GPPI Homework Set 2

Due on November 26, 2007

Problem 1. On-dimensional electrostatic wave for water-bag distribution.

Consider the homogenous one-dimensional water-bag distribution function,

$$g(v_x) = \begin{cases} A, & |v_x| < v_m, \\ 0, & |v_x| \geq v_m. \end{cases}$$

a) Calculate the value of the constant $A$ in terms of the plasma density $n_0$.

b) Use the Vlasov-Poisson equations to derive an integral expression for electrostatic electron plasma waves.

c) Evaluate the integral and obtain a dispersion relation $\omega(k)$, keeping terms to third order in the small quantity of $kv_m / \omega$. (Do not need to consider the Landau contours).

Problem 2. Ion plasma wave.

From the dispersion relation for an ion-electron plasma system

$$1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{3\omega_{pi}^2 v_{ti}^2 k^2}{\omega^4} + \frac{\omega_{pe}^2}{k^2 v_{te}^2} = 0,$$

show that when the last term $\frac{\omega_{pe}^2}{k^2 v_{te}^2}$ is not much larger than 1, the phase velocity of the eigenmode $\frac{\omega}{k}$ depends on $k$. The third term $\frac{3\omega_{pi}^2 v_{ti}^2 k^2}{\omega^4}$ can be treated as a small term, i.e., $\frac{3\omega_{pi}^2 v_{ti}^2 k^2}{\omega^4} << \frac{\omega_{pi}^2}{\omega^2}$. Such eigenmodes are called ion plasma waves.

Problem 3. Two-stream instability.

The two-stream instability can be described by a generic dispersion relation of the form

$$D(\omega, k) = 1 - \frac{\omega_{p1}^2}{\omega^2} - \frac{\omega_{p2}^2}{(\omega - kV)^2} = 0.$$

a) Assume $kV > 0$. Show that $D(\omega, k)$ has a single minimum in the interval $0 < \omega < kV$ at

$$\omega_0 = kV \frac{\omega_{p1}^{2/3}}{\omega_{p1}^{2/3} + \omega_{p2}^{2/3}}.$$

b) Show that the necessary and sufficient condition for instability is given by $D(\omega_0) > 1$, or equivalently

$$0 < k^2 V^2 < \left(\omega_{p1}^{2/3} + \omega_{p2}^{2/3}\right)^3.$$