(8)

MHD Equilibrium 1D

① Equilibrium: $\frac{\partial}{\partial t} = 0$.

② Mostly without flow, $V = 0$

$$\Bigg\} \Rightarrow \quad -\nabla p + \frac{(\nabla \times B) \times B}{4\pi} = 0$$

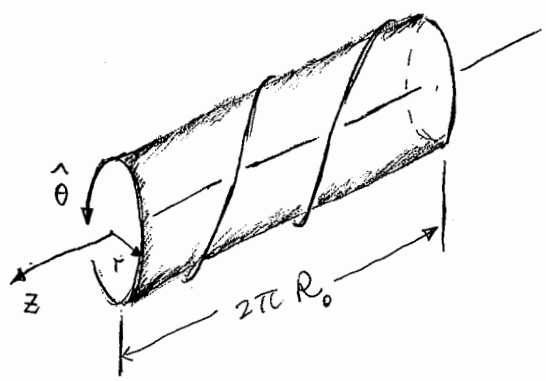
$$\nabla \cdot B = 0$$

4 Eqs. for 4 Components

Screw-pinch

$$B = B_\theta(r)\,\hat{\theta} + B_z(r)\,\hat{z}$$

$$p = p(r)$$



$$j = \frac{c}{4\pi} \nabla \times B$$

$$= J_\theta\,\hat{\theta} + J_z\,\hat{z}$$

$$J_\theta = -\frac{c}{4\pi} \frac{d}{dr} B_z(r)$$

$$J_z = \frac{c}{4\pi} \frac{1}{r} \frac{d}{dr}\left[r B_\theta(r)\right]$$

momentum Eq: $\frac{\vec{J}\times B}{c} = \nabla P$ or $\frac{(\nabla\times B)\times B}{4\pi} = \nabla P$

$\Rightarrow$

$$\frac{d}{dr}\left(P + \frac{B_\theta^2 + B_z^2}{8\pi}\right) + \frac{B_\theta^2}{4\pi r} = 0$$

$$\boxed{\vec{\kappa} = -\frac{B_\theta^2}{B^2} r\,\hat{e}_r}$$

Curvature bending force

$$\boxed{\text{Two freedoms for } P(r),\ B_\theta(r),\ B_z(r)}$$

<u>Question</u>: MHD Eqs have 8 Eqs for 8 components, why there are two freedoms for the screw pinch equilibrium?

<u>safety Factor</u>: $q(r) \equiv \frac{r B_z(r)}{R_0 B_\theta(r)}$

How many turns in $\hat{z}$ does the field line travel for one turn in $\hat{\theta}$?

(one turn in $\hat{z} \equiv 2\pi R_0$ in $\hat{z}$)

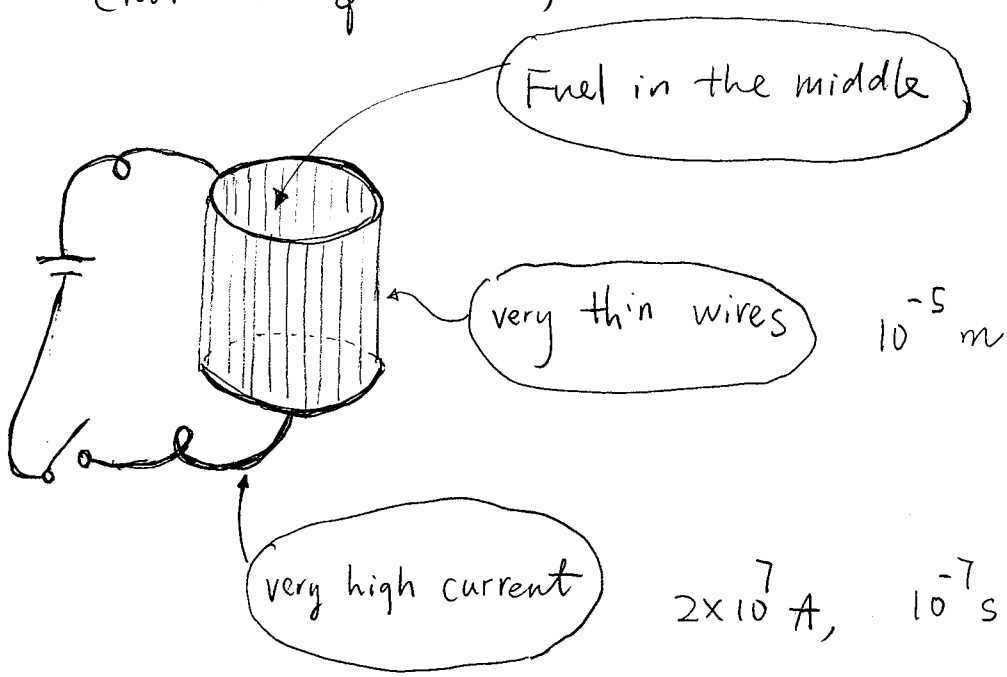
$$\frac{r\,d\theta}{dz} = \frac{B_\theta}{B_z} \Rightarrow \frac{dz}{2\pi R_0} = \frac{r B_z}{R_0 B_\theta}\frac{d\theta}{2\pi} \Rightarrow q = \frac{r}{R_0}\frac{B_z(r)}{B_\theta(r)}.$$

$\boxed{\text{Z-Pinch}}$: $J_\theta = 0$, $B_z = 0$

$q = 0$ unsafe (unstable)

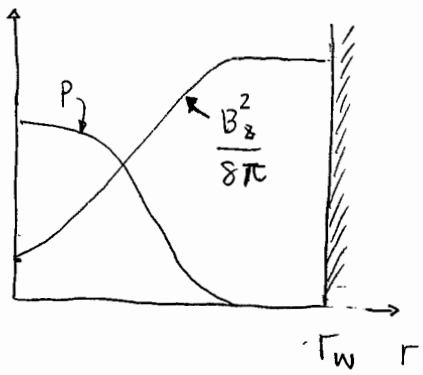
Z-pinch for inertial confinement

(NOT an equilibrium)



$\boxed{\theta\text{- Pinch}}$: $J_z = 0, \quad B_\theta = 0$

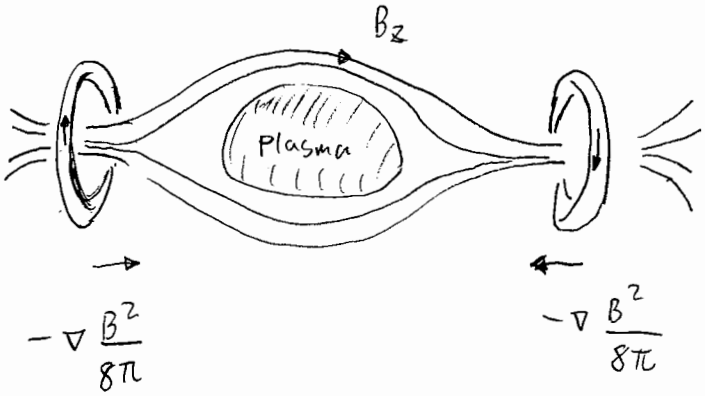
$$p + \frac{B_z^2}{8\pi} = \frac{B_0^2}{8\pi}$$



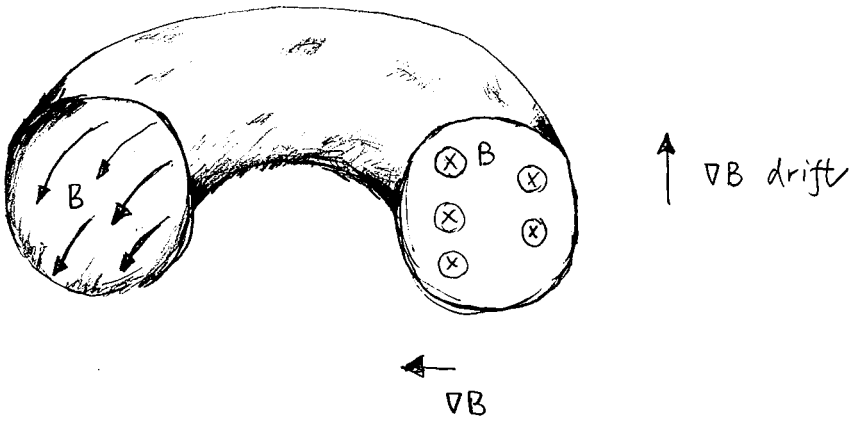
$q = \infty$, very safe (stable)

But, No axial confinement,

solution ①: make a mirror mechine. $\Rightarrow$ 2D Equilibrium



solution ②. Make a toroidal magnetic field



Need poloidal field to confine particle radially.

⇒ toroidal screw-pinch, (2D)