Kinetic Theory and Nonlinear Perturbative Particle Simulations of High-Intensity Beams

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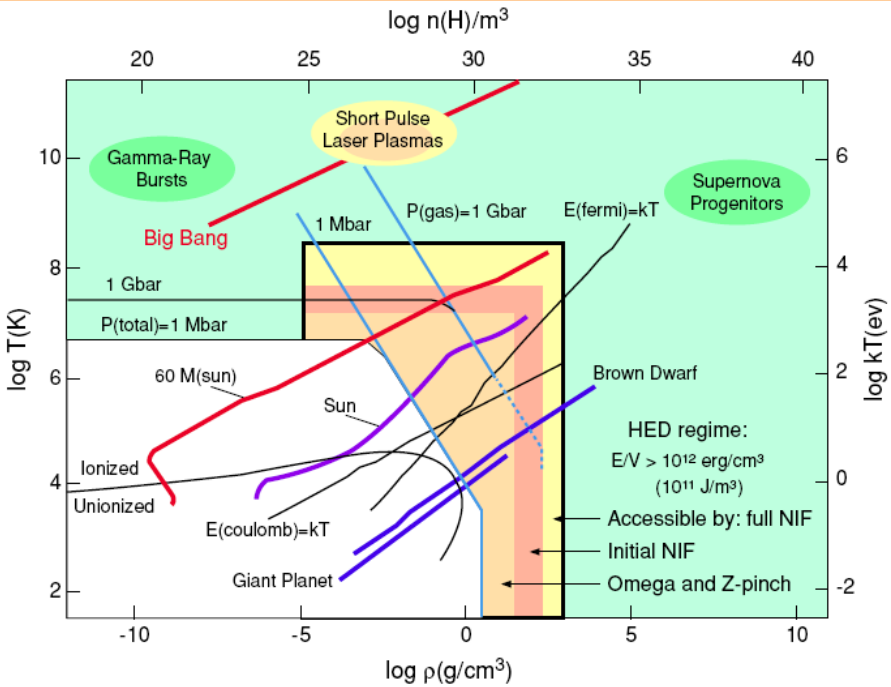
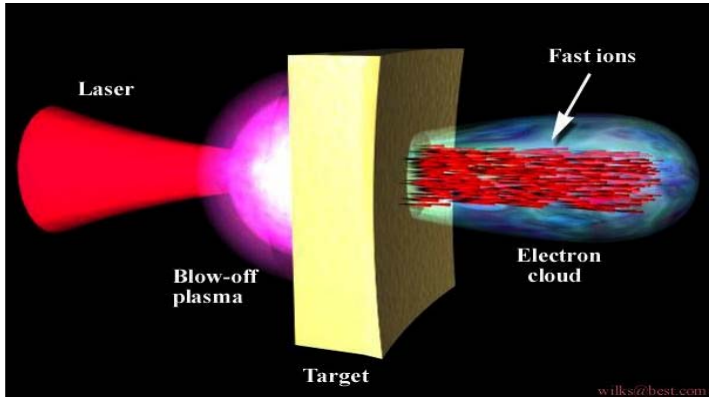
Present at

Student Seminar, PPPL

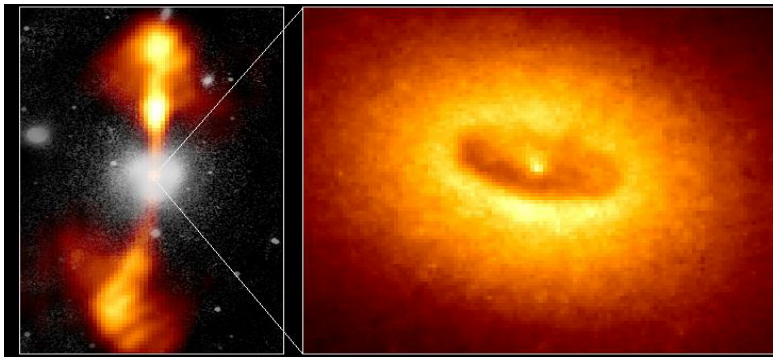
April 7, 2008

Beam physics application --- high energy density physics

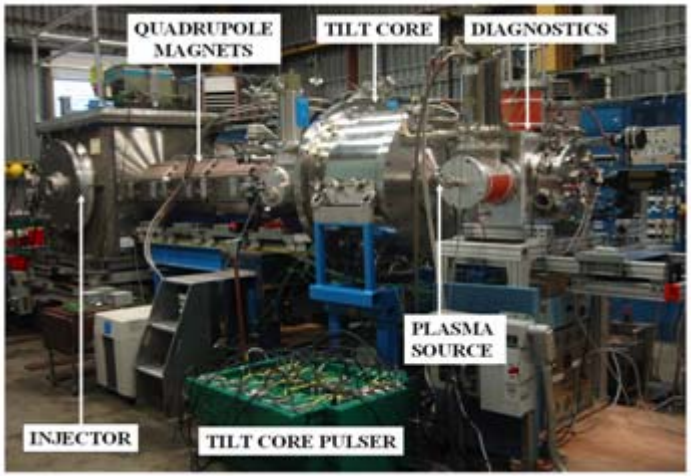




Gamma ray bursters experiment

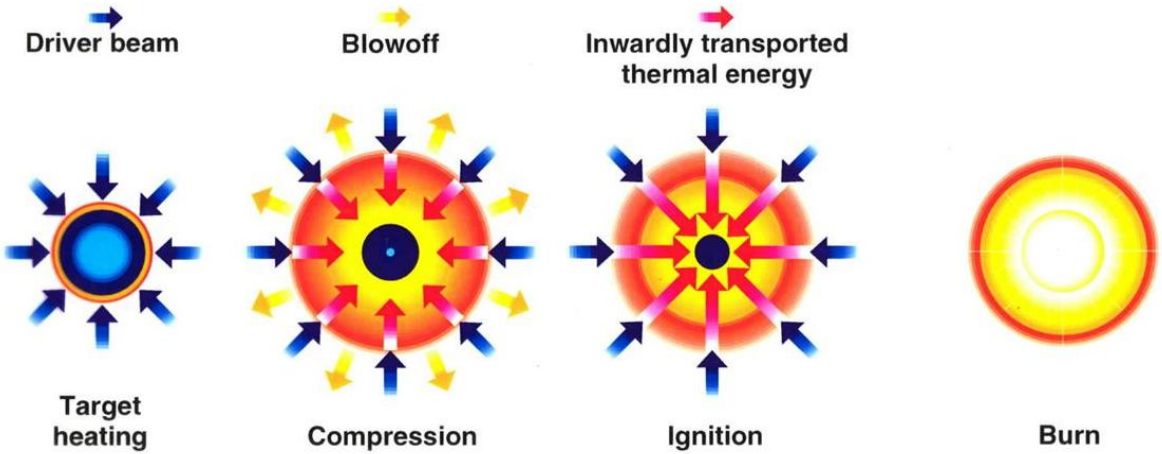


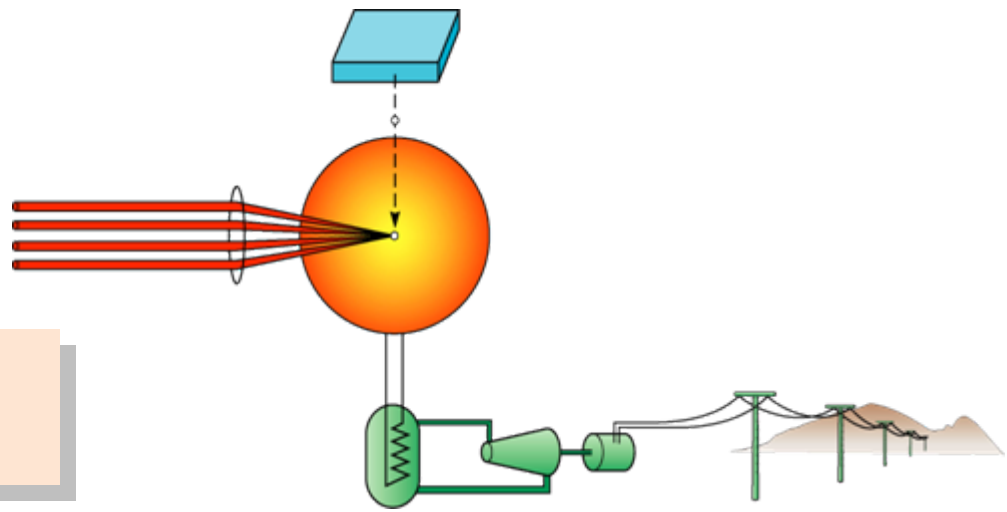
Photoionized plasmas in an accreting massive black hole



Neutralized drift compression experiment for ion beam driven HEDP

Beam physics application --- heavy ion fusion



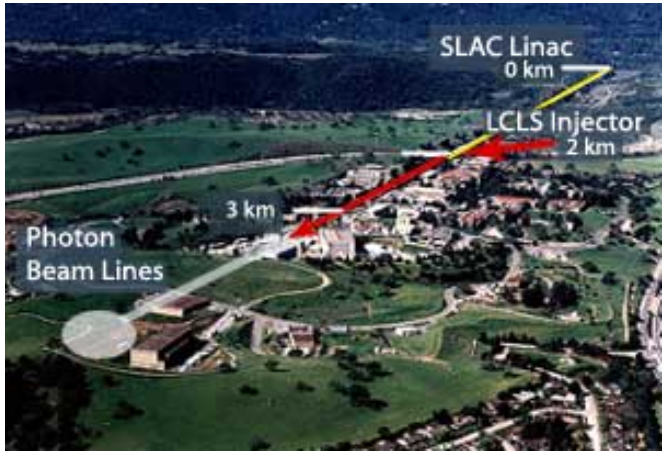


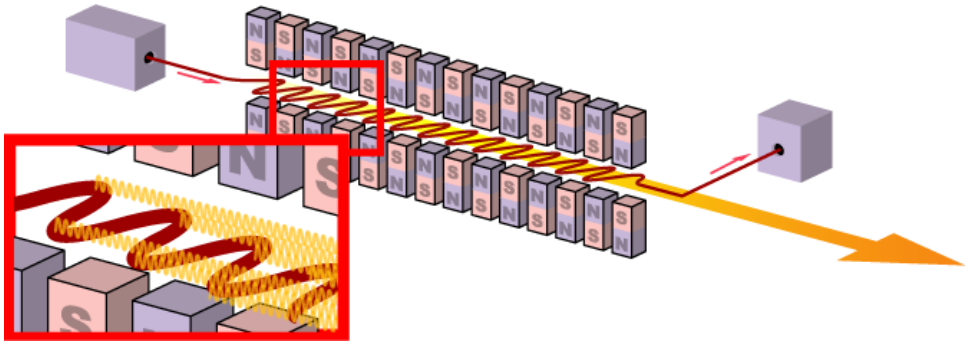
Beams strike "hohlraum," producing x-ray bath for fusion capsules.

Beam physics application --- modern high intensity accelerators

SNS
SPALLATION NEUTRON SOURCE

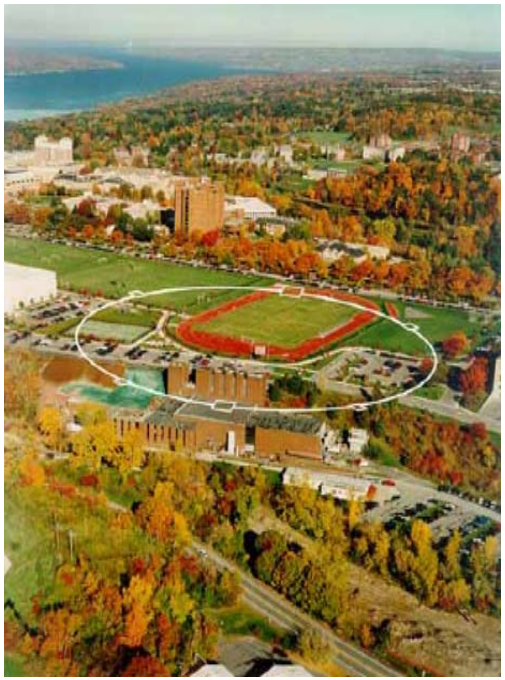
Spallation Neutron Source

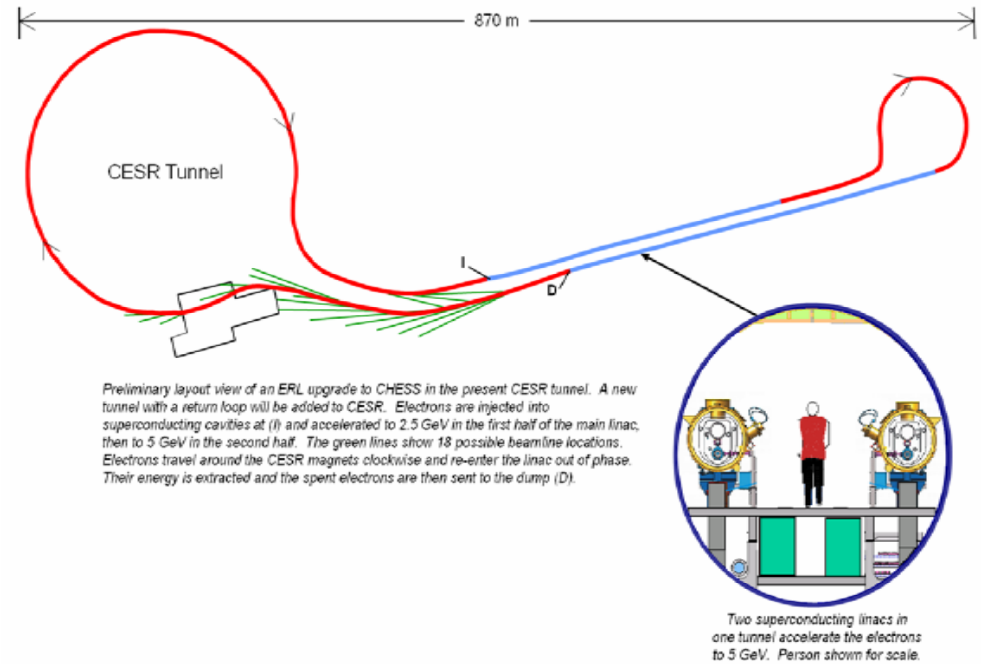




Stanford Linear Coherent Light Source 0.8 - 8 KeV

Beam physics application --- light source



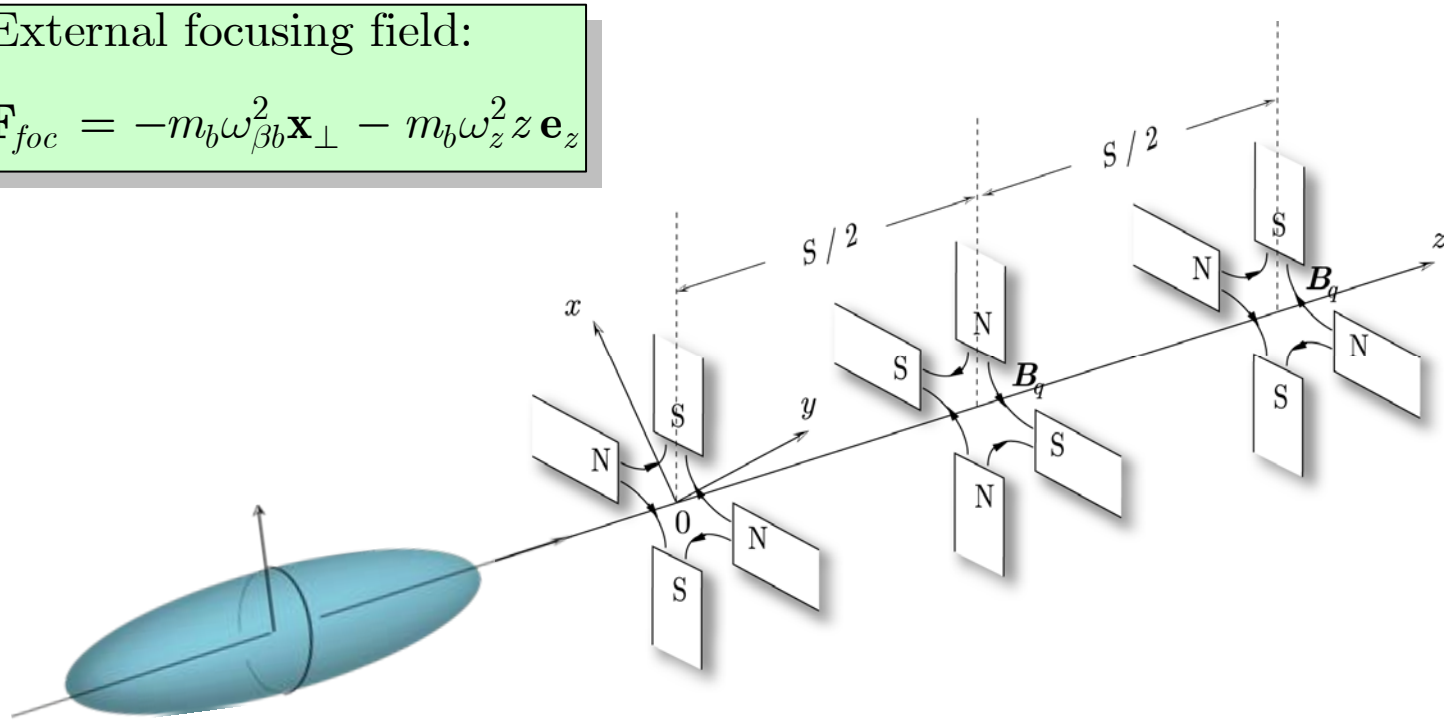


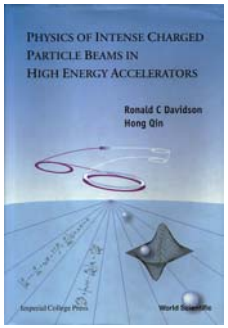
Cornell Energy Recovery Linac, 5GeV electron, 10KeV

Vlasov-Maxwell system for high intensity particle beams

External focusing field:

$$\mathbf{F}_{foc} = -m_b \omega_{\beta b}^2 \mathbf{x}_\perp - m_b \omega_z^2 z \, \mathbf{e}_z$$





An Introduction to the Physics of Intense Charged Particle Beams in High Energy Accelerators, R. C. Davidson and H. Qin, World Scientific (2001)

Interesting "nonclassical" classical dynamics

$$\ddot{u} + \kappa(t)u = 0$$

Birkhoff (1908)

Envelop equation

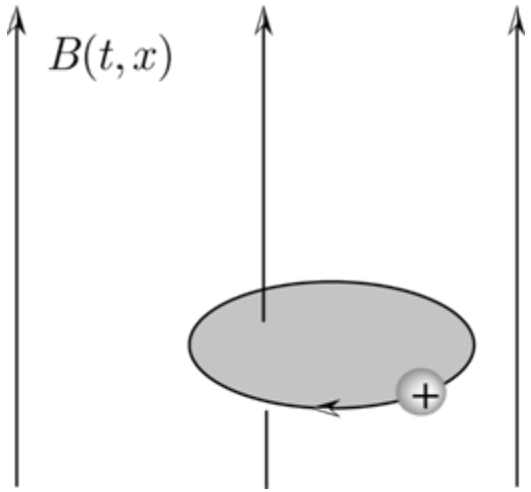
$$\ddot{w} + \kappa(t)w - \frac{e}{w^3} = 0\,.$$

$$I_{cs} = \frac{u^2}{w^2} + \left(\dot{w}u - w\dot{u}\right)^2 \text{ is an invariant.}$$

Courant (1958)

Courant-Snyder invariant

Similar to gyromotion in time-dependent, inhomogeneous field?



Conjucture:

1) There is an exact invariant.

2) Magnetic moment is asymptotic to it.

Nonlinear Vlasov-Maxwell equations for high intensity beams

Collective dynamics described by the Vlasov equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m_b\left(\omega_{\beta b}^2\mathbf{x}_\perp + \omega_z^2 z\mathbf{e}_z\right) + e_b(\nabla\phi - v_z c\nabla_\perp A_z)\right]\cdot\frac{\partial}{\partial \mathbf{p}}\right\} f(\mathbf{x},\mathbf{p},t) = 0$$

Smooth focusing field:

$$\mathbf{F}_{foc} = -m_b\omega_{\beta b}^2\mathbf{x}_\perp - m_b\omega_z^2 z\,\mathbf{e}_z$$

distribution function in phase space

Self-electric and self-magnetic fields <u>self-consistently</u> determined from Maxwell's equations.

$$\nabla^2\phi = -4\pi e_b\int d^3p f(\mathbf{x},\mathbf{p},t),$$

$$\nabla^2 A_z = -4\pi c e_b\int d^3p v_z f(\mathbf{x},\mathbf{p},t).$$

$f(\mathbf{x},\mathbf{p},t)$ and self-field (ϕ, A_z) nonlinearly coupled

δf particle simulation method reduces noise

$$f = f_0 + \delta f,$$
$$\phi = \phi_0 + \delta\phi,$$
$$A_z = A_{z0} + \delta A_z.$$

(f_0, ϕ_0, A_{z0}) – equilibrium

$(\delta f, \delta\phi, \delta A_z)$ – perturbation

Fully nonlinear

$$\frac{dw_i}{dt} = -(1 - w_i)\frac{1}{f_0}\frac{\partial f_0}{\partial \mathbf{p}} \cdot \delta\left(\frac{d\mathbf{p}_i}{dt}\right),$$
$$\delta\left(\frac{d\mathbf{p}_i}{dt}\right) \equiv -e_b\left(\nabla\delta\phi - \frac{v_{zi}}{c}\nabla_\perp \delta A_z\right),$$

Weight function $w \equiv \dfrac{\delta f}{f}$

dynamically determines δf

Statistical noise **significantly** reduced by a factor of $\dfrac{\delta f}{f}$

Beam Equilibrium Stability and Transport (BEST) code

Physics

- Perturbative particle simulation method to reduce noise.
- Linear eigenmodes and nonlinear evolution.
- 2D and 3D equilibrium structure.
- Multi-species; electrons and ions; accommodate very large mass ratio.
- Multi-time-scales, frequency span a factor of 10^5.
- 3D nonlinear perturbation.

Computation

- Message Passing Interface
  - Particle-domain-decomposition.
- Large-scale computing: particle x time-steps ~ 0.5 x 10^{12}.
- Scales linearly to 512 processors on IBM-SP3 at NERSC.
- NetCDF, HDF5 parallel I/O diagnostics.

Nonlinear equilibrium

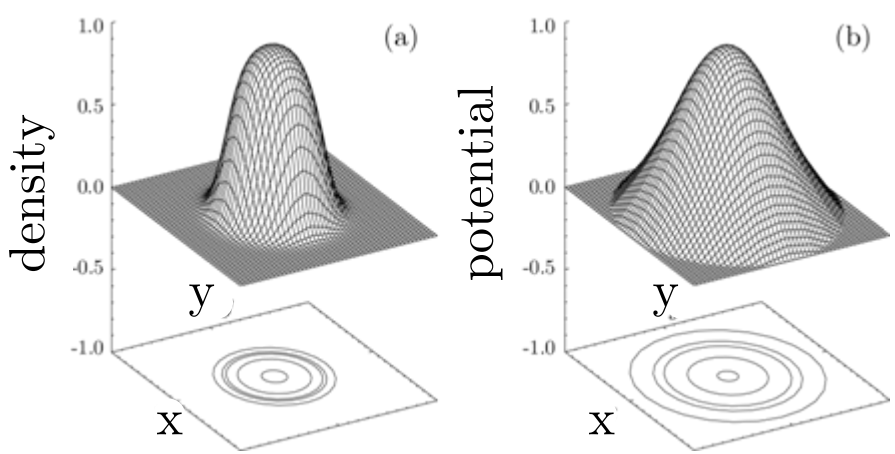
$$H = \frac{p^2}{2m_b} + e_b\phi + \frac{1}{2}m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)$$

$$f_0 = f_0(H) = \frac{\hat{n}_b}{\left(2\pi m_b T\right)^{3/2}} \exp\left(\frac{-H}{T}\right)$$

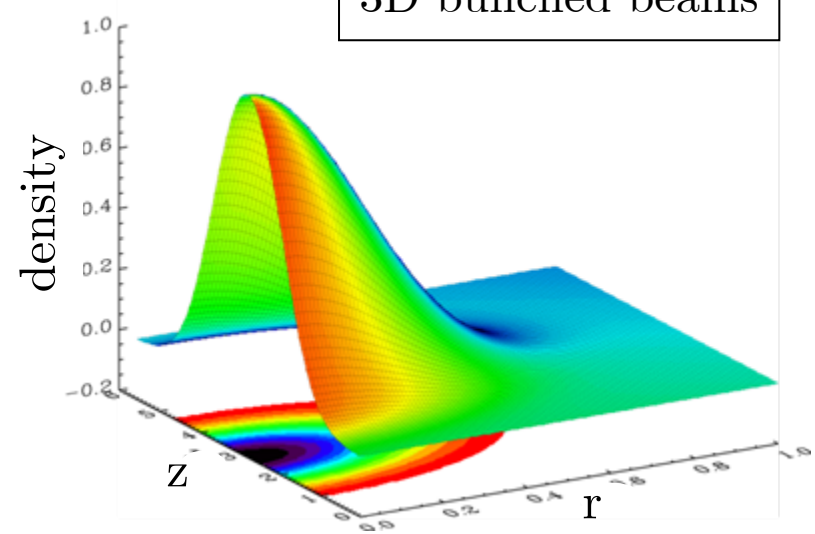
$$\nabla^2\phi_0 = -4\pi e_b \hat{n} \exp\left[-\frac{m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)}{2T} - \frac{e_b\phi_0}{T}\right]$$

Chaotic orbits.
Linear stability
theory not possible.

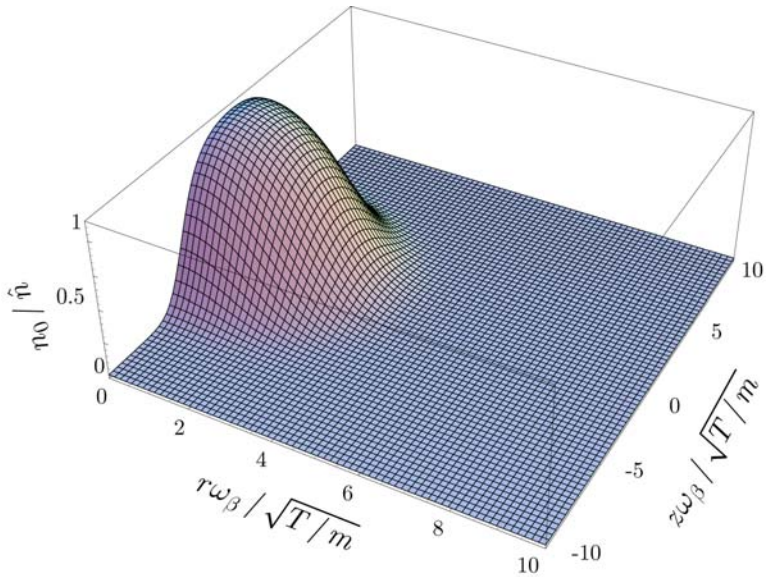
2D coasting beams



3D bunched beams



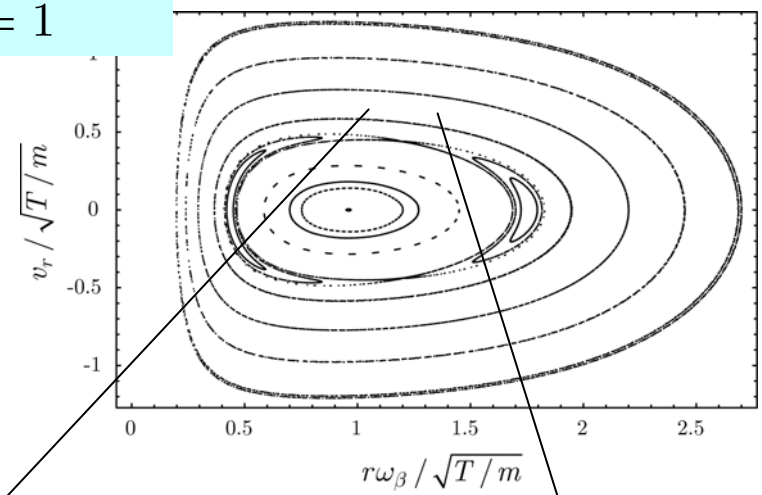
Nonintegrable particle dynamics in energy isotropic bunched beams



$$s_b = 1.1 \quad \frac{\omega_z}{\omega_\beta} = 0.50$$

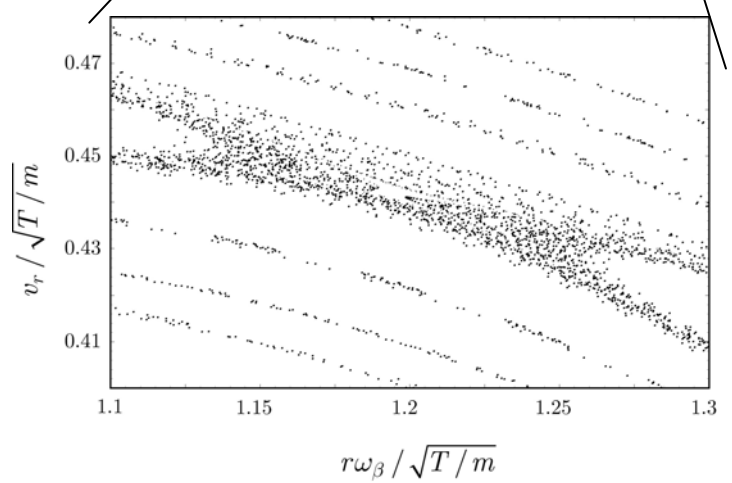
$$P_\theta \omega_\beta / T = 0.25$$

$$H / T = 1$$



Invariants: H, P_θ

No 3rd invariant.



Nonlinear stability theorem

- Global conservation constrains for the nonlinear Vlasov-Maxwell equations.
- Determine Class of beam distributions that are stable at high intensity.

A sufficient condition for linear and nonlinear stability is:

$$\frac{\partial f\left(H\right)}{\partial H} \leq 0$$

Self-field

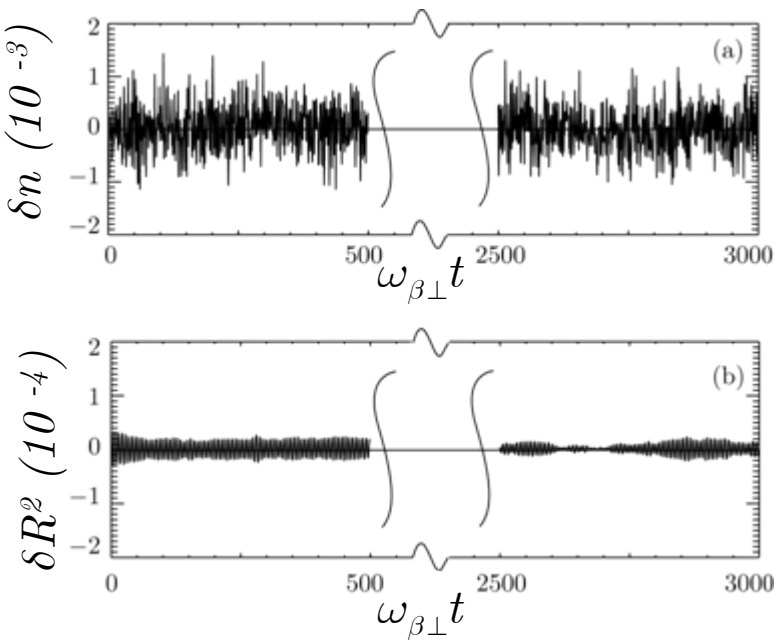
where $$H \equiv \frac{1}{2}\mathbf{p}^2 + \psi\left(\mathbf{x}\right) + e_b\phi\left(\mathbf{x}\right)$$

Focusing field

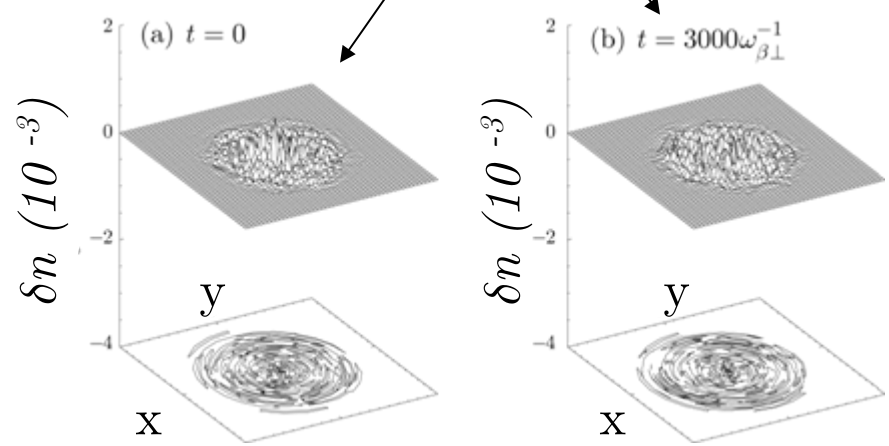
$$\psi\left(\mathbf{x}\right) = \frac{1}{2}m_b\gamma_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)$$

- R. C. Davidson, Physical Review Letters **81**, 991 (1998).
- *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, 2001), R. C. Davidson and H. Qin, Chapter 4;

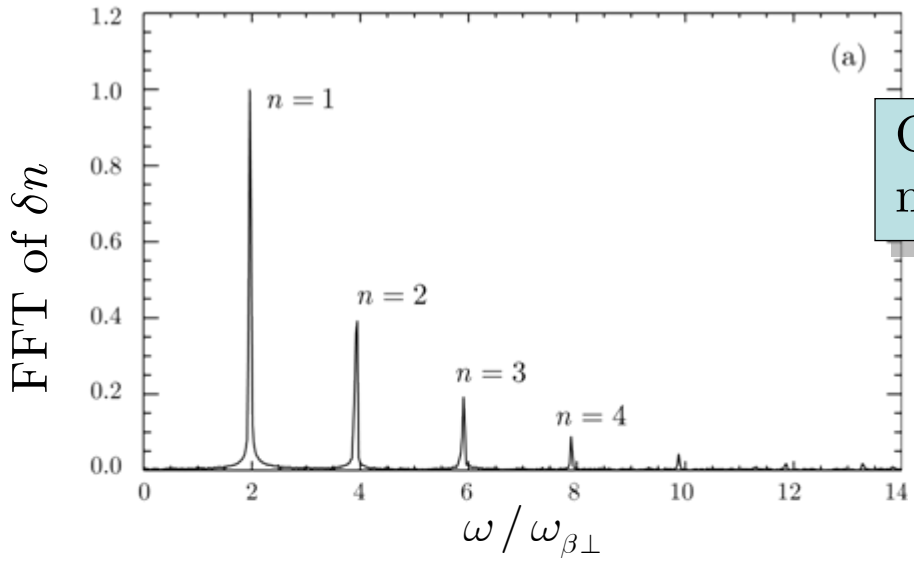
Long-time nonlinear perturbations



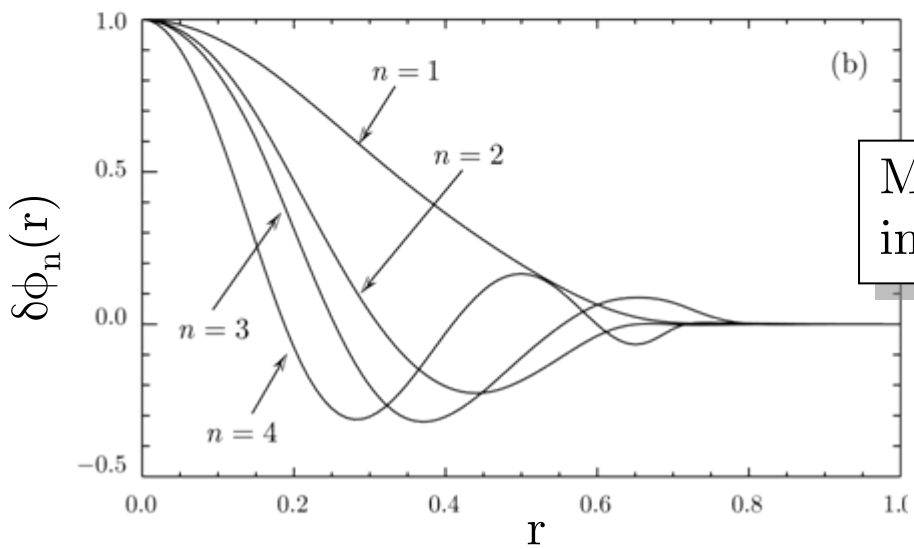
contain perturbation spectrum



Collective interior mode excitation



Collective dynamics manifest by eigenmodes

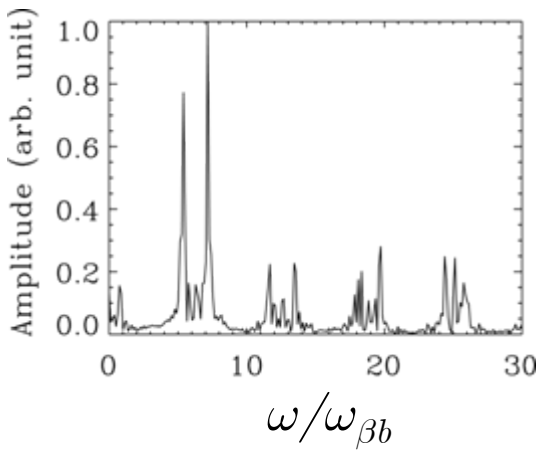


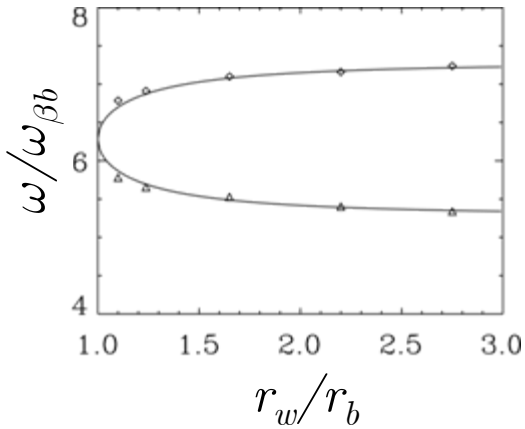
Mode structure confined inside the beam

Collective surface mode excitation

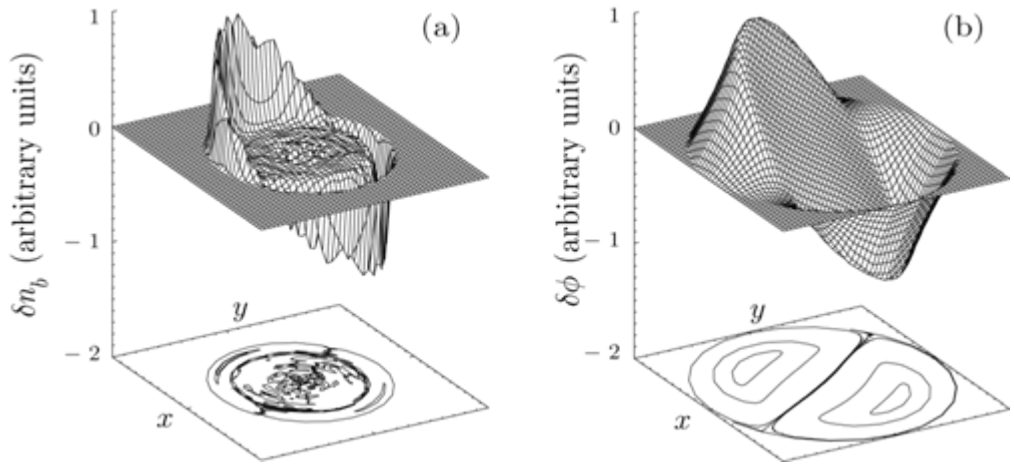
Dispersion relation:

$$\omega = k_z V_b \pm \frac{\hat{\omega}_{pb}}{\sqrt{2}\gamma_b}\sqrt{1 - \frac{r_b^2}{r_w^2}}$$





Dipole mode structure



Collective instabilities in intense charged particle beams

One-Component Beams

- Harris and Weibel instability driven by temperature anisotropy

$$T_\perp \gg T_\parallel \ .$$

- Resistive wall instability

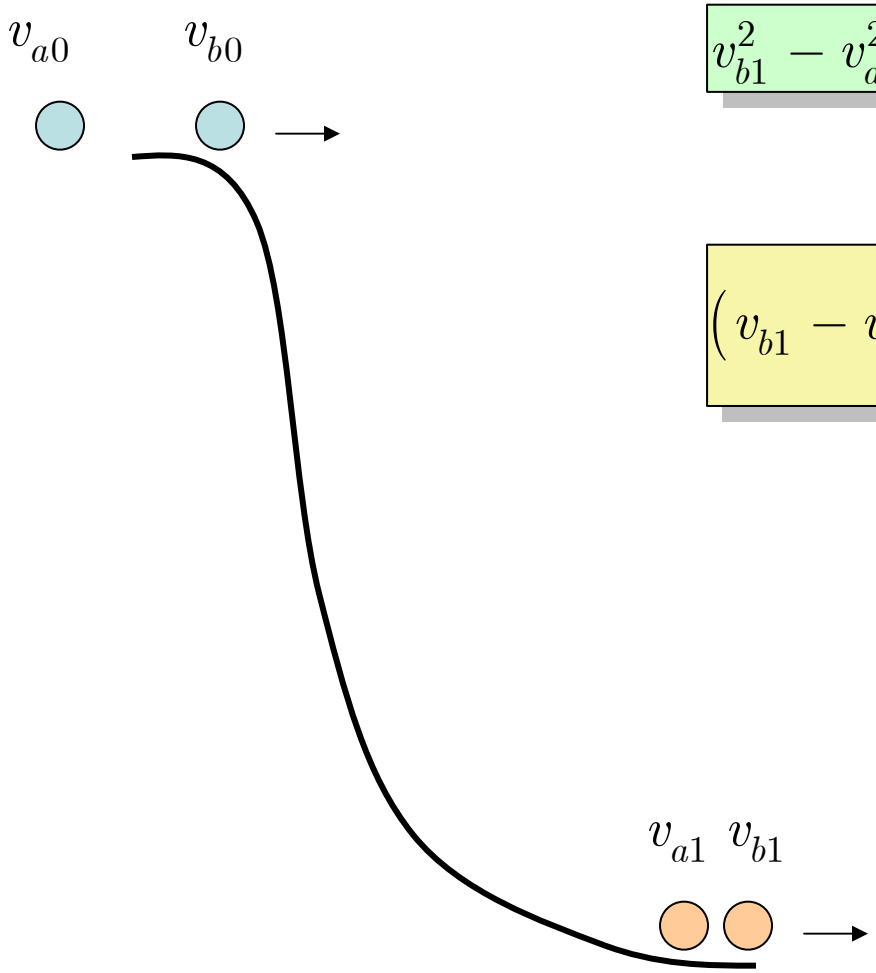
Propagation Through Background speices

- Two-stream instability
- Ion-electron (Electron cloud) instability

Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

Temperature anisotropy is natural in accelerators



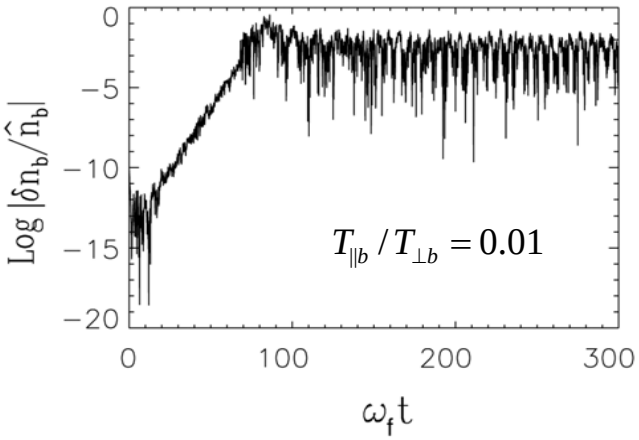
$$v_{b1}^2 - v_{a1}^2 = v_{b0}^2 - v_{a0}^2$$

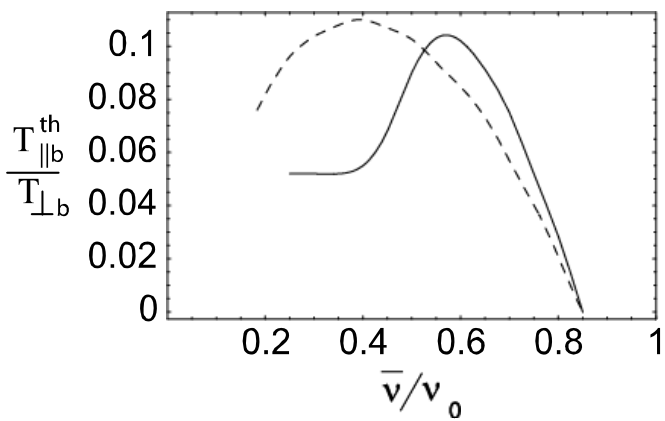
$$\downarrow$$

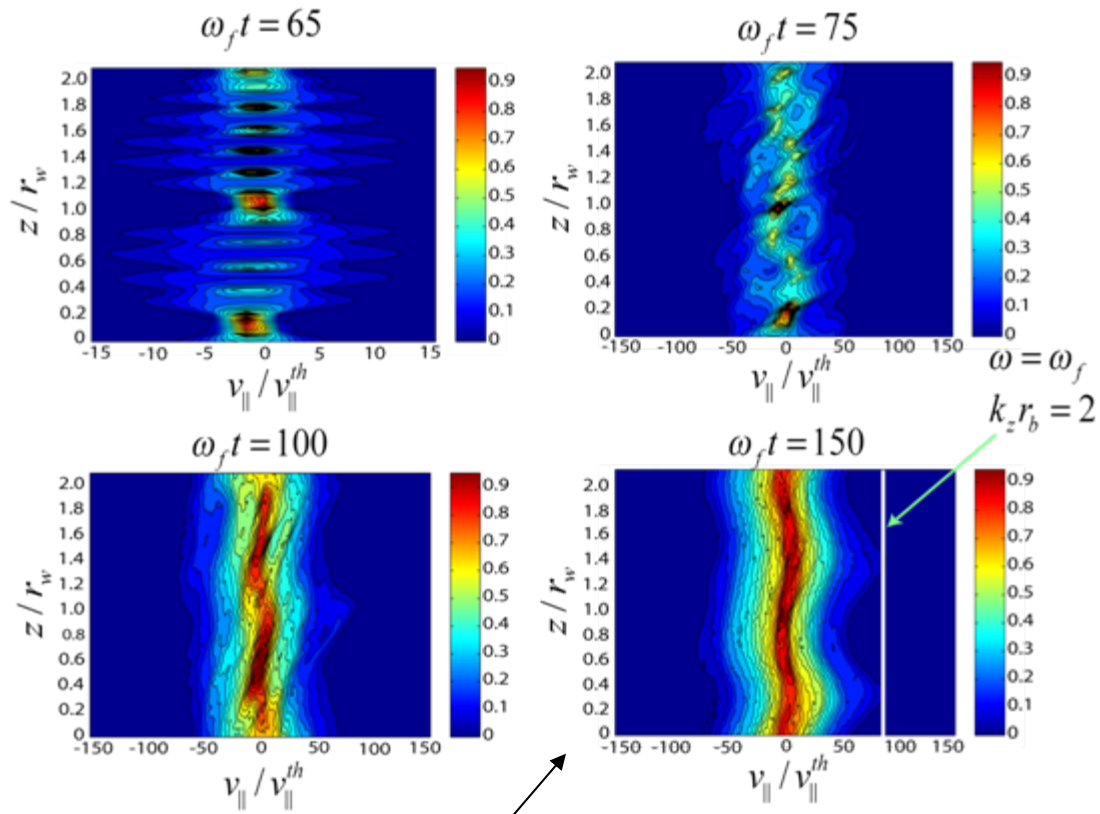
$$\left(v_{b1} - v_{a1}\right) = \left(v_{b0} - v_{a0}\right)\left(\frac{v_{b0} + v_{a0}}{v_{b1} + v_{a1}}\right)$$

$$\left(v_{b1} - v_{a1}\right) \ll \left(v_{b0} - v_{a0}\right)$$

Harris instability by large temperature anisotropy for coasting beams



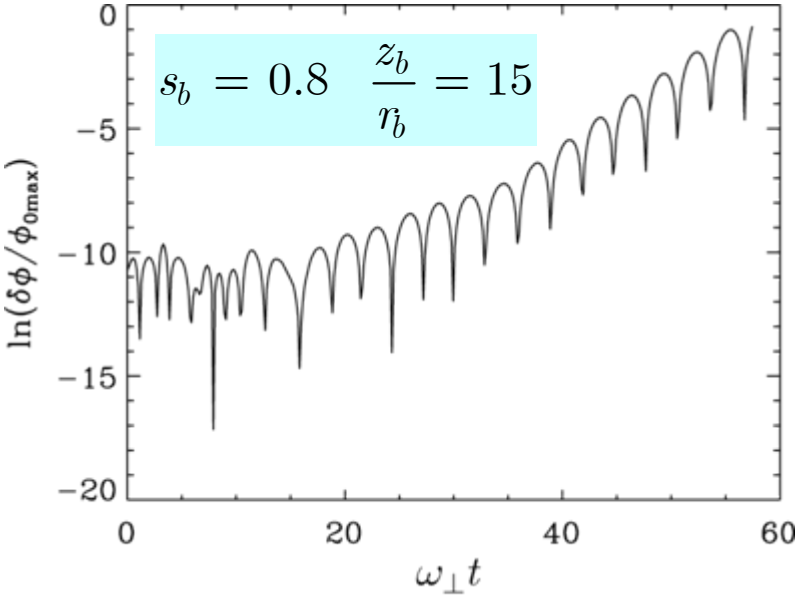


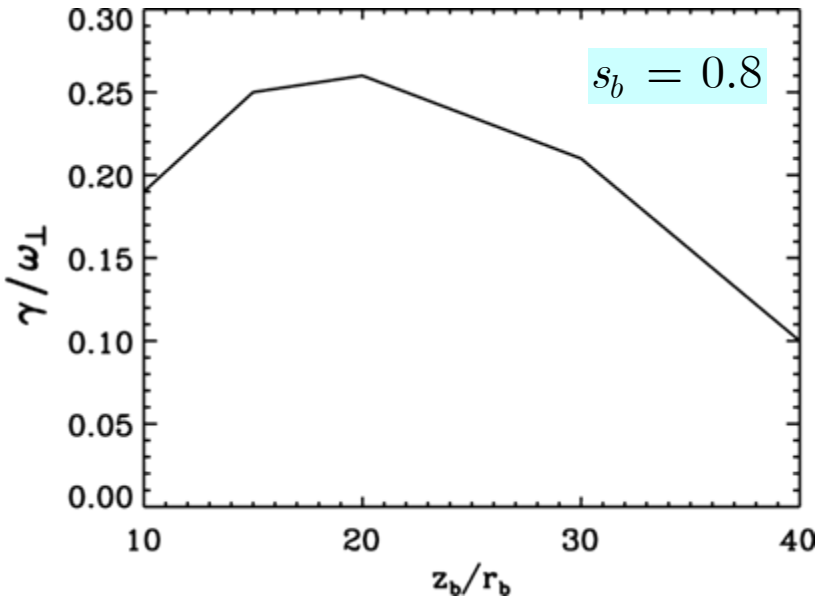


Nonlinear saturation by particle trapping

E. A. Startsev, et al., Physical Review Special Topics Accel. & Beams **8,** 124201(2005).

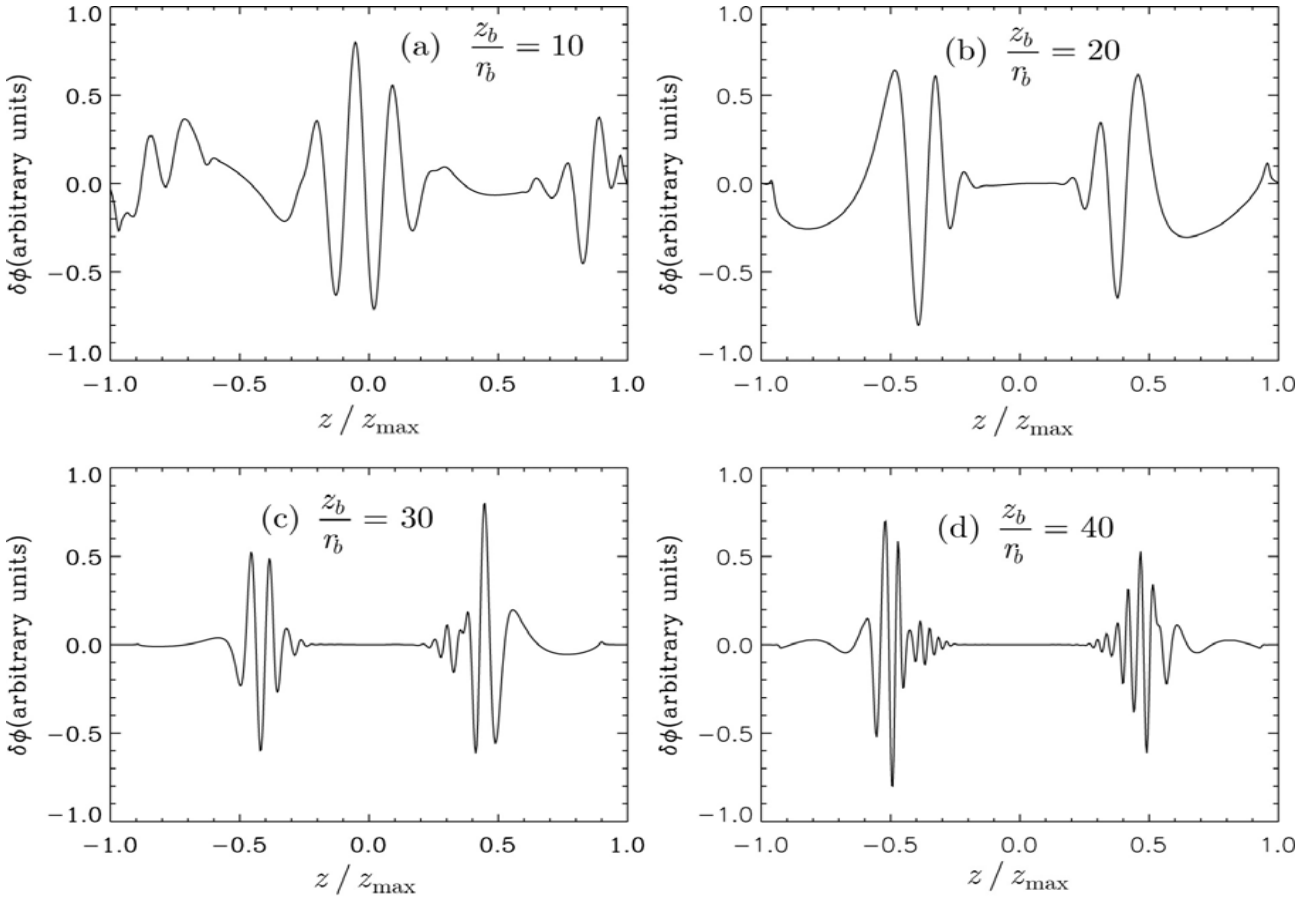
Energy anisotropic instability for bunched beams



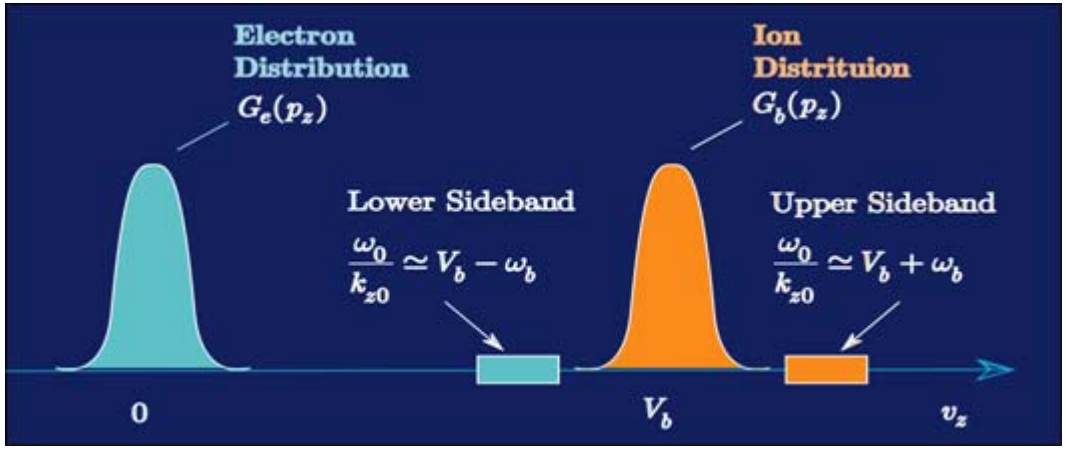


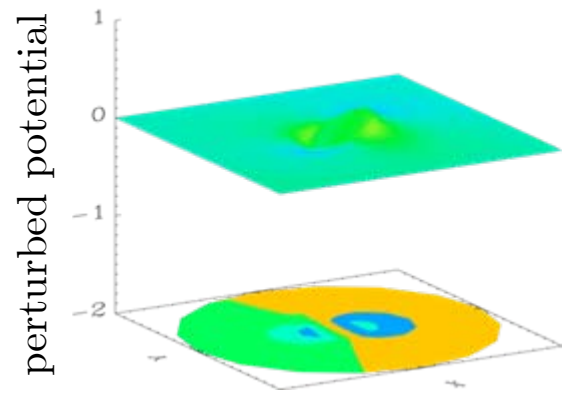
Stronger instability for shorter bunches.

Instability becomes localized for longer bunches $s_b = 0.8$



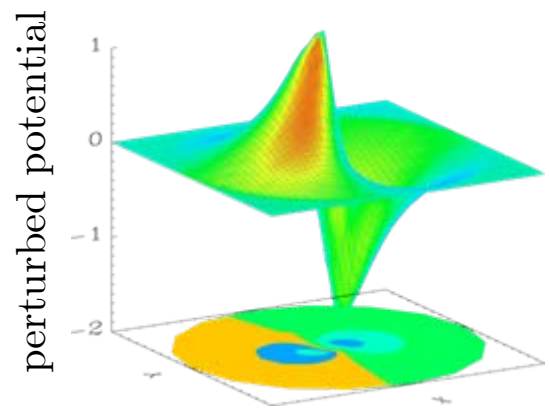
Electron-ion two-stream instability





$t = 0$

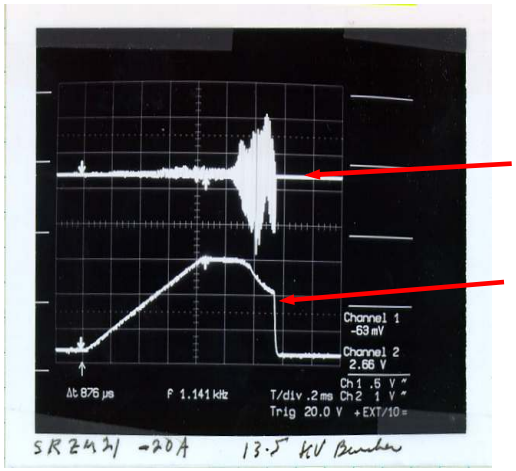
- Surface mode destabilized by background species.
- Observed in high intensity proton beams (PRS).
- Could be a show stopper for high intensity accelerators, e.g. SNS.
- Transverse geometry and dynamics are important.
- Damping mechanisms (Landau damping) are important.



$t = 200 / \omega_{\beta b}$

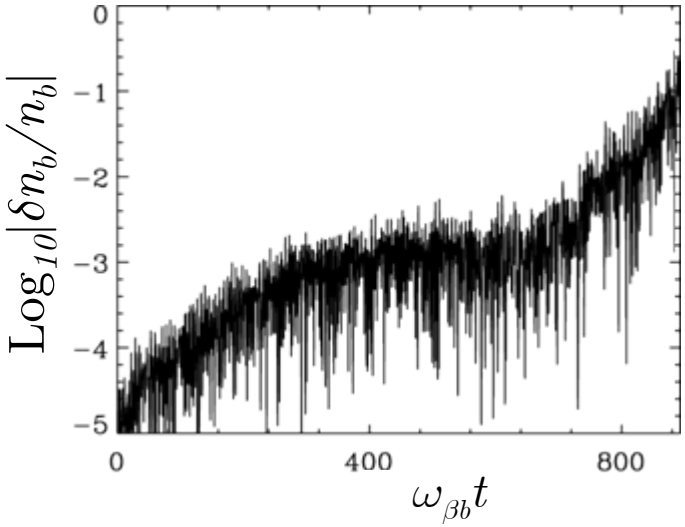
Instability properties predicted by BEST simulations

- **Agrees well with experiment observations** (Proton Storage Ring)
  - Mode structure
  - Growth rate
  - Real oscillation frequency
- Late time nonlinear growth observed.
- There are two-phases to the instability.



BPM ΔV signal

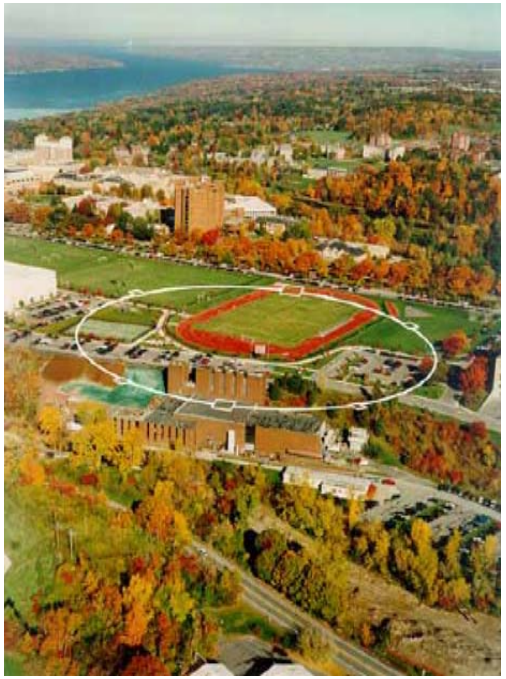
CM42 (4.2 μC) (Circulating Beam Current)



Possible two-stream instabilities for Cornell ERL

$\gamma_b \sim 10000$

- No self-space-charge effect for beam.
- Possible two-stream interactions:
  - ✓ Small mass ratio is a disadvantage.
  - ✓ Fast electron- slow electron in ERL
  - ✓ Electron-ion (background)



Fractional charge neutralization $f = 0.1\%$ (Hoffstaetter, et al, PRST-AB)

$$\omega_{pb}^2 \equiv \frac{4\pi n_b e^2}{m_b \gamma_b^3}$$

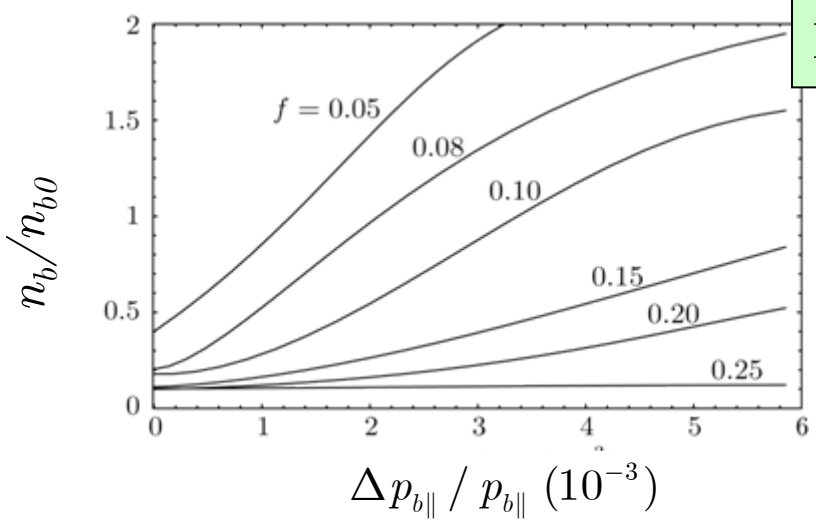
$$\frac{(\mathrm{Im}\,\omega)_{\max}}{\omega_\beta} = \frac{1}{2^{7/4}} \frac{f^{1/2}\left(\gamma_b m_b / m_i\right)^{1/4}\left(\omega_{pb}^2 / \omega_\beta^2\right)^{3/4}}{\left(1 + \frac{f}{2}\frac{\omega_{pb}^2}{\omega_\beta^2}\right)^{1/4}} \sim 0.092$$

R. C. Davidson, et al, PRST-AB **2**, 054401(1999).

Damping mechanisms for two-stream instabilities

- Landau damping by momentum spread.
- Landau damping induced by transverse turn spread.
  - Nonlinear space charge field.
  - Chromaticity induced.
- Theoretical growth rate is greatly reduced.
- Instability threshold observed both numerically and experimentally.

Instability threshold
Proton Storage Ring



Conclusions

- **Vlasov-Maxwell equations provide a very effective tool to understand collective processes in intense charged particle beams.**

- **Nonlinear δf particle simulation method significantly reduces noise.**

- **Major progress has been made in simulation studies of collective effects.**
  - Electron-cloud-induced two-stream instability.
  - Temperature anisotropy instability.

- **Exciting future task areas include:**
  - Other collective excitations and instabilities.
  - Beam-beam and beam-plasma collective interactions.

References

General references

- R. C. Davidson and H. Qin, *An Introduction to the Physics of Intense Charged Particle Beams in High Energy Accelerators*, 583 pp., World Scientific (2001).
- R. C. Davidson, Physical Review Letters **81**, 991 (1998).

Two-stream instability

- R. C. Davidson, et al, PRST-AB **2**, 054401(1999).
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