GPPI Homework Set 1

Due on November 12, 2007

Problem 1. Conservative form of the kinetic equation

From the Klimontovich equation for $N(\mathbf{x},\mathbf{v},t) = \sum \delta(\mathbf{x}-\mathbf{X}_i(t))\delta(\mathbf{v}-\mathbf{V}_i(t))$,

$$\frac{\partial N}{\partial t} + \mathbf{v}\cdot\frac{\partial N}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E}^m + \frac{1}{c}\mathbf{v}\times\mathbf{B}^m\right)\cdot\frac{\partial N}{\partial \mathbf{v}} = 0,$$

derive the Klimontovich equation in its conservative form

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial \mathbf{x}}\cdot(\mathbf{v}N) + \frac{q}{m}\frac{\partial}{\partial \mathbf{v}}\cdot\left[\left(\mathbf{E}^m + \frac{1}{c}\mathbf{v}\times\mathbf{B}^m\right)N\right] = 0.$$

Problem 2. Energy conservation of the Vlasov-Maxwell equations

Show that the Vlasov-Maxwell equations conserve the total energy, i.e.,

$$\frac{d\mathcal{E}}{dt} = 0,$$

where $\mathcal{E} \equiv \int d^3x \left[\frac{E^2 + B^2}{8\pi} + \int d^3v\, \frac{m}{2} v^2 f\right]$ is the total energy.

Problem 3. A single species plasma in an external potential

Consider a single species, non-neutral plasma of ions in equilibrium with a given external potential $u(x)$. The plasma and the external potential are homogeneous in the y and z directions. The plasma density $n(x)$, electrostatic potential $\phi(x)$ generated by the plasma, and the external potential $u(x)$ are functions of x only.

1) What are the constants of motion for a single ion particle?

2) Suppose that the plasma has a constant temperature T, and the distribution function is a thermal equilibrium distribution function. Express the $n(x)$ in terms of $\phi(x)$ and $u(x)$, and derive a ordinary differential equation for $\phi(x)$.