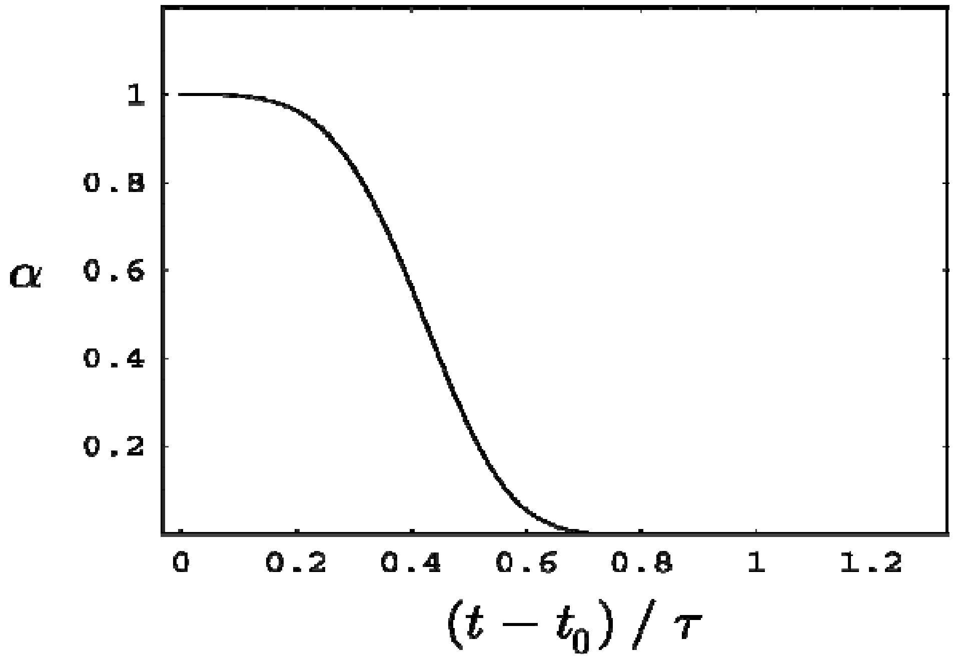
select the switching function $0 \le \alpha(t) \le 1$

$$\frac{dw}{dt} = \frac{w-g}{f_0}\frac{df_0}{dt} + \frac{w-g}{\alpha}\frac{d\alpha}{dt}$$
$$= \frac{q}{m}\left(\delta \boldsymbol{E} + \frac{\boldsymbol{v} \times \delta \boldsymbol{B}}{c}\right) \cdot \frac{(w-g)}{f_0}\frac{\partial f_0}{\partial \boldsymbol{v}} + \frac{w-g}{\alpha}\frac{d\alpha}{dt}$$

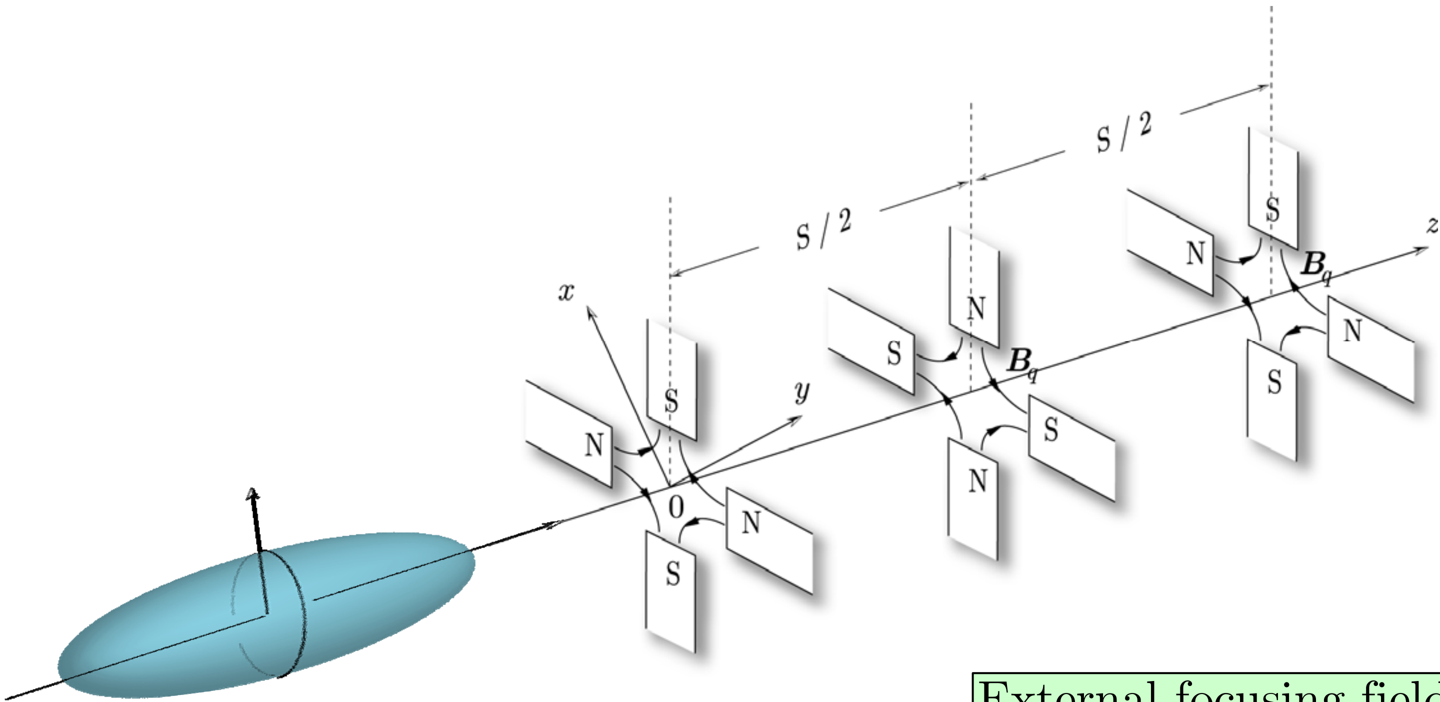
α can depend on $(\boldsymbol{x}, \boldsymbol{v})$ too.

Modified δf method with smooth switching between δf and total-f methods

$$\frac{d\ln\alpha}{dt} = \begin{cases} 0, & t - t_0 < 0, \\ -a\left(\dfrac{t - t_0}{\tau}\right)^n, & 0 \le t - t_0 < \tau, \\ -a, & \tau < t - t_0, \end{cases}$$



Vlasov-Maxwell system for high intensity particle beams

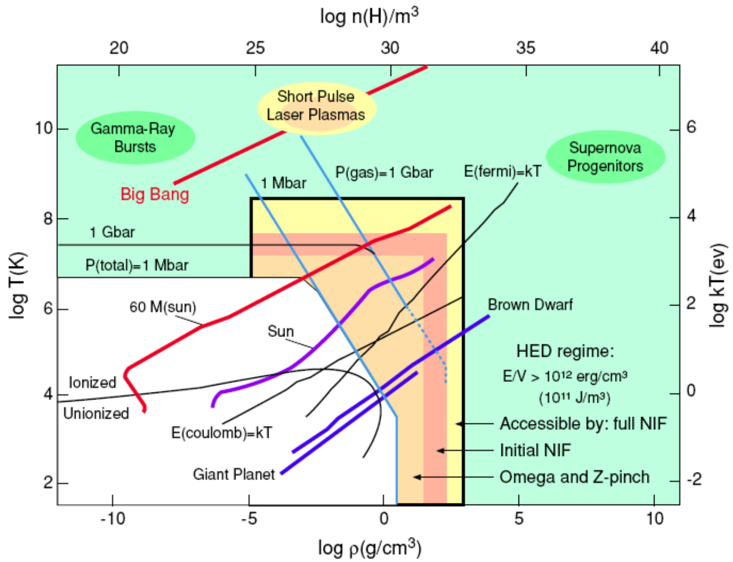


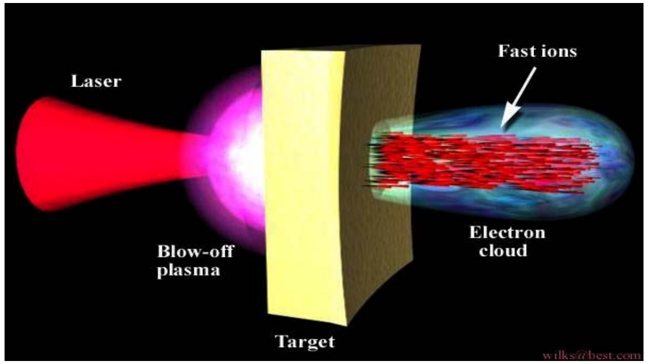
External focusing field:

$$\mathbf{F}_{foc} = -m_b \omega_{\beta b}^2 \mathbf{x}_\perp$$

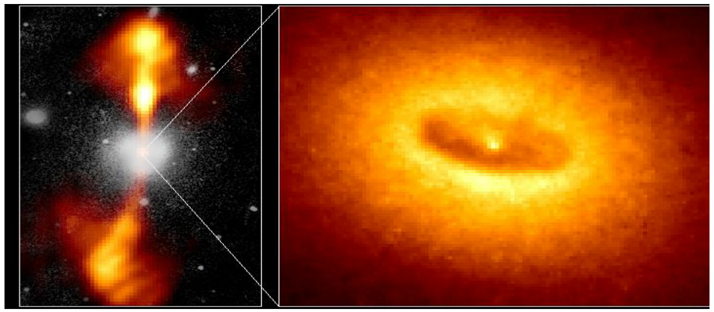
An Introduction to the Physics of Intense Charged Particle Beams in High Energy Accelerators, R. C. Davidson and H. Qin, World Scientific (2001)

Beam physics application --- high energy density physics

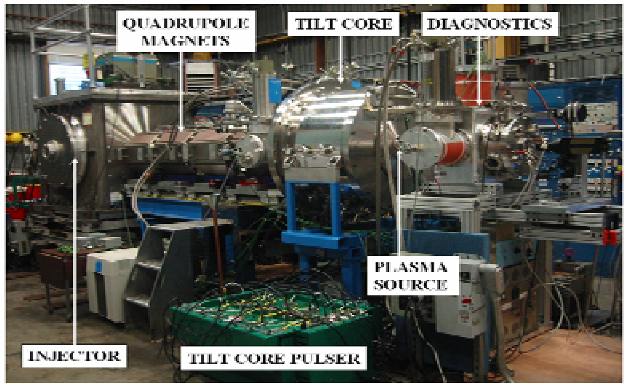




Gamma ray bursters experiment

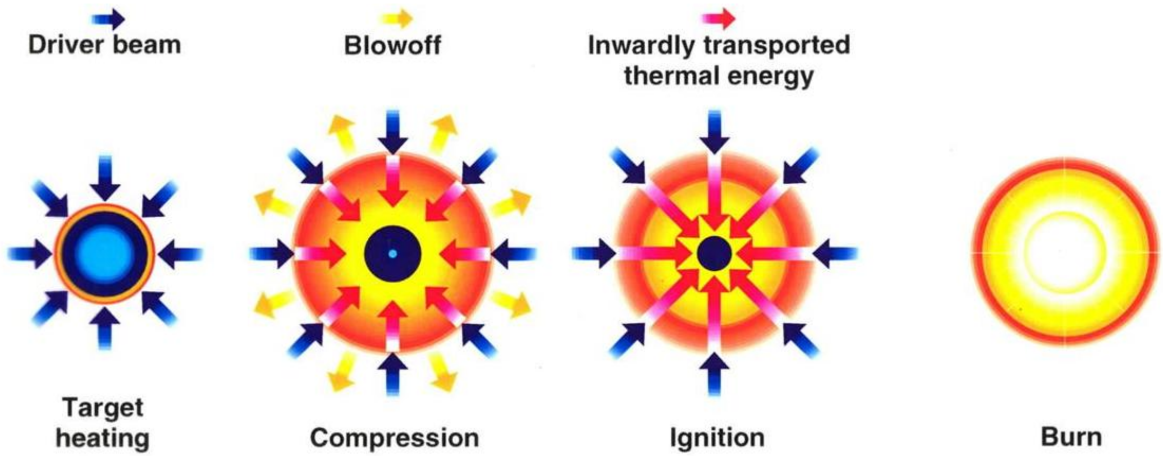


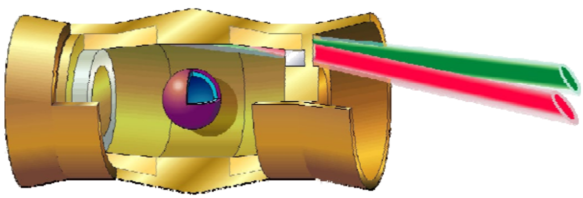
Photoionized plasmas in an accreting massive black hole

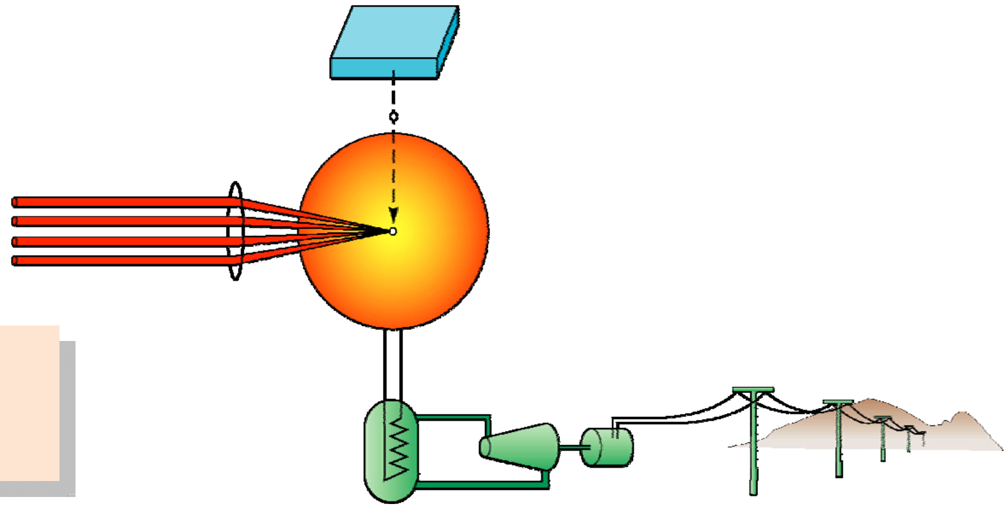


Neutralized drift compression experiment for ion beam driven HEDP

Beam physics application --- heavy ion fusion



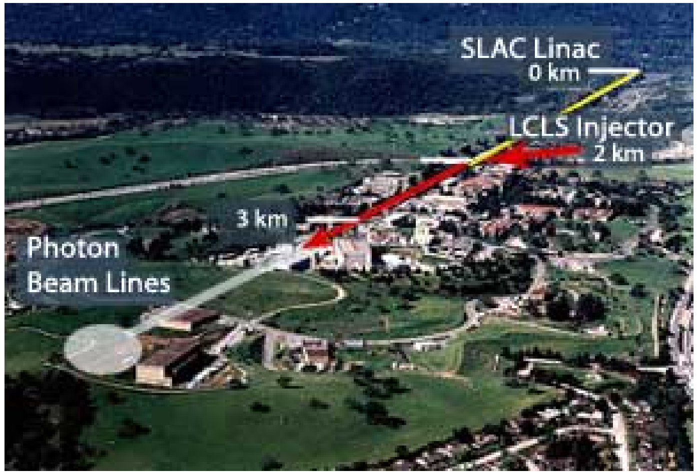


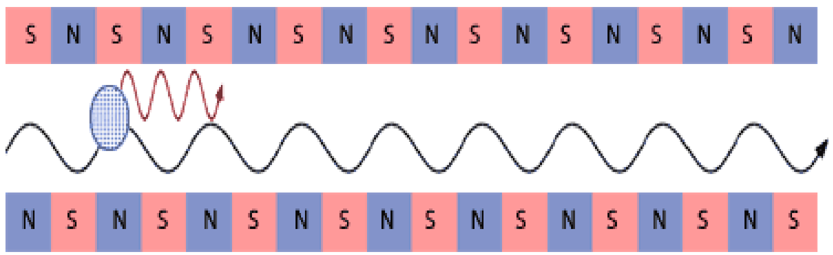


Beams strike “hohlraum,” producing x-ray bath for fusion capsules.

Beam physics application --- modern high intensity accelerators

Spallation Neutron Source





Linear Coherent Light Source

Collective instabilities in intense charged particle beams

One-Component Beams

- Harris and Weibel instability driven by temperature anisotropy

$$T_\perp \gg T_\parallel \ .$$

- Resistive wall instability

Propagation Through Background species

- Two-stream instability
- Ion-electron (Electron cloud) instability

Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

Beam Equilibrium Stability and Transport (BEST) Code

Physics

- Perturbative particle simulation method to reduce noise.
- Linear eigenmodes and nonlinear evolution.
- 2D and 3D equilibrium structure.
- Multi-species; electrons and ions; accommodate very large mass ratio.
- Multi-time-scales, frequency span a factor of 10^5.
- 3D nonlinear perturbation.

Computation

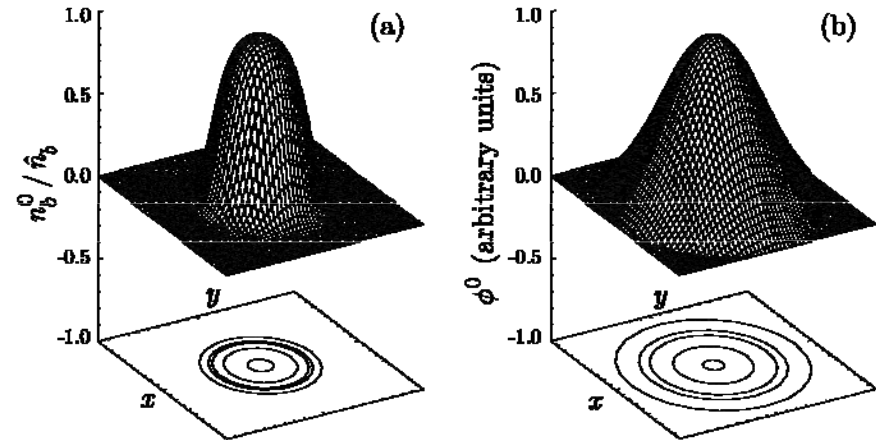
- Message Passing Interface
  - Multiple-1D domain decompositon (OpenMP by users).
- Large-scale computing: particle x time-steps ~ 0.5×10^{12}.
- Scales linearly to 512 processors on IBM-SP3 at NERSC.
- NetCDF, HDF5 parallel I/O diagnostics.

Energy anisotropic coasting beams

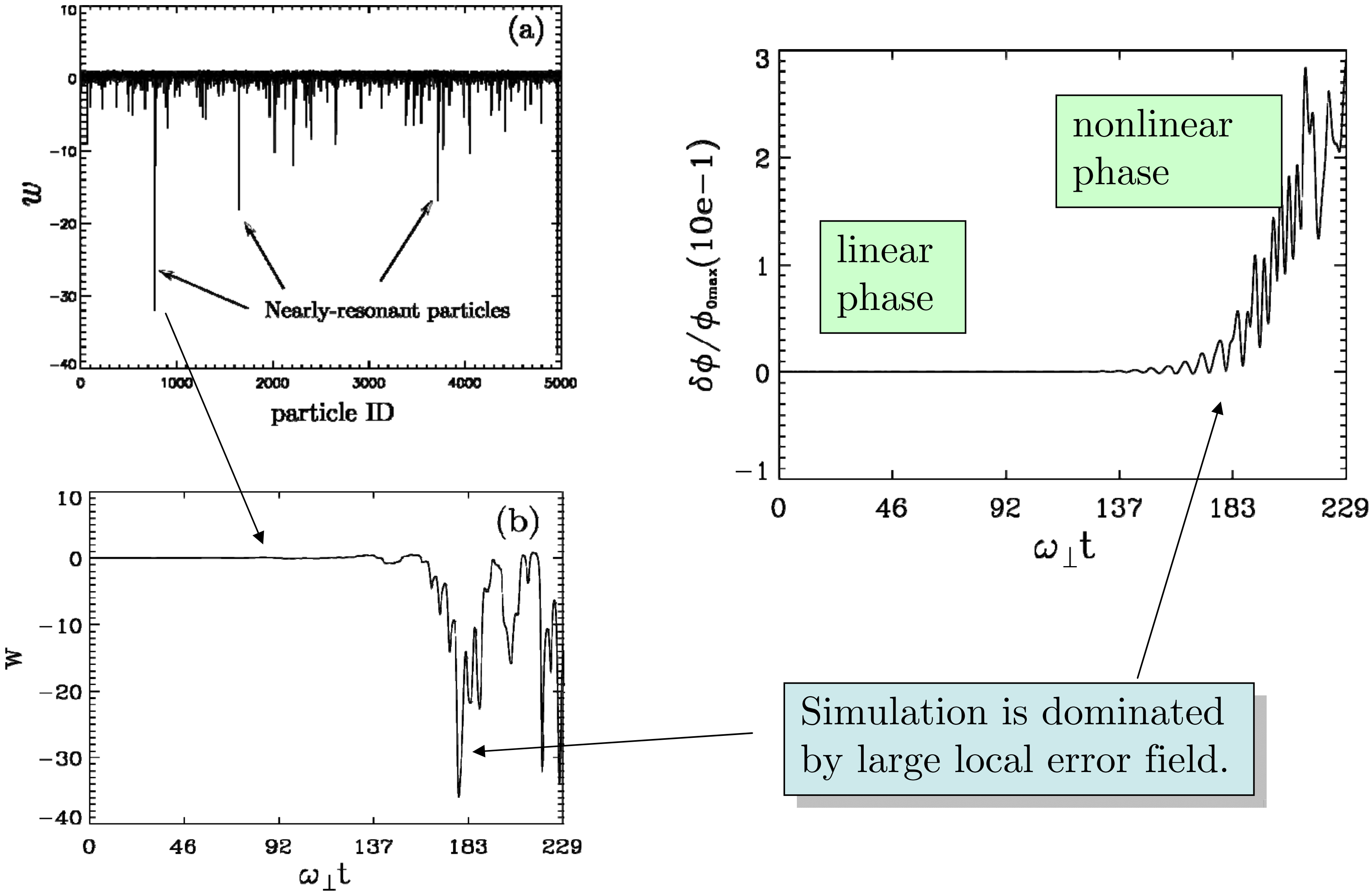
$$f_0 = \frac{\hat{n}}{(2\pi m T_\perp)(2\pi m T_z)^{1/2}} \exp\left(-\frac{H_\perp}{T_\perp} - \frac{H_z}{T_z} \right)$$

$$H_z = \frac{p_z^2}{2m}, \quad H_\perp = \frac{p_\perp^2}{2m} + \frac{m}{2}\omega_\beta^2 r^2 + e\phi_0(z)$$

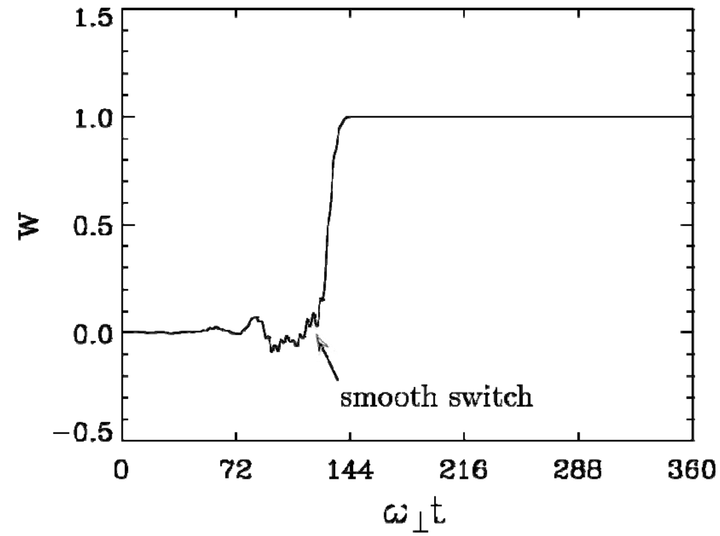
$$\nabla^2 \phi_0 = -4\pi e \hat{n} \exp\left[-\frac{m\omega_\beta^2 r^2 + 2e\phi_0(r)}{2T_\perp} \right]$$

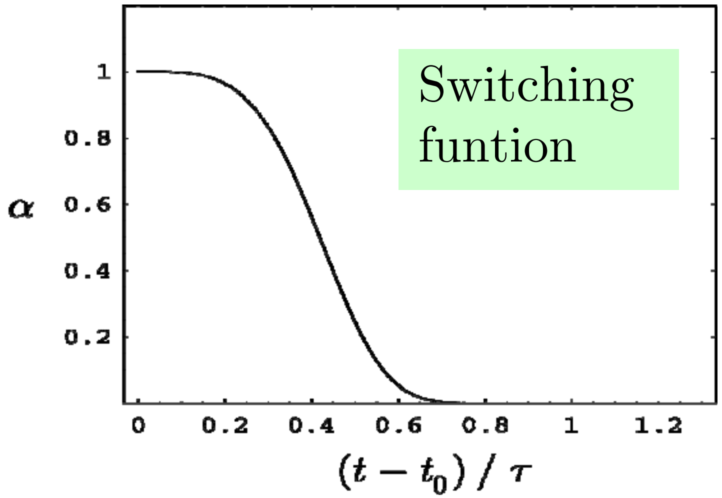


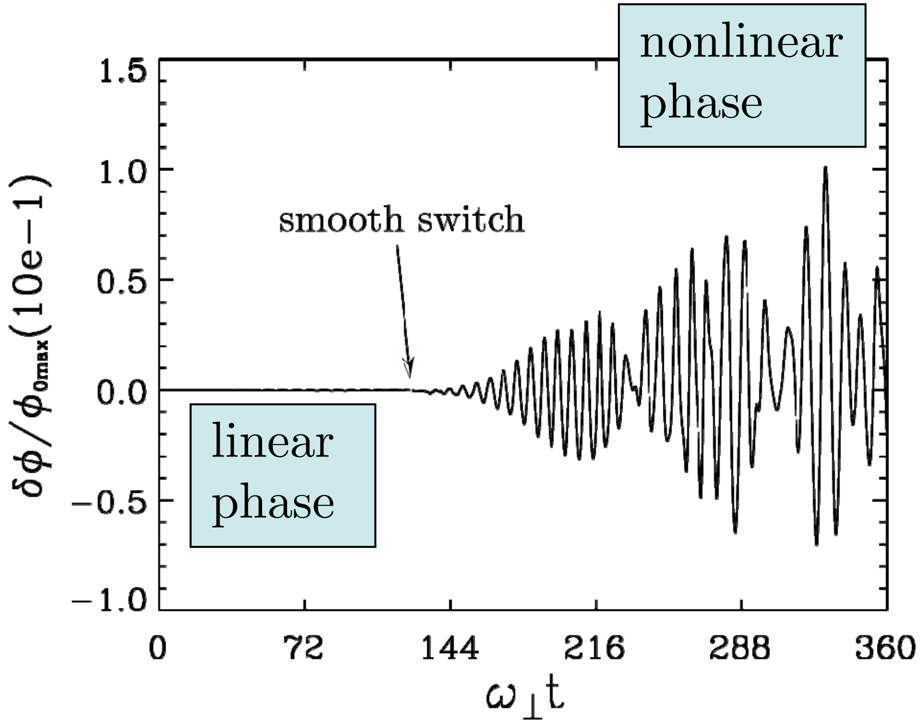
Temperature anisotropy instability with delta-f method



Valid simulation result by the smooth switching algorithm







Conclusions

- Nonlinear δf particle simulation method significantly reduces noise.
- Resonant simulation particles induce large weight growth.
- A modified δf method with smooth switching between δf and total-f methods is developed.