$\boxed{4}$ MHD Equilibrium – One Dimensional

① Equilibrium: $\frac{\partial}{\partial t} = 0$.

② Mostly without flow, $v = 0$

$$\Rightarrow \quad -\nabla p + \frac{(\nabla \times B) \times B}{4\pi} = 0$$

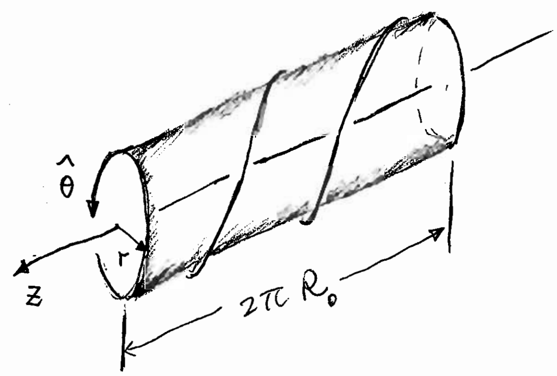
$$\nabla \cdot B = 0$$

4 Eqs. for 4 Components

Screw – pinch

$$B = B_\theta(r)\,\hat{\theta} + B_z(r)\,\hat{z}$$

$$p = p(r)$$



$$j = \frac{c}{4\pi} \nabla \times B$$

$$= J_\theta \hat{\theta} + J_z \hat{z}$$

$$J_\theta = -\frac{c}{4\pi} \frac{d}{dr} B_z(r)$$

$$J_z = \frac{c}{4\pi} \frac{1}{r} \frac{d}{dr} \left[r B_\theta(r) \right]$$

momentum Eq: $\frac{J \times B}{c} = \nabla P$ or $\frac{(\nabla \times B) \times B}{4\pi} = \nabla P$

$$\Rightarrow \quad \frac{d}{dr}\left(P + \frac{B_\theta^2 + B_z^2}{8\pi}\right) + \underbrace{\frac{B_\theta^2}{4\pi r}}_{\text{curvature bending force}} = 0$$

$\boxed{\text{Two freedoms for } P(r),\ B_\theta(r),\ B_z(r)}$

Question: MHD Eqs have 8 Eqs for 8 components, why there are two freedoms for the screw pinch equilibrium?

safety Factor: $q(r) \equiv \frac{r B_z(r)}{R_0 B_\theta(r)}$

How many turns in $\hat{z}$ does the field line travel for one turn in $\hat{\theta}$?

(one turn in $\hat{z} \equiv 2\pi R_0$ in $\hat{z}$)

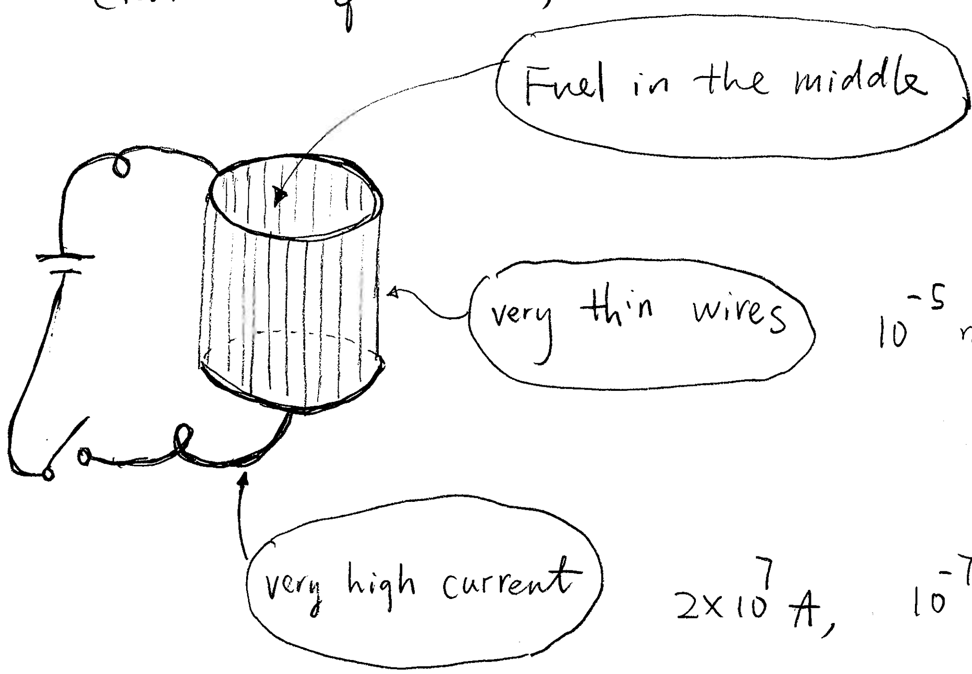
$$\frac{r\,d\theta}{dz} = \frac{B_\theta}{B_z} \Rightarrow \frac{dz}{2\pi R_0} = \frac{r B_z}{R_0 B_\theta} \frac{d\theta}{2\pi} \Rightarrow q = \frac{r}{R_0} \frac{B_z(r)}{B_\theta(r)}.$$

$\boxed{\text{Z-pinch}}$ $J_\theta = 0$, $B_z = 0$

$q = 0$ unsafe (unstable)

Z-pinch for inertial confinement

(NOT an equilibrium)

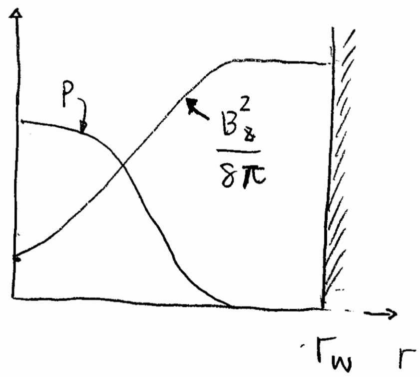


10^{-5} m

2×10^{7} A, 10^{-7} s

$\boxed{\theta\text{- Pinch}}$: $J_z = 0, \quad B_\theta = 0$

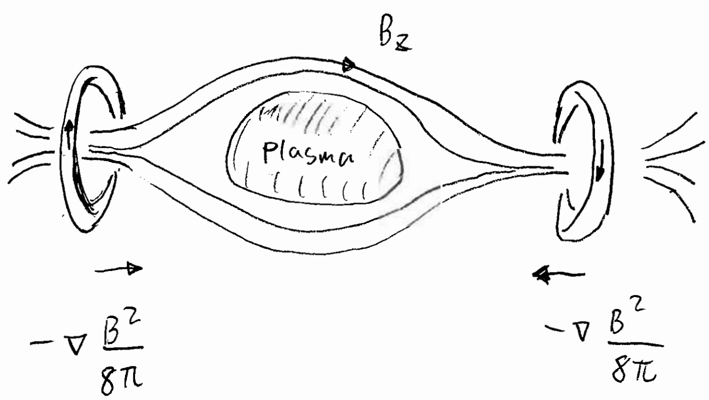
$$P + \frac{B_z^2}{8\pi} = \frac{B_0^2}{8\pi}$$



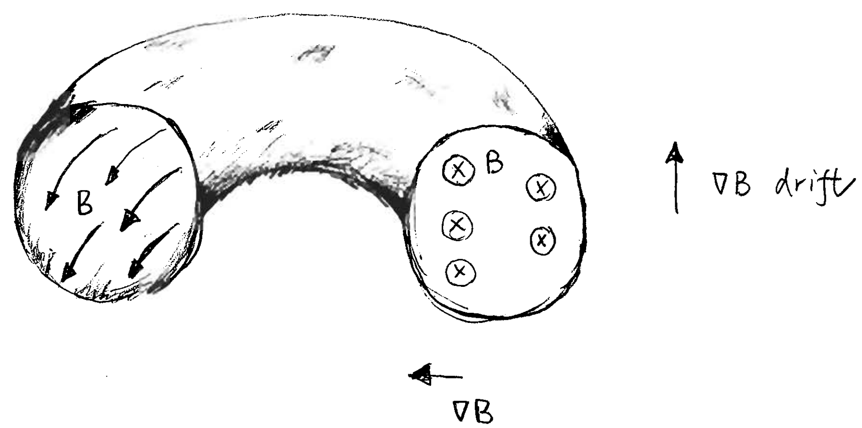
$q = \infty$, very safe (stable)

But, No axial confinement,

solution ①: make a mirror mechine. ⇒ 2D Equilibrium



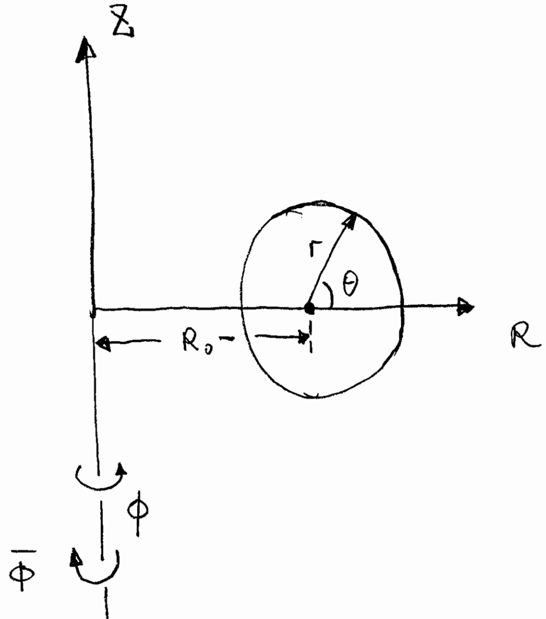
solution ②. Make a toroidal Magnetic field



Need poloidal field to confine particle radially.

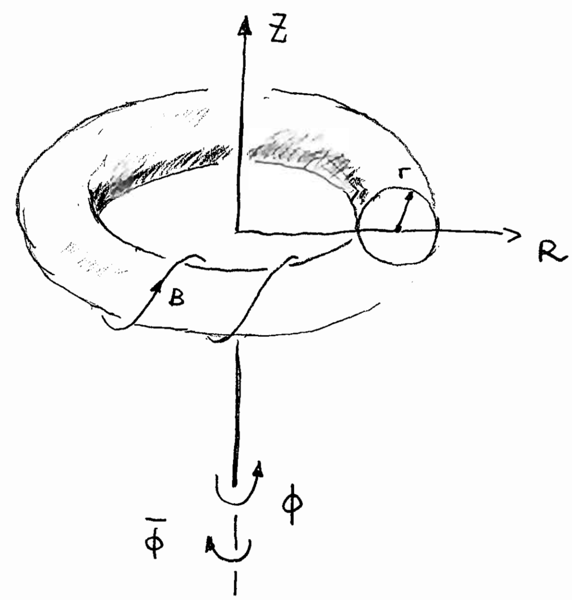
$\Rightarrow$ toroidal screw-pinch, (2D)

5 | MHD Equilibrium 2D, Grad-Shafranov Eq.



cylindrical coord. (R, ϕ, Z)

toroidal coord $(r, \theta, \bar{\Phi})$



$$\boxed{\text{Symmetry:} \quad \frac{\partial}{\partial \phi} = 0}$$

$$B = \nabla \times A$$

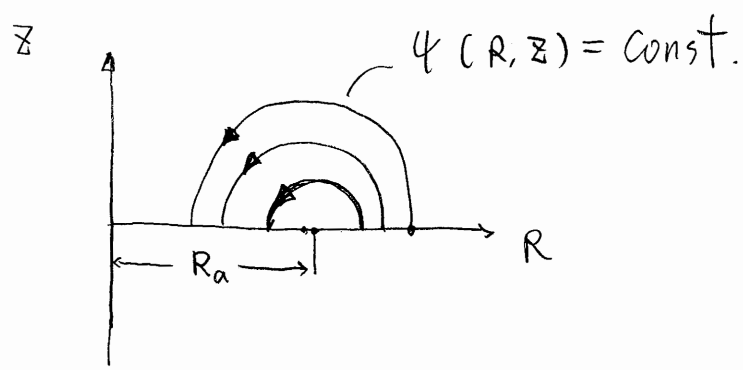
$$\Rightarrow \begin{cases} B_R = -\dfrac{\partial A_\phi}{\partial Z} = -\dfrac{1}{R}\dfrac{\partial \psi}{\partial Z} & (1) \\ B_Z = +\dfrac{1}{R}\dfrac{\partial}{\partial R}(R A_\phi) = \dfrac{1}{R}\dfrac{\partial \psi}{\partial R} & (2) \end{cases}$$

$$\psi \equiv R A_\phi$$

$$\Rightarrow \quad \vec{B} = \frac{1}{R} \nabla \psi \times e_\phi + \vec{B}_\phi \quad \cdots \quad (3)$$

$B \cdot \nabla \psi = 0 \quad \Rightarrow \quad \psi = \text{Const}$ is a flux surface

- The fundamental reason for the existance of Magnetic flux surface is the symmetry, Not $\nabla \cdot B = 0$
- More on the Hamiltonian structure of magnetic field later.



Assume: There is a magnetic axis at R_a

poloidal flux enclosed by a flux surface $\psi = \text{const}$:

$$\psi_p = 2\pi \int_{R_a}^{R} dR\ R\ B_z(R, z=0)$$

$$= 2\pi \left[\psi(R, z=0) - \psi(R_a, z=0)\right]$$

$$\frac{4\pi}{c} j = \nabla \times B = \nabla \times (B_\phi e_\phi) + \nabla \times \left(\frac{\nabla\psi}{R} \times e_\phi\right)$$

same calculation as Eq. (3)

$$\nabla \times (B_\phi e_\phi) = \frac{1}{R}\nabla(RB_\phi) \times \hat{e}_\phi$$

or

$$\nabla B_\phi \times e_\phi + B_\phi \underbrace{\nabla \times e_\phi}_{\frac{1}{R}\hat{z}}$$

$$\nabla \times \left(\frac{\nabla\psi}{R} \times e_\phi\right) = \underbrace{\frac{\nabla\psi}{R}\, \nabla\cdot e_\phi}_{0} - e_\phi\, \underbrace{\nabla\cdot\left(\frac{\nabla\psi}{R}\right)}_{\nabla\cdot\left(\frac{1}{R}\frac{\partial\psi}{\partial R}e_R + \frac{1}{R}\frac{\partial\psi}{\partial z}e_z\right)} + \underbrace{(e_\phi\cdot\nabla)\frac{\nabla\psi}{R}}_{=\frac{1}{R^2}\frac{\partial\psi}{\partial R}\underbrace{\frac{\partial e_R}{\partial\phi}}_{=e_\phi}} - \underbrace{\left(\frac{\nabla\psi}{R}\cdot\nabla\right)e_\phi}_{0}$$

$$= -e_\phi \left[\underbrace{\frac{1}{R}\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R^2}\frac{\partial\psi}{\partial R}}_{\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)} + \frac{1}{R}\frac{\partial^2\psi}{\partial z^2}\right] \quad \cdots \quad (4)$$

$$= -e_\phi \frac{\Delta^*\psi}{R}$$

$$\Delta^* = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial z^2}$$

$$\boxed{\frac{1}{4\pi}\overbrace{\left[\frac{1}{R}\nabla(RB_\phi)\times e_\phi - e_\phi\frac{\Delta^*\psi}{R}\right]}^{\frac{j}{c}} \overset{\times}{\times} \overbrace{\left[\frac{1}{R}\nabla\psi\times e_\phi + e_\phi B_\phi\right]}^{B} = \nabla p} \quad (5)$$

☆

$$0 = B\cdot\nabla p = \frac{1}{R}\, e_\phi\cdot(\underbrace{\nabla p}_{\left(\frac{\partial p}{\partial\psi}\nabla\psi + \frac{\partial p}{\partial\theta}\nabla\theta\right)}\times\nabla\psi) = \frac{1}{R}\, e_\phi\cdot\left(\frac{\partial p}{\partial\theta}\nabla\theta\times\nabla\psi\right)$$

$$\Rightarrow \quad \frac{\partial p}{\partial\theta} = 0 \quad \Rightarrow \quad \underline{p = p(\psi)}$$

assume: $e_\phi\cdot[\nabla\theta\times\nabla\psi]\neq 0$

☆

$$0 = j\cdot\nabla p = \frac{1}{R}\left[e_\phi\cdot\nabla p\times\nabla(RB_\phi)\right] = \frac{1}{R}\frac{dp}{d\psi}\left[e_\phi\cdot\nabla\psi\times\nabla(RB_\phi)\right]$$

$$\Rightarrow \quad RB_\phi = F(\psi)$$

(5) $\Rightarrow$

$$-\frac{1}{4\pi}\frac{B_\phi}{R}F'(\psi)\nabla\psi - \frac{\Delta^*\psi}{4\pi R^2}\nabla\psi = \nabla p$$

$$\Rightarrow \quad \boxed{\Delta^*\psi = -FF' - 4\pi R^2 p'} \quad (6)$$

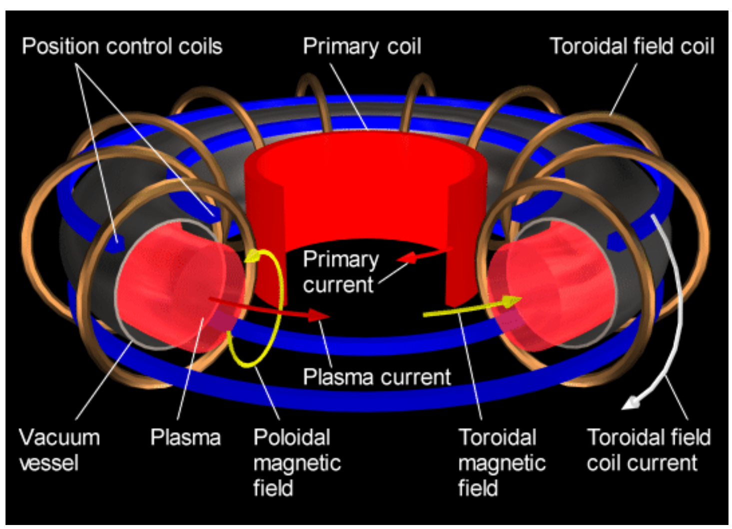
Grad-Shafranov Eq.

Harold Grad

Vitalii D. Shafranov

Grad-Shafranov Equation:

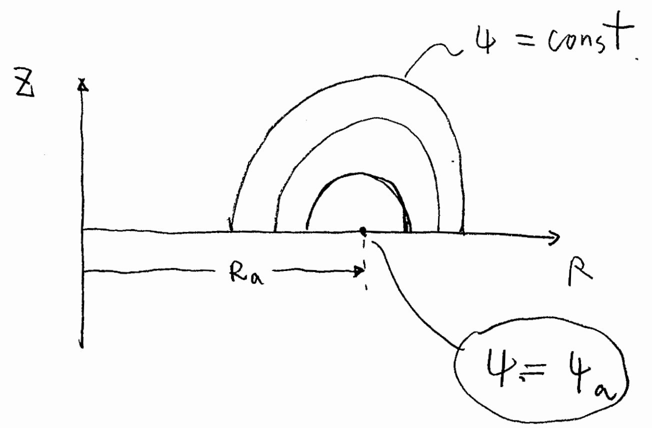
$$\Delta^{*}\psi = -FF' - 4\pi R^{2} p'$$



specify $P(\psi)$, $F(\psi)$ and B.C.s

solve for $\psi(R, Z)$. Again, two freedoms!

$\boxed{\text{what is } F(\psi)\,?}$



$$\vec{j} = \frac{c}{4\pi}\left[\frac{1}{R}\nabla F \times e_\phi - \frac{\overset{*}{\Delta}\psi}{R}\, e_\phi\right]$$

$\Rightarrow$ $\psi = \text{const}$ is a current flux surface.

poloidal current enclosed by a flux surface:

$$I_p = 2\pi \int_{R_a}^{R} dR\, R\, j_z = \frac{c}{2}\int_{R_0}^{R} dR\, \nabla F$$

$$= \frac{c}{2}\left[\underset{\substack{\parallel \\ F(\psi)}}{F(R, z=0)} - \underset{\substack{\parallel \\ F(\psi=\psi_a)}}{F(R_a, z=0)}\right]$$

Safty factor q

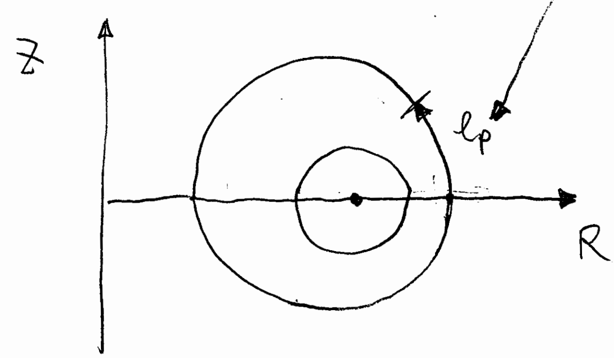
The local magnetic pitch angle $\frac{B_\phi}{B_p}$ is not a constant even on a flux surface. The safty factor q is defined as a flux function:

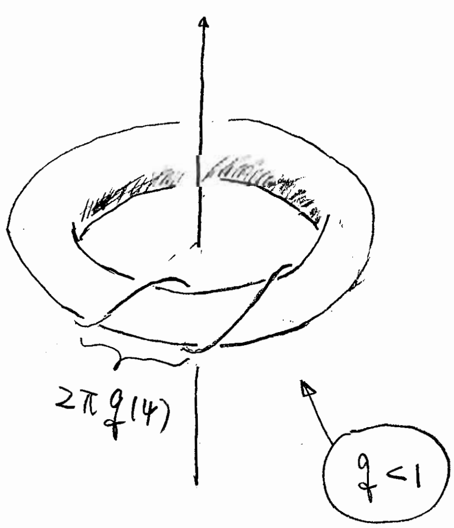
Number of toroidal transit in one poloidal transit on the flux surface

l_p: poloidal arc length on the surface

$$\frac{R\,d\phi}{B_\phi} = \frac{dl_p}{B_p}$$

$$q(\psi) = \frac{1}{2\pi}\oint \frac{B_\phi}{B_p R}\,dl_p = \frac{F(\psi)}{2\pi}\oint \frac{dl_p}{\nabla\psi\, R} \qquad (8)$$

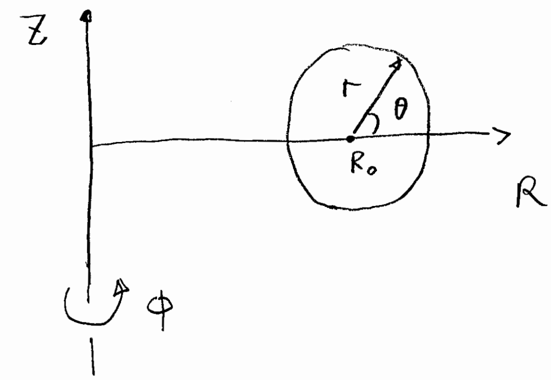




Equilibrium of low-β, large Aspect ration Tokamak; solution of Grad-Shafranov Eq.

First: cylindrical coord $\rightarrow$ toroidal coord.

$$\begin{cases} R = R_0 + r\cos\theta \\ Z = r\sin\theta \\ \phi = -\bar{\phi} \end{cases}$$



$$\begin{cases} \dfrac{\partial}{\partial r} = \cos\theta \dfrac{\partial}{\partial R} + \sin\theta \dfrac{\partial}{\partial Z} & (9) \\ \dfrac{\partial}{\partial \theta} = -r\sin\theta \dfrac{\partial}{\partial R} + r\cos\theta \dfrac{\partial}{\partial Z} & (10) \end{cases}$$

(9) $\sin\theta$ + (10) $\dfrac{\cos\theta}{r}$ $\Longrightarrow$ $\dfrac{\partial}{\partial Z} = \sin\theta \dfrac{\partial}{\partial r} + \dfrac{\cos\theta}{r}\dfrac{\partial}{\partial \theta}$ (11)

(9) $\cos\theta$ − (10) $\dfrac{\sin\theta}{r}$ $\Longrightarrow$ $\dfrac{\partial}{\partial R} = \cos\theta \dfrac{\partial}{\partial r} - \dfrac{\sin\theta}{r}\dfrac{\partial}{\partial \theta}$ (12)

$$\frac{\partial^2}{\partial z^2} = \left[\sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right]\left[\sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right]$$

$$= \sin^2\theta\frac{\partial^2}{\partial r^2} - \frac{\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta} + \frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta}$$

$$\frac{\cos^2\theta}{r}\frac{\partial}{\partial r} + \frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta}$$

$$\frac{\cos^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\sin\theta\cos\theta}{r^2}\frac{\partial}{\partial\theta}$$

$$\frac{\partial^2}{\partial R^2} = \left[\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right]\left[\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right]$$

$$= \cos^2\theta\frac{\partial^2}{\partial r^2} + \frac{\sin\theta\cos\theta}{r^2}\frac{\partial}{\partial\theta} - \frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta}$$

$$\frac{\sin^2\theta}{r}\frac{\partial}{\partial r} - \frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta}$$

$$+\frac{\sin^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\sin\theta\cos\theta}{r^2}\frac{\partial}{\partial\theta}$$

$$\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial R^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

$$\Delta^* \psi = \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2}$$

$$= \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} - \frac{1}{R}\left(\cos\theta \frac{\partial \psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial \psi}{\partial \theta}\right) \quad (13)$$

ordering:

(1) $\frac{r}{R} \sim \epsilon \ll 1$ Large aspect ratio

(2) $\frac{B_p}{B_\phi} \sim \epsilon$, $q \sim 1$ safe

(3) $\beta \sim \frac{P}{B_\phi^2} \sim \epsilon^2$ low pressure

solution:

$$\psi \simeq \psi_0(r) + \psi_1(r, \theta)$$

$$P'(\psi) = P_2'(\psi) \simeq P_2'(\psi_0) + P_2''(\psi_0)\, \psi_1$$

$$F(\psi) = F_0 + F_2(\psi) \simeq \underbrace{F_0}_{= B_0 R_0} + \underbrace{F_2(\psi_0)}_{= R_0 B_{\phi 2}(\psi_0)} + F_2'(\psi_0)\, \psi_1$$

$$F'(\psi) = F_2'(\psi) = R_0 B_{\phi 2}'(\psi_0) + R_0 B_{\phi 2}''(\psi_0)\, \psi_1$$

$\boxed{\text{order } O(\epsilon^0)}$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi_0}{\partial r} = -4\pi R_0^2 P_2'(\psi_0) - B_0 R_0^2 B_{\phi 2}'(\psi_0) \qquad (14)$$

$$B_{\theta 1} = \frac{1}{R_0}\frac{d\psi_0}{dr}$$

$$\Downarrow$$

$$\frac{d}{dr}\left(P_2 + \frac{B_0 B_{\phi 2}}{4\pi}\right) + \frac{B_{\theta 1}}{4\pi r}\frac{d}{dr}(r B_{\theta 1}) = 0$$

screw pinch

$$\frac{d(B_0 B_{\phi 2})}{dr} = \frac{\partial}{\partial r}\left(\frac{B_\phi^2}{2}\right) + O(\epsilon^2)$$

$\boxed{\text{order } O(\epsilon)}$:

$$\frac{\partial^2 \psi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \psi_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi_1}{\partial \theta^2} - \frac{1}{R_0}\left(\cos\theta \frac{\partial \psi_0}{\partial r}\right)$$

$$= -4\pi R_0^2 P_2''(\psi_0)\psi_1 - 8\pi R_0 r \cos\theta\, P_2'(\psi_0)$$

$$- R_0^2 B_0 B_{\phi 2}''(\psi_0)\psi_1 \qquad (15)$$

Let $\psi_1(r,\theta) = \bar{\psi}_1(r)\cos\theta$,

$\cos\theta$ factors out in Eq.(15)

$$\Rightarrow \quad \frac{d^2\bar{\psi}_1}{dr^2} + \frac{1}{r}\frac{d\bar{\psi}_1}{dr} - \frac{1}{r^2}\bar{\psi}_1 - \frac{1}{R_0}\frac{d\psi_0}{dr}$$

$$= -4\pi R_0^2 P_2''(\psi_0)\bar{\psi}_1 - 8\pi R_0 r P_2'(\psi_0) - R_0^2 B_0 B_{\phi 2}''(\psi_0)\bar{\psi}_1 \tag{16}$$

Eq.(14) $\Rightarrow$

$$-4\pi R_0^2 P_2''(\psi_0) - B_0 R_0^2 B_{\phi 2}''(\psi_0) = \frac{1}{\psi_0'}\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr} r\frac{d\psi_0}{dr}\right)$$

$$B_{\theta 1} = \frac{1}{R_0}\frac{d\psi_0}{dr}$$

$$\Rightarrow \quad \frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{\psi}_1}{dr}\right) - \frac{\bar{\psi}_1}{r^2} - \frac{\bar{\psi}_1}{B_{\theta 1}}\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rB_{\theta 1})\right)$$

$$= -8\pi R_0 r P_2'(\psi_0) + B_{\theta 1}$$

$\times\ r B_{\theta 1}$

$$\Rightarrow \quad B_{\theta 1}\frac{d}{dr}(r\bar{\psi}_1') - \frac{\bar{\psi}_1 B_{\theta 1}}{r} - \bar{\psi}_1 r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rB_{\theta 1})\right]$$

$$= -8\pi R_0 r^2 B_{\theta 1} P_2'(\psi_0) + r B_{\theta 1}^2 \tag{17}$$

$$\text{LHS} = B_{\theta 1}\bar{\psi}_1' + B_{\theta 1} r \bar{\psi}_1'' - \frac{\bar{\psi}_1 B_{\theta 1}}{r}$$

$$- \bar{\psi}_1 r \left[\frac{B_{\theta 1}'}{r} - \frac{B_{\theta 1}}{r^2} + B_{\theta 1}'' \right]$$

$$= \left[B_{\theta 1}\bar{\psi}_1' - \bar{\psi}_1 B_{\theta 1}' \right] + \left[B_{\theta 1}\bar{\psi}_1'' - \bar{\psi}_1 B_{\theta 1}'' \right] r$$

$$= \frac{d}{dr}\left[r \left(B_{\theta 1}\bar{\psi}_1' - \bar{\psi}_1 B_{\theta 1}' \right) \right]$$

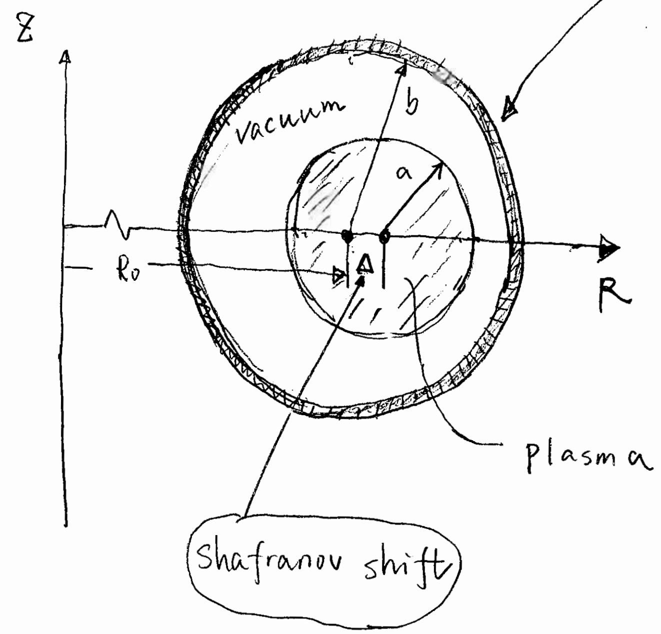
$$= \frac{d}{dr}\left[r B_{\theta 1}^2 \frac{d}{dr}\left(\frac{\bar{\psi}_1}{B_{\theta 1}} \right) \right]$$

<u>Finally $O(\epsilon)$ eq is:</u>

$$\boxed{\frac{d}{dr}\left[r B_{\theta 1}^2 \frac{d}{dr}\left(\frac{\bar{\psi}_1}{B_{\theta 1}} \right) \right] = r B_{\theta 1}^2 - 8\pi r^2 \frac{d P_2(\psi_0(r))}{dr}} \quad (18)$$

Need B.C. to solve for $\bar{\psi}_1$

B.C 1 Perfectly conducting wall at $r = b$



$$
\begin{cases}
\psi(b,\theta) = \psi_w \\
\psi_0(b) = \psi_w \\
\psi_1(b,\theta) = 0
\end{cases}
$$

$$
\boxed{\bar{\psi}_1 = B_{\theta 1} \int_r^b \frac{dx}{x^2 B_{\theta 1}(x)} \int_0^x \left[8\pi y^2 \frac{dP_2(\psi_0(y))}{dy} - y B_{\theta 1}^2(y) \right] dy} \quad (19)
$$

$$\psi(r,\theta) = \psi_0(r) + \bar{\psi}_1(r) \cos\theta$$

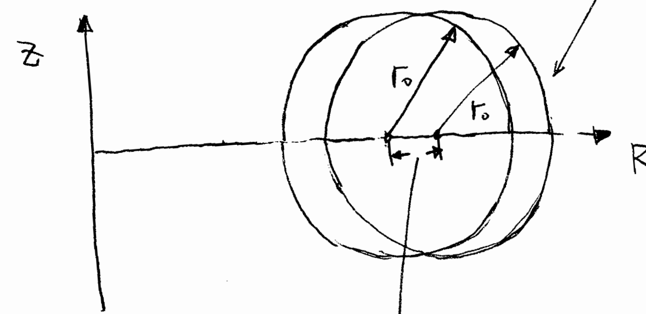
Flux surface $\psi(r,\theta) = c \Rightarrow r(\theta) = r_0 + r_1(\theta)$

$$O(\epsilon^0): \quad \psi_0(r_0) = c$$

$$O(\epsilon^1): \quad \psi_0'(r_0)\, r_1(\theta) + \bar{\psi}_1(r_0)\cos\theta = 0$$

$$\Rightarrow \quad r_1(\theta) = -\frac{\bar{\psi}_1(r_0)}{\psi_0'(r_0)}\cos\theta$$

$$\boxed{\text{shifted circle}} \longrightarrow \quad r = r_0 - \frac{\bar{\psi}_1(r_0)}{\psi_0'(r_0)}\cos\theta$$



$$-\frac{\bar{\psi}_1(r_0)}{\psi_0'(r_0)}$$

$\boxed{\text{Shafranov shift}}$: the shift of the plasma-vacuum boundary

$$r_a = a + \Delta\cos\theta$$

$$\Delta = -\frac{\bar{\psi}_1(a)}{\psi_0'(a)}$$

a Number related to β_p

a number related to l_i

$$\bar{\psi}_1(a) = B_{\theta 1}(a)\int_a^b \frac{dx}{x B_{\theta 1}(x)}\left\{\int_0^a \left[8\pi y^2 \frac{dP_2}{dy} - y B_{\theta 1}^2(y)\right]dy + \int_a^x \left[-y B_{\theta 1}^2(y)\right]dy\right\}$$

$$B_{\theta 1}(r) = B_{\theta 1}(a)\, a/r, \quad r \geq a$$

Over all:

$$\frac{\Delta}{b} = \frac{b}{2R_0}\left[\left(\beta_p + \frac{l_i - 1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right]$$

$$\beta_p \equiv \frac{\int_0^a 2\pi r \, p \, dr}{\dfrac{B_{\theta 1}^2(a)}{8\pi}\,\pi a^2}$$: poloidal beta

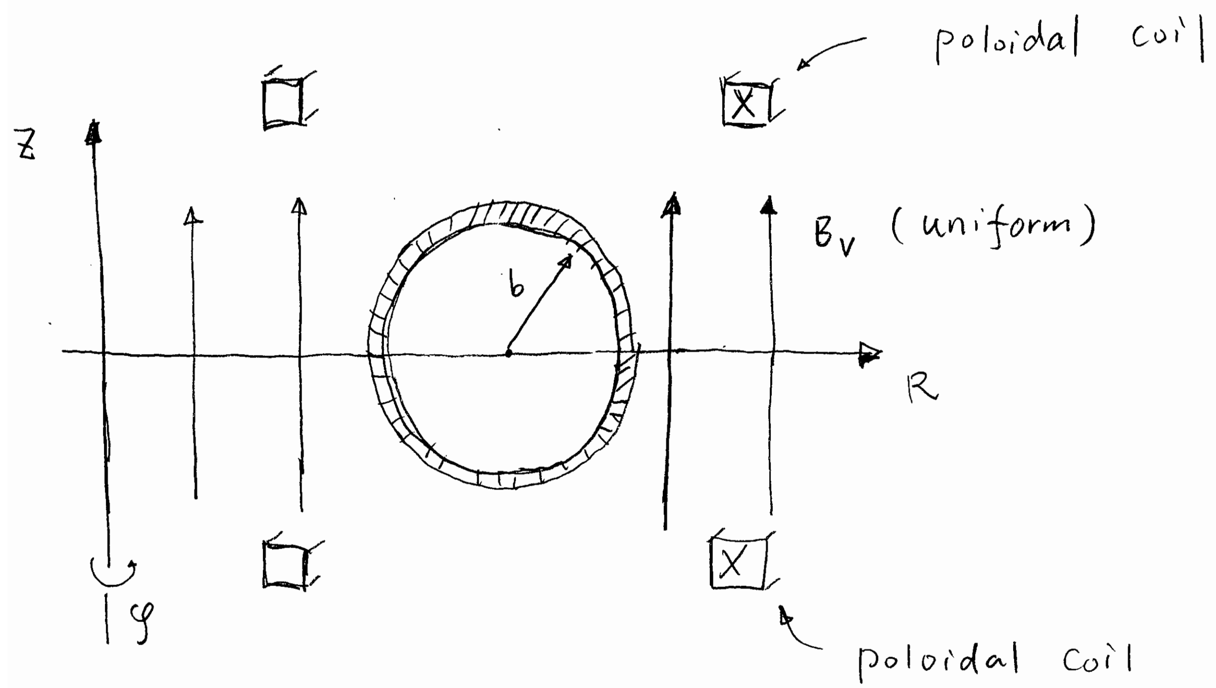
$$l_i \equiv \frac{1}{\pi a^2 B_\theta^2(a)} \int 2\pi r \, B_\theta^2 \, dr$$: internal inductance

$\boxed{\text{B.C. 2}}$: perfectly conducting wall at $r = b$,

$+$

external vacuum field B_V created by

poloidal coils.



Assume: ① The field inside the chamber does not leak out, because it's fast MHD activity

② The vacuum field B_V penetrates in because it has been set up long enough.

$$\psi_v = R_0 B_v r \cos\theta \qquad (\text{why?})$$

B.C. $\psi(b,\theta) = \psi_w + R_0 B_v \cos\theta$

$\Rightarrow$ $\psi_0(b) = \psi_w$

$\psi_1(b,\theta) = R_0 B_v b \cos\theta$

$\Rightarrow$ $\bar{\psi}_1(b) = R_0 B_v b$

$$\boxed{\bar{\psi}_1 = \text{RHS of Eq. (19)} + R_0 B_v b \frac{B_{\theta 1}(r)}{B_{\theta 1}(b)}}$$

$$\Delta \equiv -\frac{\bar{\psi}_1(a)}{\psi_0'(a)}$$

$$\Delta \longrightarrow \Delta + \Delta_v,$$

$$\Delta_v = -\frac{R_0 B_v b}{\psi_0'(a)} \frac{B_{\theta 1}(a)}{B_{\theta 1}(b)} = -\frac{B_v b}{B_{\theta 1}(b)}$$

The vacuum field pushs plasma back!