AST 552 – General Plasma Physics II, Part 1 Magnetohydrodynamics

Homework Set 3, Feb. 20, 2006

5. Bennet pinch condition

Show that for a z-pinch the relation

$$\int_0^R p(r) 2\pi r dr = \frac{I^2}{2c^2}$$

holds for arbitrary $p(r)$ with $p(R) = 0$ on the bounding wall at $r = R$. I is the total axial current inside the bounding wall.

6. Symmetry for MHD equilibrium with flow

Consider an MHD equilibrium with non-collinear $\boldsymbol{B}$ and $\boldsymbol{v}$ everywhere, i.e. $\boldsymbol{v} \times \boldsymbol{B} \neq 0$.

(i) Show that there exist flux surfaces $\psi(\boldsymbol{x}) = const.$, such that

$$\boldsymbol{B} \cdot \nabla\psi = \boldsymbol{v} \cdot \nabla\psi = 0.$$

(ii) Suppose $p(\psi)$ and $\rho(\psi)$ are functions of flux only. Show that a family of new equilibrium can be generated by the following transformation

$$\rho_1 = a^2 \rho,$$

$$p_1 = a^{2\gamma} p,$$

$$\boldsymbol{v}_1 = a^{\gamma-1} \boldsymbol{v},$$

$$\boldsymbol{B}_1 = a^{\gamma} \boldsymbol{B},$$

where $a(\psi)$ is an arbitrary function of the flux ψ.