AST 552 – General Plasma Physics II, Part 1 Magnetohydrodynamics

Homework Set 5, March 13, 2006

10. Facts related to the self-adjointness of $\boldsymbol{F}(\xi)$

Use the self-adjointness of the ideal MHD linear force operator $\boldsymbol{F}(\xi)$ to prove:

a) The eigenfrequency for linear eigenmode in ideal MHD is either imaginary (no real part) or real (no imaginary part).

b) There exists a real displacement ξ such that $\delta w(\xi,\xi) < 0$ if and only if there exists a complex displacement η such that $\delta w(\eta^*,\eta) < 0$.

11. Rayleigh-Taylor instability of neutral fluid

Consider a neutral fluid in a gravitational field. The equilibrium of the fluid is given by

$$\frac{\partial p}{\partial z} = -\rho \boldsymbol{g}\,.$$

a) Assume $\xi = 0$ on the fluid boundary ∂V. Show that the energy $\delta w(\xi,\xi)$ in the energy principle can be expressed as

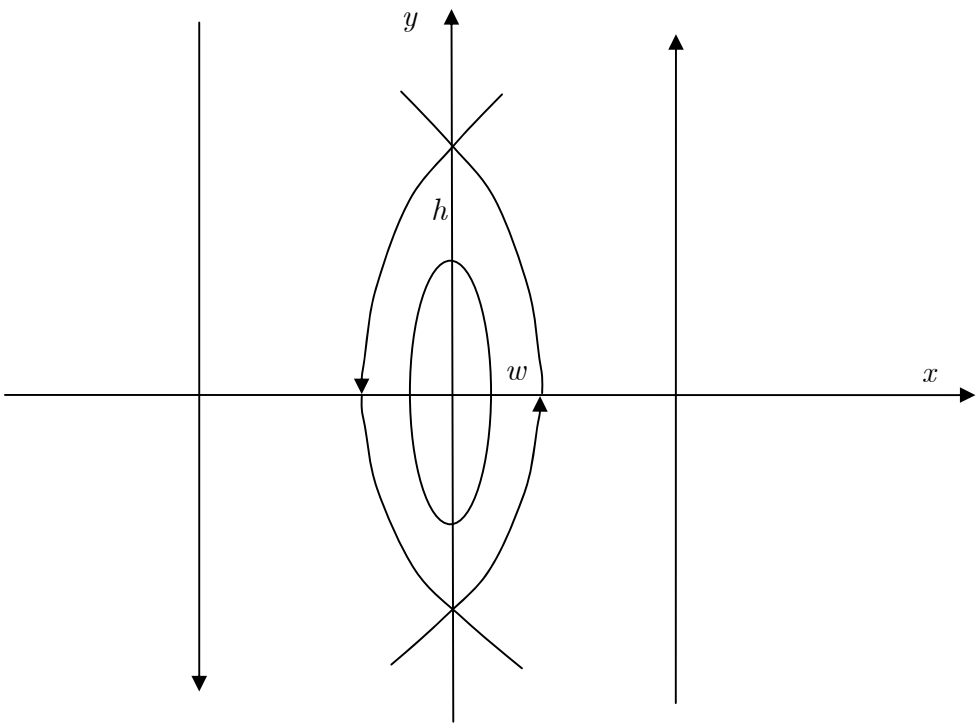
$$\delta w(\xi,\xi) = \frac{1}{2}\int_V dx^3 \left[\gamma p\,(\nabla\cdot\xi)^2 + (\xi\cdot\nabla p)\,\nabla\cdot\xi + \nabla\cdot(\rho\xi)\,\xi\cdot\boldsymbol{g}\right].$$

b) Show that a necessary and sufficient condition for stability is that

$$\frac{d}{dz}\left(\frac{p}{\rho^\gamma}\right) > 0 \quad \text{everywhere.}$$

12. Magnetic island of a tearing mode

Consider an unstable 2D tearing mode with a single wave number k in the parallel direction and real perturbed field

$$B_{1x} = B_{1x}(x)e^{\gamma t}\sin(ky).$$

Show that the perturbed field lines form an island structure around the resistive layer at $x = 0$. What are the height h and width w of the island (in terms of k, γ, B_{1x}, and B_{0y})? In which direction does the island grow?