Response to "Comment on 'A new derivation of the plasma susceptibility tensor for a hot magnetized plasma without infinite sums of products of Bessel functions'" [Phys. Plasmas 15, 024701 (2008)]

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We welcome the Comment by Lerche *et al.* on our recent paper titled "A new derivation of the plasma susceptibility tensor for a hot magnetized plasma without infinite sums of products of Bessel functions."[1] The Comment brings up additional historical facts about previous research on the infinite sums of products of Bessel functions appearing in the plasma susceptibility.

First of all, we would like to emphasize that the main purpose of our paper[1] is to show that it is not necessary to introduce the infinite sums from the very beginning in the derivation of the plasma susceptibility tensor for a hot magnetized plasma, and there is no sum rule needed at all.

If one had to follow the previous redundant approach, which first introduces the infinite sums and then invokes certain sum rules to remove the infinite sums, then the question of who first discovered these sum rules would become scholarly relevant. On this note, we believe that the Comment makes a valid point by pointing out that the sum rule

$$\sum_{n=-\infty}^{\infty} \frac{J_n^2(z)}{a-n} = \frac{\pi}{\sin \pi a} J_{-a}(z) J_a(z) \tag{1}$$

was derived by Lerche in 1966.[2] A more generalized sum rule

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\alpha-n}(z) J_{\beta+n}(z)}{n+\mu} = \frac{\pi}{\sin(\pi\mu)} J_{\alpha+\mu}(z) J_{\beta-\mu}(z) \tag{2}$$

was shown by Lerche in 1974.[3] What is now called "Newberger's sum rule" by the plasma physics community[4,5] takes the form of

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\alpha-\gamma n}(z) J_{\beta+\gamma n}(z)}{n+\mu} = \frac{\pi}{\sin \pi\mu} J_{\alpha+\gamma\mu}(z) J_{\beta-\gamma\mu}(z) \tag{3}$$

for μ nonintegral, $\mathrm{Re}(\alpha+\beta) > -1$, $0 < \gamma \leq 1$. It was first derived by Newberger in 1982.[6,7] Obviously, Eq. (3) includes Eqs. (1) and (2) as special cases. Furthermore, Eq. (3) is actually only a subset of the most general sum rule discovered by Newberger [See Eq. (2.17) in Ref. 6]. It indeed took 16 years before Newberger rediscovered and generalized Eqs. (1) and (2), and it took yet another 25 years for Lerche *et al.* to discover that Eqs. (1) and (2) had been rediscovered and generalized. This certainly underscores the importance of communication among different research communities, even in the age of electronic publishing and the high-speed Internet. Based on these facts, we propose that the so-called "Newberger's sum rule" [Eq. (3)] be renamed to "Lerche–Newberger sum rule."

Finally, after studying carefully the available literature, we have reconfirmed our original conclusion[1] that it was Swanson[4] who first explicitly showed that every infinite sum in the 3×3 susceptibility tensor of a magnetized plasma can be reduced to one or two single terms using Eq. (1) and its variations. We have not found this fact being clearly stated or demonstrated in any publications prior to Ref. 4, including Refs. 2, 3, and 8.

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